Exercise 6.5 Page No: 6.32

Using factor theorem, factorize each of the following polynomials:

Question 1: $x^3 + 6x^2 + 11x + 6$

Solution:

Let $f(x) = x^3 + 6x^2 + 11x + 6$

Step 1: Find the factors of constant term

Here constant term = 6

Factors of 6 are ±1, ±2, ±3, ±6

Step 2: Find the factors of f(x)

Let x + 1 = 0

=> x = -1

Put the value of x in f(x)

$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$

= -1 + 6 -11 + 6

= 12 – 12

= 0

So, (x + 1) is the factor of f(x)

Let x + 2 = 0

=> x = -2

Put the value of x in f(x)

$f(-2) = (-2)^3 + 6(-2)^2 + 11(-2) + 6 = -8 + 24 - 22 + 6 = 0$

So, (x + 2) is the factor of f(x)

Let x + 3 = 0

=> $x = -3$

Put the value of x in f(x)

$f(-3) = (-3)^3 + 6(-3)^2 + 11(-3) + 6 = -27 + 54 - 33 + 6 = 0$

So, (x + 3) is the factor of f(x)

Hence, $f(x) = (x + 1)(x + 2)(x + 3)$

Question 2: $x^3 + 2x^2 - x - 2$
Solution:
Let $f(x) = x^3 + 2x^2 - x - 2$
Constant term = -2

Factors of -2 are ±1, ±2

Let $x - 1 = 0$

=> $x = 1$

Put the value of x in f(x)

$f(1) = (1)^3 + 2(1)^2 - 1 - 2 = 1 + 2 - 1 - 2 = 0$

So, (x – 1) is factor of f(x)

Let $x + 1 = 0$

=> $x = -1$

Put the value of x in f(x)

$f(-1) = (-1)^3 + 2(-1)^2 - 1 - 2 = -1 + 2 + 1 - 2 = 0$

(x + 1) is a factor of f(x)

Let $x + 2 = 0$

=> $x = -2$

Put the value of x in f(x)

$f(-2) = (-2)^3 + 2(-2)^2 - (-2) - 2 = -8 + 8 + 2 - 2 = 0$

$(x + 2)$ is a factor of $f(x)$

Let $x - 2 = 0$

$\Rightarrow x = 2$

Put the value of x in $f(x)$

$f(2) = (2)^3 + 2(2)^2 - 2 - 2 = 8 + 8 - 2 - 2 = 12 \neq 0$

$(x - 2)$ is not a factor of $f(x)$

Hence $f(x) = (x + 1)(x- 1)(x+2)$

Question 3: $x^3 - 6x^2 + 3x + 10$
Solution:
Let $f(x) = x^3 - 6x^2 + 3x + 10$
Constant term = 10
Factors of 10 are ±1, ±2, ±5, ±10

Let $x + 1 = 0$ or $x = -1$
$f(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10 = 10 - 10 = 0$
$f(-1) = 0$

Let $x + 2 = 0$ or $x = -2$
$f(-2) = (-2)^3 - 6(-2)^2 + 3(-2) + 10 = -8 - 24 - 6 + 10 = -28$
$f(-2) \neq 0$

Let $x - 2 = 0$ or $x = 2$
$f(2) = (2)^3 - 6(2)^2 + 3(2) + 10 = 8 - 24 + 6 + 10 = 0$
$f(2) = 0$

Let $x - 5 = 0$ or $x = 5$
$f(5) = (5)^3 - 6(5)^2 + 3(5) + 10 = 125 - 150 + 15 + 10 = 0$
$f(5) = 0$
Therefore, $(x + 1)$, $(x - 2)$ and $(x-5)$ are factors of $f(x)$

Hence $f(x) = (x + 1) (x - 2) (x-5)$

Question 4: $x^4 – 7x^3 + 9x^2 + 7x- 10$
Solution:
Let $f(x) = x^4 – 7x^3 + 9x^2 + 7x- 10$

Constant term = -10
Factors of -10 are ±1, ±2, ±5, ±10

Let x - 1 = 0 or x = 1
$f(1) = (1)^4 – 7(1)^3 + 9(1)^2 + 7(1) – 10 = 1 – 7 + 9 + 7 -10 = 0$
f(1) = 0

Let x + 1 = 0 or x = -1
$f(-1) = (-1)^4 – 7(-1)^3 + 9(-1)^2 + 7(-1) – 10 = 1 + 7 + 9 - 7 -10 = 0$
f(-1) = 0

Let x - 2 = 0 or x = 2
$f(2) = (2)^4 – 7(2)^3 + 9(2)^2 + 7(2) – 10 = 16 – 56 + 36 + 14 – 10 = 0$
f(2) = 0

Let x - 5 = 0 or x = 5
$f(5) = (5)^4 – 7(5)^3 + 9(5)^2 + 7(5) – 10 = 625 – 875 + 225 + 35 – 10 = 0$
f(5) = 0

Therefore, (x - 1), (x + 1), (x – 2) and (x-5) are factors of f(x)

Hence f(x) = (x - 1) (x + 1) (x – 2) (x-5)

Question 5: $x^4 – 2x^3 – 7x^2 + 8x + 12$
Solution:
$f(x) = x^4 – 2x^3 – 7x^2 + 8x + 12$
Constant term = 12
Factors of 12 are ±1, ±2, ±3, ±4, ±6, ±12

Let x - 1 = 0 or x = 1
$f(1) = (1)^4 – 2(1)^3 - 7(1)^2 + 8(1) + 12 = 1 – 2 – 7 + 8 + 12 = 12$
f(1) ≠ 0

Let x + 1 = 0 or x = -1
$f(-1) = (-1)^4 – 2(-1)^3 - 7(-1)^2 + 8(-1) + 12 = 1 + 2 – 7 - 8 + 12 = 0$
f(-1) = 0

Let x +2 = 0 or x = -2
$f(-2) = (-2)^4 – 2(-2)^3 - 7(-2)^2 + 8(-2) + 12 = 16 + 16 – 28 - 16 + 12 = 0$
f(-2) = 0

Let x - 2 = 0 or x = 2
$f(2) = (2)^4 – 2(2)^3 - 7(2)^2 + 8(2) + 12 = 16 - 16 – 28 + 16 + 12 = 0$
f(2) = 0

Let x - 3 = 0 or x = 3
$f(3) = (3)^4 – 2(3)^3 - 7(3)^2 + 8(3) + 12 = 0$
f(3) = 0

Therefore, (x + 1), (x + 2), (x – 2) and (x-3) are factors of f(x)

Hence f(x) = (x + 1)(x + 2) (x – 2) (x-3)

Question 6: $x^4 + 10x^3 + 35x^2 + 50x + 24$
Solution:
Let $f(x) = x^4 + 10x^3 + 35x^2 + 50x + 24$
Constant term = 24
Factors of 24 are ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24

Let x + 1 = 0 or x = -1
$f(-1) = (-1)^4 + 10(-1)^3 + 35(-1)^2 + 50(-1) + 24 = 1 – 10 + 35 – 50 + 24 = 0$
f(1) = 0
(x + 1) is a factor of f(x)
Likewise, (x + 2),(x + 3),(x + 4) are also the factors of f(x)

Hence f(x) = (x + 1) (x + 2)(x + 3)(x + 4)

Question 7: $2x^4 – 7x^3 – 13x^2 + 63x – 45$
Solution:
Let $f(x) = 2x^4 – 7x^3 – 13x^2 + 63x – 45$
Constant term = -45
Factors of -45 are ±1, ±3, ±5, ±9, ±15, ±45
Here coefficient of x^4 is 2. So possible rational roots of f(x) are

±1, ±3, ±5, ±9, ±15, ±45, ±1/2,±3/2,±5/2,±9/2,±15/2,±45/2

Let x - 1 = 0 or x = 1
$f(1) = 2(1)^4 - 7(1)^3 - 13(1)^2 + 63(1) – 45 = 2 – 7 – 13 + 63 – 45 = 0$
f(1) = 0

f(x) can be written as,
$f(x) = (x-1)(2x^3 - 5x^2 - 18x + 45)$

or $f(x) = (x-1)g(x)$ …(1)

Let $x - 3 = 0$ or $x = 3$
$f(3) = 2(3)^4 - 7(3)^3 - 13(3)^2 + 63(3) - 45 = = 162 - 189 - 117 + 189 - 45 = 0$
$f(3) = 0$

Now, we are available with 2 factors of f(x), $(x - 1)$ and $(x - 3)$

Here $g(x) = 2x^2(x-3) + x(x-3) - 15(x-3)$
Taking $(x-3)$ as common
$= (x-3)(2x^2 + x - 15)$

$=(x-3)(2x^2+6x - 5x - 15)$

$= (x-3)(2x-5)(x+3)$

$= (x-3)(x+3)(2x-5)$ ….(2)

From (1) and (2)

$f(x) = (x-1)(x-3)(x+3)(2x-5)$