Exercise 6.1 Page No: 6.2

Question 1: Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer:
(i) $3x^2 – 4x + 15$
(ii) $y^2 + 2\sqrt{3}$
(iii) $3\sqrt{x} + \sqrt{2}x$
(iv) $x – 4/x$
(v) $x^{12} + y^3 + t^{50}$

Solution:
(i) $3x^2 – 4x + 15$
It is a polynomial of x.

(ii) $y^2 + 2\sqrt{3}$
It is a polynomial of y.

(iii) $3\sqrt{x} + \sqrt{2}x$
It is not a polynomial since the exponent of $3\sqrt{x}$ is a rational term.

(iv) $x – 4/x$
It is not a polynomial since the exponent of $– 4/x$ is not a positive term.

(v) $x^{12} + y^3 + t^{50}$

It is a three variable polynomial, x, y and t.

Question 2: Write the coefficient of x^2 in each of the following:
(i) $17 – 2x + 7x^2$
(ii) $9 – 12x + x^3$
(iii) $\prod/6\ x^2 – 3x + 4$
(iv) $\sqrt{3}x – 7$

Solution:
(i) $17 – 2x + 7x^2$
Coefficient of $x^2 = 7$

(ii) $9 – 12x + x^3$
Coefficient of $x^2 =0$

(iii) ∏/6 $x^2 – 3x + 4$
Coefficient of x^2 = ∏/6

(iv) √3x – 7
Coefficient of $x^2 = 0$

Question 3: Write the degrees of each of the following polynomials:
(i) $7x^3 + 4x^2 – 3x + 12$
(ii) $12 – x + 2x^3$
(iii) 5y – √2
(iv) 7
(v) 0

Solution:
As we know, degree is the highest power in the polynomial
(i) Degree of the polynomial $7x^3 + 4x^2 – 3x + 12$ is 3
(ii) Degree of the polynomial $12 – x + 2x^3$ is 3
(iii) Degree of the polynomial 5y – √2 is 1
(iv) Degree of the polynomial 7 is 0
(v) Degree of the polynomial 0 is undefined.

Question 4: Classify the following polynomials as linear, quadratic, cubic and biquadratic polynomials:
(i) $x + x^2 + 4$
(ii) $3x – 2$
(iii) $2x + x^2$
(iv) 3y
(v) $t^2 + 1$
(vi) $7t^4 + 4t^3 + 3t – 2$

Solution:
(i) $x + x^2 + 4$: It is a quadratic polynomial as its degree is 2.
(ii) $3x – 2$: It is a linear polynomial as its degree is 1.
(iii) $2x + x^2$: It is a quadratic polynomial as its degree is 2.
(iv) 3y: It is a linear polynomial as its degree is 1.
(v) $t^2+ 1$: It is a quadratic polynomial as its degree is 2.
(vi) $7t^4 + 4t^3 + 3t – 2$: It is a biquadratic polynomial as its degree is 4.

Exercise 6.2 Page No: 6.8

Question 1: If $f(x) = 2x^3 – 13x^2 + 17x + 12$, find
(i) f (2)
(ii) f (-3)
(iii) f(0)
Solution:
$f(x) = 2x^3 – 13x^2 + 17x + 12$

(i) $f(2) = 2(2)^3 – 13(2)^2 + 17(2) + 12$
= 2 x 8 - 13 x 4 + 17 x 2 + 12
= 16 - 52 + 34 + 12
= 62 – 52
= 10

(ii) $f(-3) = 2(-3)^3 – 13(-3)^2 + 17 \times (-3) + 12$
= 2 x (-27) – 13 x 9 + 17 x (-3) + 12
= -54 - 117 -51 + 12
= -222 + 12
= -210

(iii) $f(0) = 2 \times (0)^3 – 13(0)^2 + 17 \times 0 + 12$
= 0-0 + 0+ 12
= 12

Question 2: Verify whether the indicated numbers are zeros of the polynomials corresponding to them in the following cases:
(i) f(x) = 3x + 1, x = −1/3
(ii) $f(x) = x^2 – 1$, x = 1,−1
(iii) $g(x) = 3x^2 – 2$, x = 2/√3 , −2/√3
(iv) $p(x) = x^3 – 6x^2 + 11x – 6$, x = 1, 2, 3
(v) f(x) = 5x – π, x = 4/5
(vi) $f(x) = x^2$, x = 0
(vii) f(x) = lx + m, x = −m/l
(viii) f(x) = 2x + 1, x = 1/2

Solution:

(i) f(x) = 3x + 1, x = −1/3

f(x) = 3x + 1

Substitute $x = -1/3$ in $f(x)$

$f(-1/3) = 3(-1/3) + 1$

$= -1 + 1$

$= 0$

Since, the result is 0, so $x = -1/3$ is the root of $3x + 1$

(ii) $f(x) = x^2 - 1, x = 1,-1$

$f(x) = x^2 - 1$

Given that $x = (1 , -1)$

Substitute $x = 1$ in $f(x)$

$f(1) = 1^2 - 1$

$= 1 - 1$

$= 0$

Now, substitute $x = (-1)$ in $f(x)$

$f(-1) = (-1)^2 - 1$

$= 1 - 1$

$= 0$

Since , the results when $x = 1$ and $x = -1$ are 0, so $(1 , -1)$ are the roots of the polynomial $f(x) = x^2 - 1$

(iii) $g(x) = 3x^2 - 2 , x = 2/\sqrt{3} , -2/\sqrt{3}$

$g(x) = 3x^2 - 2$

Substitute $x = 2/\sqrt{3}$ in $g(x)$

$g(2/\sqrt{3}) = 3(2/\sqrt{3})^2 - 2$

= 3(4/3) – 2

= 4 – 2

= 2 ≠ 0

Now, Substitute x = −2/√3 in g(x)

g(2/√3) = $3(-2/\sqrt{3})^2 - 2$

= 3(4/3) – 2

= 4 – 2

= 2 ≠ 0

Since, the results when x = 2/√3 and x = −2/√3) are not 0. Therefore (2/√3 , −2/√3) are not zeros of $3x^2 – 2$.

(iv) p(x) = $x^3 – 6x^2 + 11x – 6$, x = 1, 2, 3

p(1) = $1^3 – 6(1)^2$ + 11x 1 – 6 = 1 – 6 + 11 – 6 = 0

p(2) = $2^3 – 6(2)^2$ + 11x2 – 6 = 8 – 24 + 22 – 6 = 0

p(3) = $3^3 – 6(3)^2$ + 11x3 – 6 = 27 – 54 + 33 – 6 = 0

Therefore, x = 1, 2, 3 are zeros of p(x).

(v) f(x) = 5x – π, x = 4/5

f(4/5) = 5 x 4/5 – π = 4 – π ≠ 0

Therefore, x = 4/5 is not a zeros of f(x).

(vi) f(x) = x^2 , x = 0

f(0) = 0^2 = 0

Therefore, x = 0 is a zero of f(x).

(vii) f(x) = lx + m, x = −m/l

f(−m/l) = l x −m/l + m = -m + m = 0

Therefore, x = −m/l is a zero of f(x).

(viii) f(x) = 2x + 1, x = ½

f(1/2) = 2x 1/2 + 1 = 1 + 1 = 2 ≠ 0

Therefore, x = ½ is not a zero of f(x).

Exercise 6.3 Page No: 6.14

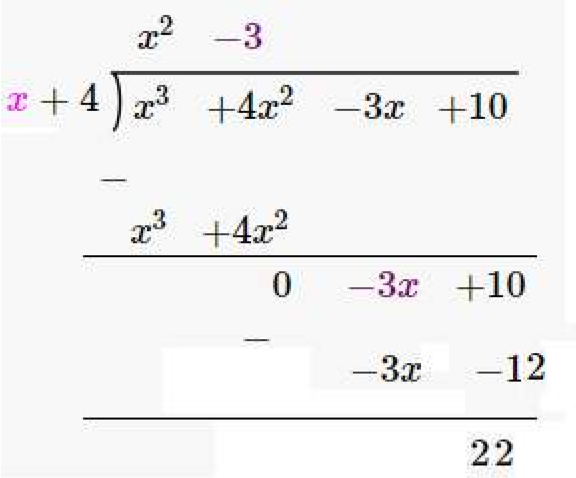
In each of the following, using the remainder theorem, find the remainder when f(x) is divided by g(x) and verify the by actual division : (1 – 8)

Question 1: f(x) = $x^3 + 4x^2 - 3x + 10$, g(x) = x + 4
Solution:
f(x) = $x^3 + 4x^2 - 3x + 10$, g(x) = x + 4

Put g(x) =0
=> x + 4 = 0 or x = -4

Remainder = f(-4)
Now,
f(-4) = $(-4)^3 + 4(-4)^2 - 3(-4) + 10 = -64 + 64 + 12 + 10 = 22$

Actual Division:



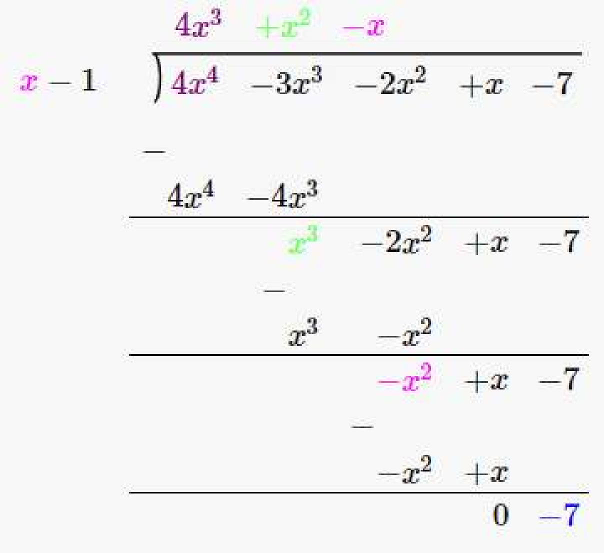
$$\begin{array}{r|rrrr} & x^2 & -3 & & \\ \hline x+4 & x^3 & +4x^2 & -3x & +10 \\ - & x^3 & +4x^2 & & \\ \hline & & 0 & -3x & +10 \\ & & - & -3x & -12 \\ \hline & & & & 22 \end{array}$$

Question 2: f(x) = $4x^4 - 3x^3 - 2x^2 + x - 7$, g(x) = x – 1
Solution:

f(x) = $4x^4 - 3x^3 - 2x^2 + x - 7$
Put g(x) =0
=> x – 1 = 0 or x = 1

Remainder = f(1)
Now,
f(1) = $4(1)^4 - 3(1)^3 - 2(1)^2 + (1) - 7 = 4 - 3 - 2 + 1 - 7 = -7$

Actual Division:



$$\begin{array}{rrrrrrr}
 & 4x^3 & +x^2 & -x & & & \\
x-1 & \big) 4x^4 & -3x^3 & -2x^2 & +x & -7 & \\
 & - & & & & & \\
 & 4x^4 & -4x^3 & & & & \\
\hline
 & & x^3 & -2x^2 & +x & -7 & \\
 & & - & & & & \\
 & & x^3 & -x^2 & & & \\
\hline
 & & & -x^2 & +x & -7 & \\
 & & & - & & & \\
 & & & -x^2 & +x & & \\
\hline
 & & & & 0 & -7 &
\end{array}$$

Question 3: f(x) = $2x^4 - 6X^3 + 2x^2 - x + 2$, g(x) = x + 2

Solution:

f(x) = $2x^4 - 6X^3 + 2x^2 - x + 2$, g(x) = x + 2

Put g(x) = 0

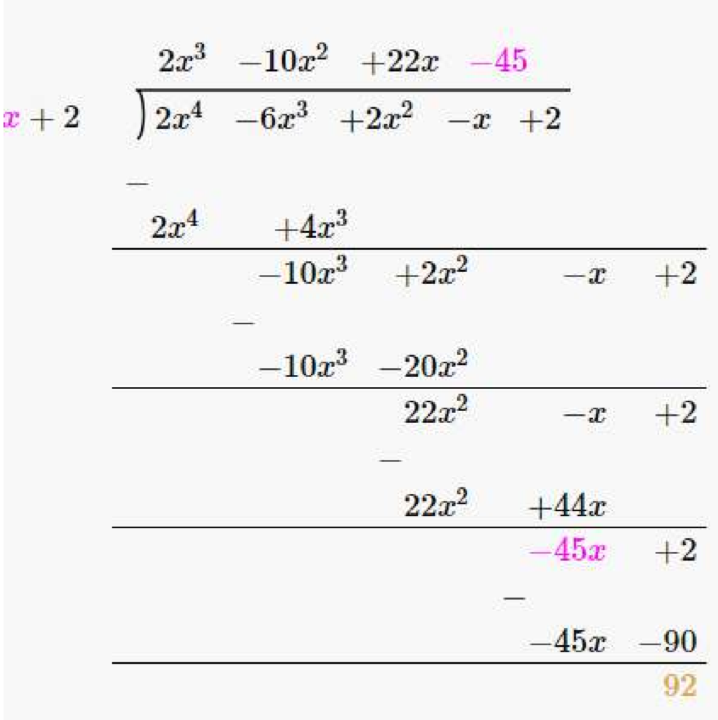
=> x + 2 = 0 or x = -2

Remainder = f(-2)

Now,

$f(-2) = 2(-2)^4 - 6(-2)^3 + 2(-2)^2 - (-2) + 2 = 32 + 48 + 8 + 2 + 2 = 92$

Actual Division:



$$\begin{array}{r|llllll}
 & 2x^3 & -10x^2 & +22x & -45 & \\
x+2 & 2x^4 & -6x^3 & +2x^2 & -x & +2 \\
 & - & & & & \\
 & 2x^4 & +4x^3 & & & \\ \hline
 & & -10x^3 & +2x^2 & -x & +2 \\
 & & - & & & \\
 & & -10x^3 & -20x^2 & & \\ \hline
 & & & 22x^2 & -x & +2 \\
 & & & - & & \\
 & & & 22x^2 & +44x & \\ \hline
 & & & & -45x & +2 \\
 & & & & - & \\
 & & & & -45x & -90 \\ \hline
 & & & & & 92
\end{array}$$

Question 4: f(x) = 4x^3 – 12x^2 + 14x – 3, g(x) = 2x – 1

Solution:

f(x) = 4x^3 – 12x^2 + 14x – 3, g(x) = 2x – 1

Put g(x) =0

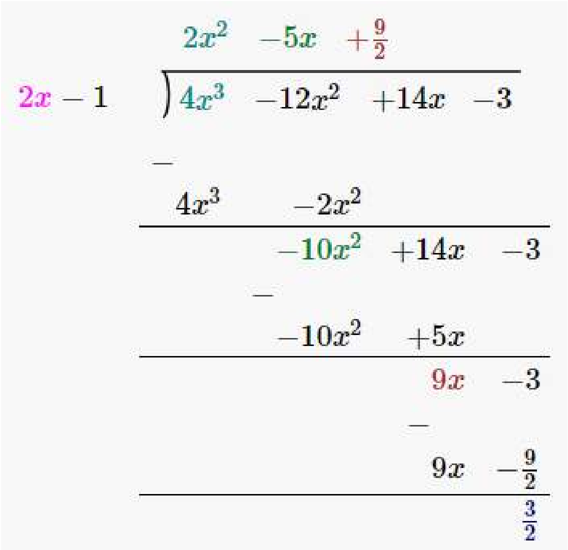
=> 2x -1 =0 or x = 1/2

Remainder = f(1/2)

Now,

f(1/2) = 4(1/2)^3 – 12(1/2)^2 + 14(1/2) – 3 = ½ - 3 + 7 – 3 = 3/2

Actual Division:



$$\begin{array}{r|rrrr}
 & 2x^2 & -5x & +\frac{9}{2} & \\
\hline
2x-1 & 4x^3 & -12x^2 & +14x & -3 \\
- & & & & \\
 & 4x^3 & -2x^2 & & \\
\hline
 & & -10x^2 & +14x & -3 \\
 & - & & & \\
 & & -10x^2 & +5x & \\
\hline
 & & & 9x & -3 \\
 & & & - & \\
 & & & 9x & -\frac{9}{2} \\
\hline
 & & & & \frac{3}{2}
\end{array}$$

Question 5: f(x) = x^3 – 6x^2 + 2x – 4, g(x) = 1 – 2x

Solution:

f(x) = x^3 – 6x^2 + 2x – 4, g(x) = 1 – 2x

Put g(x) = 0

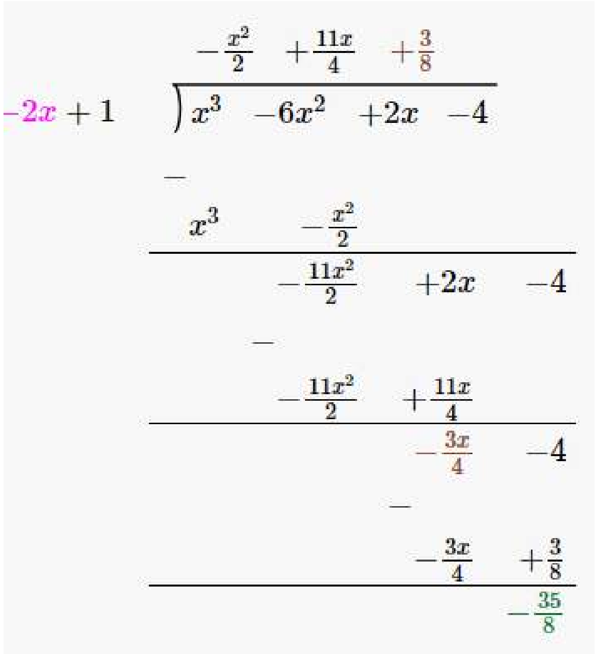
=> 1 – 2x = 0 or x = 1/2

Remainder = f(1/2)

Now,

f(1/2) = $(1/2)^3$ – 6$(1/2)^2$ + 2(1/2) – 4 = 1 + 1/8 – 4 - 3/2 = -35/8

Actual Division:



$$\begin{array}{rrrrr}
 & & -\frac{x^2}{2} & +\frac{11x}{4} & +\frac{3}{8} \\
\cline{2-5}
-2x+1 & \big) \; x^3 & -6x^2 & +2x & -4 \\
 & - & & & \\
 & x^3 & -\frac{x^2}{2} & & \\
\hline
 & & -\frac{11x^2}{2} & +2x & -4 \\
 & & - & & \\
 & & -\frac{11x^2}{2} & +\frac{11x}{4} & \\
\hline
 & & & -\frac{3x}{4} & -4 \\
 & & & - & \\
 & & & -\frac{3x}{4} & +\frac{3}{8} \\
\hline
 & & & & -\frac{35}{8}
\end{array}$$

Question 6: f(x) = $x^4 - 3x^2 + 4$, g(x) = x – 2

Solution:

f(x) = $x^4 - 3x^2 + 4$, g(x) = x – 2

Put g(x) = 0

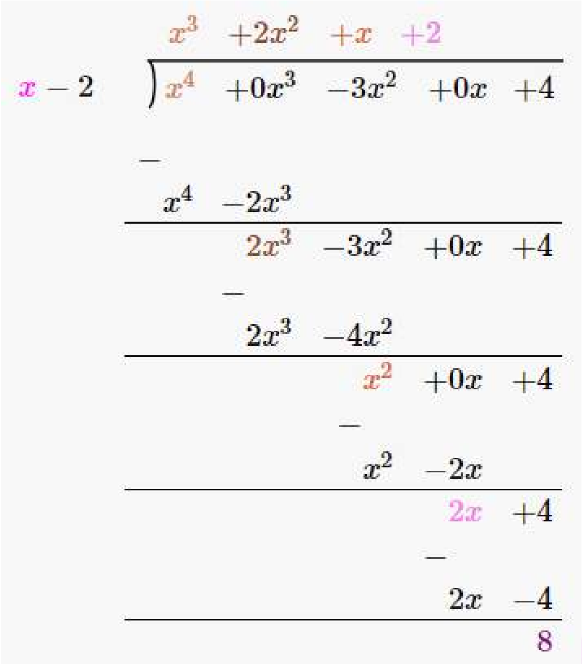
=> x - 2 = 0 or x = 2

Remainder = f(2)

Now,

f(2) = $(2)^4 - 3(2)^2 + 4 = 16 - 12 + 4 = 8$

Actual Division:



$$\begin{array}{cllllll}
 & & x^3 & +2x^2 & +x & +2 & \\
\hline
x-2 &) & x^4 & +0x^3 & -3x^2 & +0x & +4 \\
 & - & & & & & \\
 & & x^4 & -2x^3 & & & \\
\hline
 & & & 2x^3 & -3x^2 & +0x & +4 \\
 & & - & & & & \\
 & & & 2x^3 & -4x^2 & & \\
\hline
 & & & & x^2 & +0x & +4 \\
 & & & - & & & \\
 & & & & x^2 & -2x & \\
\hline
 & & & & & 2x & +4 \\
 & & & & - & & \\
 & & & & & 2x & -4 \\
\hline
 & & & & & & 8
\end{array}$$

Question 7: f(x) = $9x^3 - 3x^2 + x - 5$, g(x) = x – 2/3

Solution:

f(x) = $9x^3 - 3x^2 + x - 5$, g(x) = x – 2/3

Put g(x) = 0

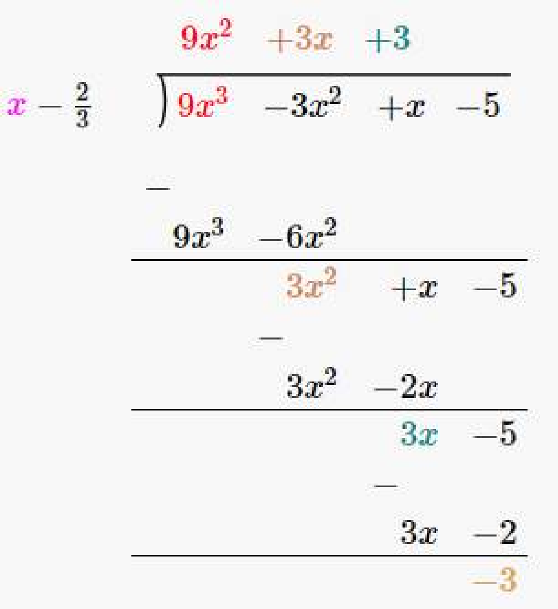
=> x - 2/3 = 0 or x = 2/3

Remainder = f(2/3)

Now,

f(2/3) = $9(2/3)^3 - 3(2/3)^2 + (2/3) - 5$ = 8/3 – 4/3 + 2/3 – 5/1 = -3

Actual Division:



$$\begin{array}{rrrrrr}
 & 9x^2 & +3x & +3 & & \\
\cline{2-5}
x-\frac{2}{3} & \big)\,9x^3 & -3x^2 & +x & -5 & \\
 & - & & & & \\
 & 9x^3 & -6x^2 & & & \\
\hline
 & & 3x^2 & +x & -5 & \\
 & & - & & & \\
 & & 3x^2 & -2x & & \\
\hline
 & & & 3x & -5 & \\
 & & & - & & \\
 & & & 3x & -2 & \\
\hline
 & & & & -3 &
\end{array}$$

Exercise 6.4

In each of the following, use factor theorem to find whether polynomial g(x) is a factor of polynomial f(x) or, not: (1-7)

Question 1: f(x) = $x^3 – 6x^2 + 11x – 6$; g(x) = x – 3
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = x -3 = 0

or x = 3
Remainder = f(3)

Now,
f(3) = $(3)^3 – 6(3)^2$ +11 x 3 – 6
= 27 - 54 + 33 - 6
= 60 – 60
= 0
Therefore, g(x) is a factor of f(x)

Question 2: f(x) = $3X^4 + 17x^3 + 9x^2 – 7x – 10$; g(x) = x + 5
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = x + 5 = 0, then x = -5

Remainder = f(-5)

Now,
f(3) = $3(-5)^4 + 17(-5)^3 + 9(-5)^2 – 7(-5) – 10$
= 3 x 625 + 17 x (-125) + 9 x (25) – 7 x (-5) – 10
= 1875 -2125 + 225 + 35 – 10
= 0
Therefore, g(x) is a factor of f(x).

Question 3: f(x) = $x^5 + 3x^4 – x^3 – 3x^2 + 5x + 15$, g(x) = x + 3
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = x + 3 = 0, then x = -3

Remainder = f(-3)

Now,
$f(-3) = (-3)^5 + 3(-3)^4 – (-3)^3 – 3(-3)^2 + 5(-3) + 15$
= -243 + 3 x 81 -(-27)-3 x 9 + 5(-3) + 15
= -243 +243 + 27-27- 15 + 15
= 0

Therefore, g(x) is a factor of f(x).

Question 4: $f(x) = x^3 – 6x^2 – 19x + 84$, $g(x) = x – 7$
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.
g(x) = x – 7 = 0, then x = 7
Remainder = f(7)
Now,
$f(7) = (7)^3 – 6(7)^2 – 19 x 7 + 84$
= 343 – 294 – 133 + 84
= 343 + 84 – 294 – 133
= 0
Therefore, g(x) is a factor of f(x).

Question 5: $f(x) = 3x^3 + x^2 – 20x + 12$, $g(x) = 3x – 2$
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.
g(x) = 3x – 2 = 0, then x = 2/3
Remainder = f(2/3)
Now,
$f(2/3) = 3(2/3)^3 + (2/3)^2 – 20(2/3) + 12$
= 3 x 8/27 + 4/9 – 40/3 + 12
= 8/9 + 4/9 – 40/3 + 12
= 0/9
= 0
Therefore, g(x) is a factor of f(x).

Question 6: $f(x) = 2x^3 – 9x^2 + x + 12$, $g(x) = 3 – 2x$
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.
g(x) = 3 – 2x = 0, then x = 3/2
Remainder = f(3/2)
Now,
$f(3/2) = 2(3/2)^3 – 9(3/2)^2 + (3/2) + 12$

= 2 x 27/8 – 9 x 9/4 + 3/2 + 12

= 27/4 – 81/4 + 3/2 + 12

= 0/4
= 0

Therefore, g(x) is a factor of f(x).

Question 7: f(x) = $x^3 – 6x^2 + 11x – 6$, g(x) = $x^2 – 3x + 2$
Solution:
If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.
g(x) = 0
or $x^2 – 3x + 2 = 0$
$x^2 – x – 2x + 2 = 0$
x(x – 1) – 2(x – 1) = 0
(x – 1) (x – 2) = 0
Therefore x = 1 or x = 2

Now,
f(1) = $(1)^3 – 6(1)^2 + 11(1) – 6$ = 1-6+11-6= 12- 12 = 0

f(2) = $(2)^3 – 6(2)^2 + 11(2) - 6$ = 8 – 24 + 22 – 6 = 30 – 30 = 0

=> f(1) = 0 and f(2) = 0

Which implies g(x) is factor of f(x).

Question 8: Show that (x – 2), (x + 3) and (x – 4) are factors of $x^3 – 3x^2 – 10x + 24$.
Solution:

Let f(x) = $x^3 – 3x^2 – 10x + 24$
If x – 2 = 0, then x = 2,
If x + 3 = 0 then x = -3,
and If x – 4 = 0 then x = 4

Now,
f(2) = $(2)^3 – 3(2)^2$ – 10 x 2 + 24 = 8 – 12 – 20 + 24 = 32 – 32 = 0

f(-3) = $(-3)^3 – 3(-3)^2$ – 10 (-3) + 24 = -27 -27 + 30 + 24 = -54 + 54 = 0

f(4) = $(4)^3 – 3(4)^2$ – 10 x 4 + 24 = 64-48 -40 + 24 = 88 – 88 = 0

f(2) = 0
f(-3) = 0
f(4) = 0

Hence (x – 2), (x + 3) and (x – 4) are the factors of f(x)

Question 9: Show that (x + 4), (x – 3) and (x – 7) are factors of $x^3 - 6x^2 - 19x + 84$.
Solution:
Let $f(x) = x^3 - 6x^2 - 19x + 84$
If x + 4 = 0, then x = -4
If x – 3 = 0, then x = 3
and if x – 7 = 0, then x = 7

Now,
$f(-4) = (-4)^3 - 6(-4)^2 - 19(-4) + 84 = -64 - 96 + 76 + 84 = 160 - 160 = 0$
f(-4) = 0

$f(3) = (3)^3 - 6(3)^2 - 19 \times 3 + 84 = 27 - 54 - 57 + 84 = 111 - 111 = 0$
f(3) = 0

$f(7) = (7)^3 - 6(7)^2 - 19 \times 7 + 84 = 343 - 294 - 133 + 84 = 427 - 427 = 0$

f(7) = 0

Hence (x + 4), (x – 3), (x – 7) are the factors of f(x).

Exercise 6.5

Using factor theorem, factorize each of the following polynomials:

Question 1: $x^3 + 6x^2 + 11x + 6$

Solution:

Let $f(x) = x^3 + 6x^2 + 11x + 6$

Step 1: Find the factors of constant term

Here constant term = 6

Factors of 6 are ±1, ±2, ±3, ±6

Step 2: Find the factors of f(x)

Let $x + 1 = 0$

$\Rightarrow x = -1$

Put the value of x in f(x)

$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$

$= -1 + 6 - 11 + 6$

$= 12 - 12$

$= 0$

So, (x + 1) is the factor of f(x)

Let $x + 2 = 0$

$\Rightarrow x = -2$

Put the value of x in f(x)

$f(-2) = (-2)^3 + 6(-2)^2 + 11(-2) + 6 = -8 + 24 - 22 + 6 = 0$

So, (x + 2) is the factor of f(x)

Let $x + 3 = 0$

=> x = -3

Put the value of x in f(x)

$f(-3) = (-3)^3 + 6(-3)^2 + 11(-3) + 6 = -27 + 54 - 33 + 6 = 0$

So, (x + 3) is the factor of f(x)

Hence, f(x) = (x + 1)(x + 2)(x + 3)

Question 2: $x^3 + 2x^2 - x - 2$
Solution:
Let $f(x) = x^3 + 2x^2 - x - 2$
Constant term = -2

Factors of -2 are ±1, ±2

Let x – 1 = 0

=> x = 1

Put the value of x in f(x)

$f(1) = (1)^3 + 2(1)^2 - 1 - 2 = 1 + 2 - 1 - 2 = 0$

So, (x – 1) is factor of f(x)

Let x + 1 = 0

=> x = -1

Put the value of x in f(x)

$f(-1) = (-1)^3 + 2(-1)^2 - 1 - 2 = -1 + 2 + 1 - 2 = 0$

(x + 1) is a factor of f(x)

Let x + 2 = 0

=> x = -2

Put the value of x in f(x)

$f(-2) = (-2)^3 + 2(-2)^2 – (-2) – 2 = -8 + 8 + 2 – 2 = 0$

(x + 2) is a factor of f(x)

Let x - 2 = 0

=> x = 2

Put the value of x in f(x)

$f(2) = (2)^3 + 2(2)^2 – 2 – 2 = 8 + 8 – 2 – 2 = 12 \neq 0$

(x - 2) is not a factor of f(x)

Hence f(x) = (x + 1)(x- 1)(x+2)

Question 3: $x^3 – 6x^2 + 3x + 10$
Solution:
Let $f(x) = x^3 – 6x^2 + 3x + 10$
Constant term = 10
Factors of 10 are ±1, ±2, ±5, ±10

Let x + 1 = 0 or x = -1
$f(-1) = (-1)^3 – 6(-1)^2 + 3(-1) + 10 = 10 – 10 = 0$
f(-1) = 0

Let x + 2 = 0 or x = -2
$f(-2) = (-2)^3 – 6(-2)^2 + 3(-2) + 10 = -8 – 24 – 6 + 10 = -28$
$f(-2) \neq 0$

Let x - 2 = 0 or x = 2
$f(2) = (2)^3 – 6(2)^2 + 3(2) + 10 = 8 – 24 + 6 + 10 = 0$
f(2) = 0

Let x - 5 = 0 or x = 5
$f(5) = (5)^3 – 6(5)^2 + 3(5) + 10 = 125 – 150 + 15 + 10 = 0$
f(5) = 0
Therefore, (x + 1), (x – 2) and (x-5) are factors of f(x)

Hence f(x) = (x + 1) (x – 2) (x-5)

Question 4: $x^4 – 7x^3 + 9x^2 + 7x- 10$

Solution:

Let f(x) = $x^4 – 7x^3 + 9x^2 + 7x- 10$

Constant term = -10

Factors of -10 are ±1, ±2, ±5, ±10

Let x - 1 = 0 or x = 1

$f(1) = (1)^4 – 7(1)^3 + 9(1)^2 + 7(1) – 10 = 1 – 7 + 9 + 7 -10 = 0$

f(1) = 0

Let x + 1 = 0 or x = -1

$f(-1) = (-1)^4 – 7(-1)^3 + 9(-1)^2 + 7(-1) – 10 = 1 + 7 + 9 - 7 -10 = 0$

f(-1) = 0

Let x - 2 = 0 or x = 2

$f(2) = (2)^4 – 7(2)^3 + 9(2)^2 + 7(2) – 10 = 16 – 56 + 36 + 14 – 10 = 0$

f(2) = 0

Let x - 5 = 0 or x = 5

$f(5) = (5)^4 – 7(5)^3 + 9(5)^2 + 7(5) – 10 = 625 – 875 + 225 + 35 – 10 = 0$

f(5) = 0

Therefore, (x - 1), (x + 1), (x – 2) and (x-5) are factors of f(x)

Hence f(x) = (x - 1) (x + 1) (x – 2) (x-5)

Question 5: $x^4 – 2x^3 – 7x^2 + 8x + 12$

Solution:

f(x) = $x^4 – 2x^3 – 7x^2 + 8x + 12$

Constant term = 12

Factors of 12 are ±1, ±2, ±3, ±4, ±6, ±12

Let x - 1 = 0 or x = 1

$f(1) = (1)^4 – 2(1)^3 - 7(1)^2 + 8(1) + 12 = 1 – 2 – 7 + 8 + 12 = 12$

f(1) ≠ 0

Let x + 1 = 0 or x = -1

$f(-1) = (-1)^4 – 2(-1)^3 - 7(-1)^2 + 8(-1) + 12 = 1 + 2 – 7 - 8 + 12 = 0$

f(-1) = 0

Let x +2 = 0 or x = -2
$f(-2) = (-2)^4 – 2(-2)^3 - 7(-2)^2 + 8(-2) + 12 = 16 + 16 – 28 - 16 + 12 = 0$
f(-2) = 0

Let x - 2 = 0 or x = 2
$f(2) = (2)^4 – 2(2)^3 - 7(2)^2 + 8(2) + 12 = 16 - 16 – 28 + 16 + 12 = 0$
f(2) = 0

Let x - 3 = 0 or x = 3
$f(3) = (3)^4 – 2(3)^3 - 7(3)^2 + 8(3) + 12 = 0$
f(3) = 0

Therefore, (x + 1), (x + 2), (x – 2) and (x-3) are factors of f(x)

Hence f(x) = (x + 1)(x + 2) (x – 2) (x-3)

Question 6: $x^4 + 10x^3 + 35x^2 + 50x + 24$
Solution:
Let $f(x) = x^4 + 10x^3 + 35x^2 + 50x + 24$
Constant term = 24
Factors of 24 are ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24

Let x + 1 = 0 or x = -1
$f(-1) = (-1)^4 + 10(-1)^3 + 35(-1)^2 + 50(-1) + 24 = 1 – 10 + 35 – 50 + 24 = 0$
f(1) = 0
(x + 1) is a factor of f(x)
Likewise, (x + 2),(x + 3),(x + 4) are also the factors of f(x)

Hence f(x) = (x + 1) (x + 2)(x + 3)(x + 4)

Question 7: $2x^4 – 7x^3 – 13x^2 + 63x – 45$
Solution:
Let $f(x) = 2x^4 – 7x^3 – 13x^2 + 63x – 45$
Constant term = -45
Factors of -45 are ±1, ±3, ±5, ±9, ±15, ±45
Here coefficient of x^4 is 2. So possible rational roots of f(x) are

±1, ±3, ±5, ±9, ±15, ±45, ±1/2,±3/2,±5/2,±9/2,±15/2,±45/2

Let x - 1 = 0 or x = 1
$f(1) = 2(1)^4 - 7(1)^3 - 13(1)^2 + 63(1) – 45 = 2 – 7 – 13 + 63 – 45 = 0$
f(1) = 0

f(x) can be written as,
$f(x) = (x-1) (2x^3 – 5x^2 -18x +45)$

or $f(x) =(x-1)g(x)$ …(1)

Let $x - 3 = 0$ or $x = 3$
$f(3) = 2(3)^4 - 7(3)^3 - 13(3)^2 + 63(3) – 45 = = 162 – 189 – 117 + 189 – 45= 0$
$f(3) = 0$

Now, we are available with 2 factors of f(x), (x – 1) and (x – 3)

Here $g(x) = 2x^2 (x-3) + x(x-3) -15(x-3)$
Taking (x-3) as common
$= (x-3)(2x^2 + x – 15)$

$=(x-3)(2x^2+6x – 5x -15)$

$= (x-3)(2x-5)(x+3)$

$= (x-3)(x+3)(2x-5)$ ….(2)

From (1) and (2)

$f(x) =(x-1) (x-3)(x+3)(2x-5)$

Exercise VSAQs

Question 1: Define zero or root of a polynomial

Solution:

zero or root, is a solution to the polynomial equation, f(y) = 0.

It is that value of y that makes the polynomial equal to zero.

Question 2: If x = 1/2 is a zero of the polynomial f(x) = 8x^3 + ax^2 - 4x + 2, find the value of a.

Solution:

If x = 1/2 is a zero of the polynomial f(x), then f(1/2) = 0

8(1/2)^3 + a(1/2)^2 - 4(1/2) + 2 = 0

8 x 1/8 + a/4 - 2 + 2 = 0

1 + a/4 = 0

a = -4

Question 3: Write the remainder when the polynomial f(x) = x^3 + x^2 - 3x + 2 is divided by x + 1.

Solution:

Using factor theorem,

Put x + 1 = 0 or x = -1

f(-1) is the remainder.

Now,

f(-1) = (-1)^3 + (-1)^2 - 3(-1) + 2

= -1 + 1 + 3 + 2

= 5

Therefore 5 is the remainder.

Question 4: Find the remainder when x^3 + 4x^2 + 4x-3 if divided by x

Solution:

Using factor theorem,

Put x = 0

f(0) is the remainder.

Now,
f(0) = 0^3 + 4(0)^2 + 4x0 -3 = -3
Therefore -3 is the remainder.

Question 5: If x+1 is a factor of x^3 + a, then write the value of a.
Solution:
Let f(x) = x^3 + a
If x+1 is a factor of x^3 + a then f(-1) = 0
(-1)^3 + a = 0
-1 + a = 0
or a = 1

Question 6: If f(x) = x^4 - 2x^3 + 3x^2 – ax – b when divided by x – 1, the remainder is 6, then find the value of a+b.

Solution:

From the statement, we have f(1) = 6

(1)^4 - 2(1)^3 + 3(1)^2 – a(1) – b = 6

1 – 2 + 3 – a - b = 6

2 – a - b = 6

a + b = -4