

Exercise 5.3

Page No: 5.17

Question 1: Factorize $64a^3 + 125b^3 + 240a^2b + 300ab^2$

Solution:

$$64a^3 + 125b^3 + 240a^2b + 300ab^2$$

$$= (4a)^3 + (5b)^3 + 3(4a)^2(5b) + 3(4a)(5b)^2, \text{ which is similar to } a^3 + b^3 + 3a^2b + 3ab^2$$

We know that, $a^3 + b^3 + 3a^2b + 3ab^2 = (a+b)^3$

$$= (4a+5b)^3$$

Question 2: Factorize $125x^3 - 27y^3 - 225x^2y + 135xy^2$

Solution:

$$125x^3 - 27y^3 - 225x^2y + 135xy^2$$

Above expression can be written as $(5x)^3 - (3y)^3 - 3(5x)^2(3y) + 3(5x)(3y)^2$

Using: $a^3 - b^3 - 3a^2b + 3ab^2 = (a-b)^3$

$$= (5x - 3y)^3$$

Question 3: Factorize $\frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x$

Solution:

$$\frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x$$

$$= \left(\frac{2}{3}x\right)^3 + 1^3 + 3 \times \left(\frac{2}{3}x\right)^2 \times 1 + 3(1)^2 \times \left(\frac{2}{3}x\right)$$

$$\left[\because x^3 + b^3 + 3x^2b + 3xb^2 = (x + b)^3 \right]$$

$$\therefore \frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x = \left(\frac{2}{3}x + 1\right)^3$$

Question 4: Factorize $8x^3 + 27y^3 + 36x^2y + 54xy^2$

Solution:

$$8x^3 + 27y^3 + 36x^2y + 54xy^2$$

Above expression can be written as $(2x)^3 + (3y)^3 + 3 \times (2x)^2 \times 3y + 3 \times (2x)(3y)^2$

Which is similar to $a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3$

Here $a = 2x$ and $b = 3y$

$$= (2x+3y)^3$$

Therefore, $8x^3 + 27y^3 + 36x^2y + 54xy^2 = (2x+3y)^3$

Question 5: Factorize $a^3 - 3a^2b + 3ab^2 - b^3 + 8$

Solution:

$$a^3 - 3a^2b + 3ab^2 - b^3 + 8$$

Using: $a^3 - b^3 - 3a^2b + 3ab^2 = (a-b)^3$

$$= (a-b)^3 + 2^3$$

Again, Using: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$= (a-b+2)((a-b)^2 - (a-b) \times 2 + 2^2)$$

$$= (a-b+2)(a^2+b^2-2ab-2(a-b)+4)$$

$$= (a-b+2)(a^2+b^2-2ab-2a+2b+4)$$

$$a^3 - 3a^2b + 3ab^2 - b^3 + 8 = (a-b+2)(a^2+b^2-2ab-2a+2b+4)$$