

Exercise 5.4

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Factorize each of the following expressions:

Question 1: $a^3 + 8b^3 + 64c^3 - 24abc$

Solution:

$$a^3 + 8b^3 + 64c^3 - 24abc$$

$$= (a)^3 + (2b)^3 + (4c)^3 - 3 \times a \times 2b \times 4c$$

$$[\text{Using } a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)]$$

$$= (a+2b+4c)(a^2 + (2b)^2 + (4c)^2 - a \times 2b - 2b \times 4c - 4c \times a)$$

$$= (a+2b+4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ac)$$

$$\text{Therefore, } a^3 + 8b^3 + 64c^3 - 24abc = (a+2b+4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ac)$$

Question 2: $x^3 - 8y^3 + 27z^3 + 18xyz$

Solution:

$$= x^3 - (2y)^3 + (3z)^3 - 3 \times x \times (-2y) \times (3z)$$

$$= (x + (-2y) + 3z)(x^2 + (-2y)^2 + (3z)^2 - x(-2y) - (-2y)(3z) - 3z(x))$$

$$[\text{using } a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)]$$

$$= (x - 2y + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$$

Question 3: $27x^3 - y^3 - z^3 - 9xyz$

Solution:

$$27x^3 - y^3 - z^3 - 9xyz$$

$$= (3x)^3 - y^3 - z^3 - 3(3xyz)$$

$$[\text{Using } a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$$

Here $a = 3x$, $b = -y$ and $c = -z$

$$= (3x - y - z)\{(3x)^2 + (-y)^2 + (-z)^2 + 3xy - yz + 3xz\}$$

$$= (3x - y - z)\{9x^2 + y^2 + z^2 + 3xy - yz + 3xz\}$$

Question 4: $\frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz$

Solution:

$$\begin{aligned} & \frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz \\ &= \left(\frac{x}{3}\right)^3 + (-y)^3 + (5z)^3 - 3 \times \frac{x}{3} \times (-y) \times (5z) \end{aligned}$$

$$[\text{Using } a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$$

$$= \left(\frac{x}{3} + (-y) + 5z\right) \left(\left(\frac{x}{3}\right)^2 + (-y)^2 + (5z)^2 - \frac{x}{3}(-y) - (-y)5z - 5z\left(\frac{x}{3}\right)\right)$$

$$= \left(\frac{x}{3} - y + 5z\right) \left(\frac{x^2}{9} + y^2 + 25z^2 + \frac{xy}{3} + 5yz - 5zx\right)$$

Question 5: $8x^3 + 27y^3 - 216z^3 + 108xyz$

Solution:

$$8x^3 + 27y^3 - 216z^3 + 108xyz$$

$$= (2x)^3 + (3y)^3 + (-6z)^3 - 3(2x)(3y)(-6z)$$

$$= (2x + 3y + (-6z)) \{ (2x)^2 + (3y)^2 + (-6z)^2 - 2x \times 3y - 3y(-6z) - (-6z)2x \}$$

$$= (2x + 3y - 6z) \{ 4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx \}$$

Question 6: $125 + 8x^3 - 27y^3 + 90xy$

Solution:

$$\begin{aligned} & 125 + 8x^3 - 27y^3 + 90xy \\ &= (5)^3 + (2x)^3 + (-3y)^3 - 3 \times 5 \times 2x \times (-3y) \end{aligned}$$

$$= (5 + 2x + (-3y)) (5^2 + (2x)^2 + (-3y)^2 - 5(2x) - 2x(-3y) - (-3y)5)$$

$$= (5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$$

Question 7: $(3x-2y)^3 + (2y-4z)^3 + (4z-3x)^3$

Solution:

$$(3x-2y)^3 + (2y-4z)^3 + (4z-3x)^3$$

$$\text{Let } (3x-2y) = a, (2y-4z) = b, (4z-3x) = c$$

$$a + b + c = 3x - 2y + 2y - 4z + 4z - 3x = 0$$

$$\text{We know, } a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\Rightarrow a^3 + b^3 + c^3 - 3abc = 0$$

$$\text{or } a^3 + b^3 + c^3 = 3abc$$

$$\Rightarrow (3x-2y)^3 + (2y-4z)^3 + (4z-3x)^3 = 3(3x-2y)(2y-4z)(4z-3x)$$

Question 8: $(2x-3y)^3 + (4z-2x)^3 + (3y-4z)^3$

Solution:

$$(2x-3y)^3 + (4z-2x)^3 + (3y-4z)^3$$

$$\text{Let } 2x - 3y = a, 4z - 2x = b, 3y - 4z = c$$

$$a + b + c = 2x - 3y + 4z - 2x + 3y - 4z = 0$$

$$\begin{aligned} \text{We know, } a^3 + b^3 + c^3 - 3abc &= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \\ \Rightarrow a^3 + b^3 + c^3 - 3abc &= 0 \end{aligned}$$

$$(2x-3y)^3 + (4z-2x)^3 + (3y-4z)^3 = 3(2x-3y)(4z-2x)(3y-4z)$$