Exercise 6.4 Page No: 6.24

In each of the following, use factor theorem to find whether polynomial g(x) is a factor of polynomial f(x) or, not: (1-7)

Question 1: $f(x) = x^3 – 6x^2 + 11x – 6; g(x) = x – 3$
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = x -3 = 0

or x = 3
Remainder = f(3)

Now,
$f(3) = (3)^3 – 6(3)^2 +11 \times 3 – 6$
= 27 - 54 + 33 - 6
= 60 – 60
= 0
Therefore, g(x) is a factor of f(x)

Question 2: $f(x) = 3X^4 + 17x^3 + 9x^2 – 7x – 10; g(x) = x + 5$
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = x + 5 = 0, then x = -5

Remainder = f(-5)

Now,
$f(3) = 3(-5)^4 + 17(-5)^3 + 9(-5)^2 – 7(-5) – 10$
= 3 x 625 + 17 x (-125) + 9 x (25) – 7 x (-5) – 10
= 1875 -2125 + 225 + 35 – 10
= 0
Therefore, g(x) is a factor of f(x).

Question 3: $f(x) = x^5 + 3x^4 – x^3 – 3x^2 + 5x + 15, g(x) = x + 3$
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = x + 3 = 0, then x = -3

Remainder = f(-3)

Now,
$f(-3) = (-3)^5 + 3(-3)^4 - (-3)^3 - 3(-3)^2 + 5(-3) + 15$
$= -243 + 3 \times 81 - (-27) - 3 \times 9 + 5(-3) + 15$
$= -243 + 243 + 27 - 27 - 15 + 15$
$= 0$

Therefore, g(x) is a factor of f(x).

Question 4: $f(x) = x^3 - 6x^2 - 19x + 84$, $g(x) = x - 7$
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.
$g(x) = x - 7 = 0$, then $x = 7$
Remainder = f(7)
Now,
$f(7) = (7)^3 - 6(7)^2 - 19 \times 7 + 84$
$= 343 - 294 - 133 + 84$
$= 343 + 84 - 294 - 133$
$= 0$
Therefore, g(x) is a factor of f(x).

Question 5: $f(x) = 3x^3 + x^2 - 20x + 12$, $g(x) = 3x - 2$
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.
$g(x) = 3x - 2 = 0$, then $x = 2/3$
Remainder = f(2/3)
Now,
$f(2/3) = 3(2/3)^3 + (2/3)^2 - 20(2/3) + 12$
$= 3 \times 8/27 + 4/9 - 40/3 + 12$
$= 8/9 + 4/9 - 40/3 + 12$
$= 0/9$
$= 0$
Therefore, g(x) is a factor of f(x).

Question 6: $f(x) = 2x^3 - 9x^2 + x + 12$, $g(x) = 3 - 2x$
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.
$g(x) = 3 - 2x = 0$, then $x = 3/2$
Remainder = f(3/2)
Now,
$f(3/2) = 2(3/2)^3 - 9(3/2)^2 + (3/2) + 12$

= 2 x 27/8 – 9 x 9/4 + 3/2 + 12

= 27/4 – 81/4 + 3/2 + 12

= 0/4
= 0

Therefore, g(x) is a factor of f(x).

Question 7: $f(x) = x^3 - 6x^2 + 11x - 6$, $g(x) = x^2 - 3x + 2$
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.
g(x) = 0
or $x^2 - 3x + 2 = 0$
$x^2 - x - 2x + 2 = 0$
x(x – 1) – 2(x – 1) = 0
(x – 1) (x – 2) = 0
Therefore x = 1 or x = 2

Now,
$f(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1-6+11-6 = 12 - 12 = 0$

$f(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 30 - 30 = 0$

=> f(1) = 0 and f(2) = 0

Which implies g(x) is factor of f(x).

Question 8: Show that (x – 2), (x + 3) and (x – 4) are factors of $x^3 - 3x^2 - 10x + 24$.
Solution:

Let $f(x) = x^3 - 3x^2 - 10x + 24$
If x – 2 = 0, then x = 2,
If x + 3 = 0 then x = -3,
and If x – 4 = 0 then x = 4

Now,
$f(2) = (2)^3 - 3(2)^2 - 10 \times 2 + 24 = 8 - 12 - 20 + 24 = 32 - 32 = 0$

$f(-3) = (-3)^3 - 3(-3)^2 - 10(-3) + 24 = -27 - 27 + 30 + 24 = -54 + 54 = 0$

$f(4) = (4)^3 - 3(4)^2 - 10 \times 4 + 24 = 64 - 48 - 40 + 24 = 88 - 88 = 0$

f(2) = 0
f(-3) = 0
f(4) = 0

Hence (x – 2), (x + 3) and (x – 4) are the factors of f(x)

Question 9: Show that (x + 4), (x – 3) and (x – 7) are factors of $x^3 - 6x^2 - 19x + 84$.
Solution:
Let $f(x) = x^3 - 6x^2 - 19x + 84$
If x + 4 = 0, then x = -4
If x – 3 = 0, then x = 3
and if x – 7 = 0, then x = 7

Now,
$f(-4) = (-4)^3 - 6(-4)^2 - 19(-4) + 84 = -64 - 96 + 76 + 84 = 160 - 160 = 0$
f(-4) = 0

$f(3) = (3)^3 - 6(3)^2 - 19 \times 3 + 84 = 27 - 54 - 57 + 84 = 111 - 111 = 0$
f(3) = 0

$f(7) = (7)^3 - 6(7)^2 - 19 \times 7 + 84 = 343 - 294 - 133 + 84 = 427 - 427 = 0$

f(7) = 0

Hence (x + 4), (x – 3), (x – 7) are the factors of f(x).