Exercise 8.1 Page No: 8.7

Question 1: Write the complement of each of the following angles:
(i)20^0
(ii)35^0
(iii)90^0
(iv) 77^0
(v)30^0

Solution:
(i) The sum of an angle and its complement = 90^0
Therefore, the complement of $20^0 = 90^0 – 20^0 = 70^0$

(ii) The sum of an angle and its complement = 90^0
Therefore, the complement of 35° = 90° – 35° = 55

(iii) The sum of an angle and its complement = 90^0
Therefore, the complement of $90^0 = 90^0 – 90^0 = 0^0$

(iv) The sum of an angle and its complement = 90^0
Therefore, the complement of $77^0 = 90° – 77^0 = 13^0$

(v) The sum of an angle and its complement = 90^0
Therefore, the complement of $30^0 = 90^0 – 30^0 = 60^0$

Question 2 : Write the supplement of each of the following angles:
(i) 54^0
(ii) 132^0
(iii) 138^0

Solution:
(i) The sum of an angle and its supplement = 180^0.
Therefore supplement of angle $54^0 = 180^0 – 54^0 = 126^0$

(ii) The sum of an angle and its supplement = 180^0.
Therefore supplement of angle $132^0 = 180^0 – 132^0 = 48^0$

(iii) The sum of an angle and its supplement = 180^0.
Therefore supplement of angle $138^0 = 180^0 – 138^0 = 42^0$

Question 3: If an angle is 28^0 less than its complement, find its measure?
Solution:
Let the measure of any angle is ' a ' degrees
Thus, its complement will be $(90 – a)^0$
So, the required angle = Complement of a – 28
a = (90 – a) – 28
2a = 62
a = 31
Hence, the angle measured is 31^0.

Question 4 : If an angle is 30° more than one half of its complement, find the measure of the angle?
Solution:
Let an angle measured by ' a ' in degrees
Thus, its complement will be $(90 - a)^0$

Required Angle = 30^0 + complement/2

$a = 30^0 + (90 – a)^0 / 2$

$a + a/2 = 30^0 + 45^0$

$3a/2 = 75^0$

$a = 50^0$

Therefore, the measure of required angle is 50^0.

Question 5 : Two supplementary angles are in the ratio 4:5. Find the angles?
Solution:
Two supplementary angles are in the ratio 4:5.
Let us say, the angles are 4a and 5a (in degrees)
Since angle are supplementary angles;
Which implies, $4a + 5a = 180^0$
$9a = 180^0$
$a = 20^0$

Therefore, $4a = 4 (20) = 80^0$ and
$5(a) = 5 (20) = 100^0$

Hence, required angles are 80° and 100^0.

Question 6 : Two supplementary angles differ by 48^0. Find the angles?

Solution: Given: Two supplementary angles differ by 48^0.

Consider a^0 be one angle then its supplementary angle will be equal to $(180 – a)^0$

According to the question;

(180 – a) – x = 48

(180 – 48) = 2a

132 = 2a

132/2 = a

Or a = 66^0

Therefore, 180 – a = 114^0

Hence, the two angles are 66^0 and 114^0.

Question 7: An angle is equal to 8 times its complement. Determine its measure?

Solution: Given: Required angle = 8 times of its complement

Consider a^0 be one angle then its complementary angle will be equal to $(90 – a)^0$

According to the question;

a = 8 times of its complement

a = 8 (90 – a)

a = 720 – 8a

a + 8a = 720

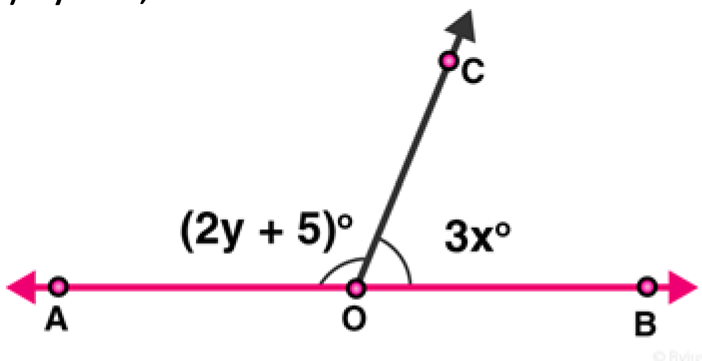
9a = 720

a = 80

Therefore, the required angle is 80^0.

Exercise 8.2 Page No: 8.13

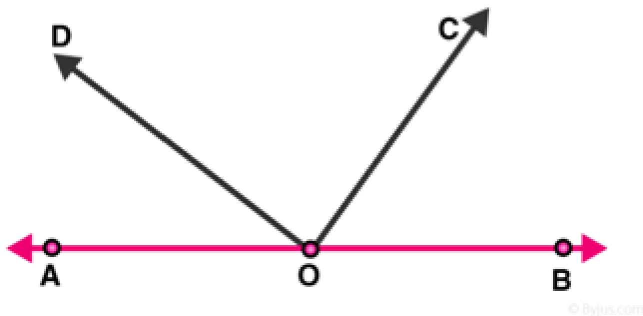
Question 1: In the below Fig. OA and OB are opposite rays:
(i) If $x = 25^0$, what is the value of y?
(ii) If $y = 35^0$, what is the value of x?



Solution:
(i) Given: x = 25
From figure: ∠AOC and ∠BOC form a linear pair
Which implies, ∠AOC + ∠BOC = 180^0
From the figure, ∠AOC = 2y + 5 and ∠BOC = 3x
∠AOC + ∠BOC = 180^0
(2y + 5) + 3x = 180
(2y + 5) + 3 (25) = 180
2y + 5 + 75 = 180
2y + 80 = 180
2y = 100
y = 100/2 = 50
Therefore, y = 50^0.

(ii) Given: y = 35^0
From figure: ∠AOC + ∠BOC = 180° (Linear pair angles)
(2y + 5) + 3x = 180
(2(35) + 5) + 3x = 180
75 + 3x = 180
3x = 105
x = 35
Therefore, x = 35^0

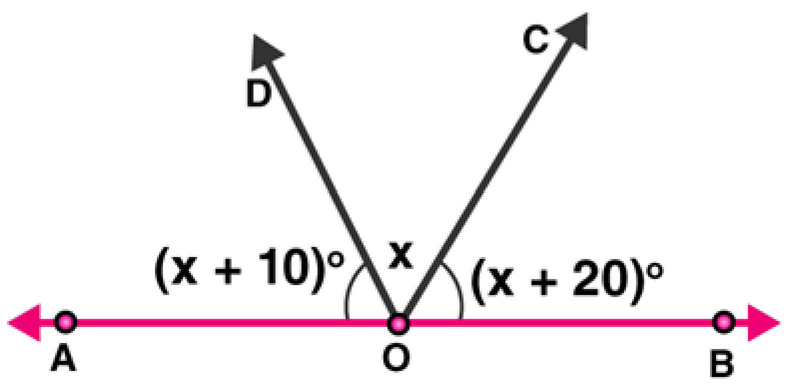
Question 2: In the below figure, write all pairs of adjacent angles and all the linear pairs.



Solution: From figure, pairs of adjacent angles are :
(∠AOC, ∠COB) ; (∠AOD, ∠BOD) ; (∠AOD, ∠COD) ; (∠BOC, ∠COD)

And Linear pair of angles are (∠AOD, ∠BOD) and (∠AOC, ∠BOC).
[As ∠AOD + ∠BOD = 180^0 and ∠AOC+ ∠BOC = 180^0.]

Question 3 : In the given figure, find x. Further find ∠BOC , ∠COD and ∠AOD.



Solution:
From figure, ∠AOD and ∠BOD form a linear pair,
Therefore, ∠AOD+ ∠BOD = 180^0

Also, ∠AOD + ∠BOC + ∠COD = 180^0

Given: ∠AOD = $(x+10)^0$, ∠COD = x^0 and ∠BOC = $(x + 20)^0$

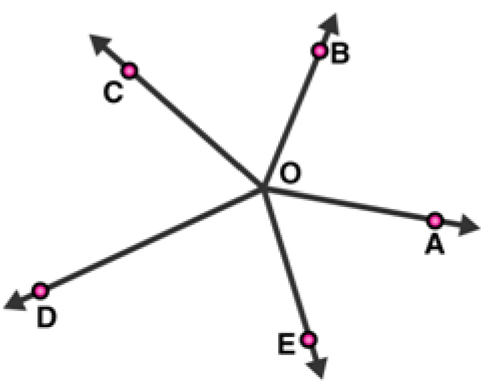
(x + 10) + x + (x + 20) = 180
3x + 30 = 180
3x = 180 – 30

x = 150/3
x = 50^0

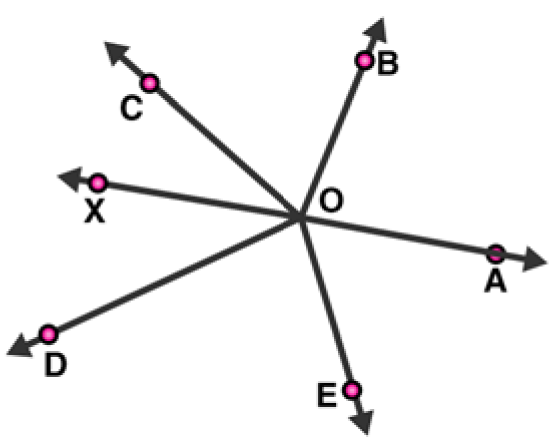
Now,
$\angle AOD = (x+10) = 50 + 10 = 60$
$\angle COD = x = 50$
$\angle BOC = (x+20) = 50 + 20 = 70$

Hence, $\angle AOD = 60^0$, $\angle COD = 50^0$ and $\angle BOC = 70^0$

Question 4: In figure, rays OA, OB, OC, OD and OE have the common end point 0. Show that $\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360°$.



Solution:
Given: Rays OA, OB, OC, OD and OE have the common endpoint O.
Draw an opposite ray OX to ray OA, which make a straight line AX.



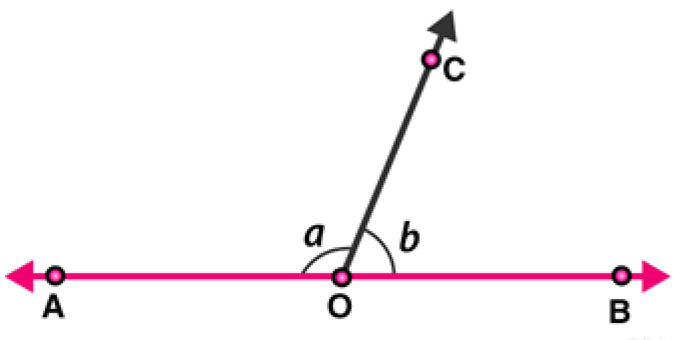
From figure:
$\angle AOB$ and $\angle BOX$ are linear pair angles, therefore,
$\angle AOB + \angle BOX = 180^0$
Or, $\angle AOB + \angle BOC + \angle COX = 180^0$ ————(1)

Also,
∠AOE and ∠EOX are linear pair angles, therefore,
∠AOE+∠EOX =180°
Or, ∠AOE + ∠DOE + ∠DOX = 180^0 ——(2)

By adding equations, (1) and (2), we get;
∠AOB + ∠BOC + ∠COX + ∠AOE + ∠DOE + ∠DOX = 180^0 + 180^0
∠AOB + ∠BOC + ∠COD + ∠DOE + ∠EOA = 360^0

Hence Proved.

Question 5 : In figure, ∠AOC and ∠BOC form a linear pair. If a – 2b = 30°, find a and b?

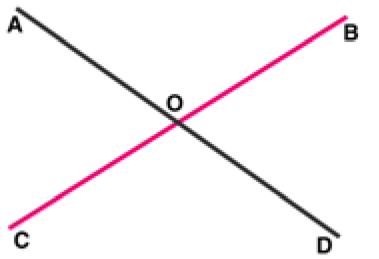


Solution:
Given : ∠AOC and ∠BOC form a linear pair.
=> a + b = 180^0 .....(1)
a – 2b = 30^0 ...(2) (given)

On subtracting equation (2) from (1), we get
a + b – a + 2b = 180 – 30
3b = 150
b = 150/3
b = 50^0
Since, a – 2b = 30^0
a – 2(50) = 30
a = 30 + 100
a = 130^0
Therefore, the values of a and b are 130° and 50° respectively.

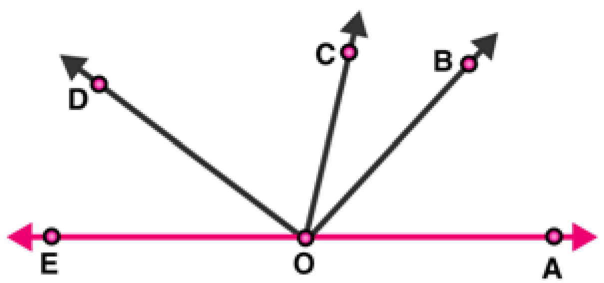
Question 6: How many pairs of adjacent angles are formed when two lines intersect at a point?
Solution: Four pairs of adjacent angles are formed when two lines intersect each other at a single point.

For example, Let two lines AB and CD intersect at point O.



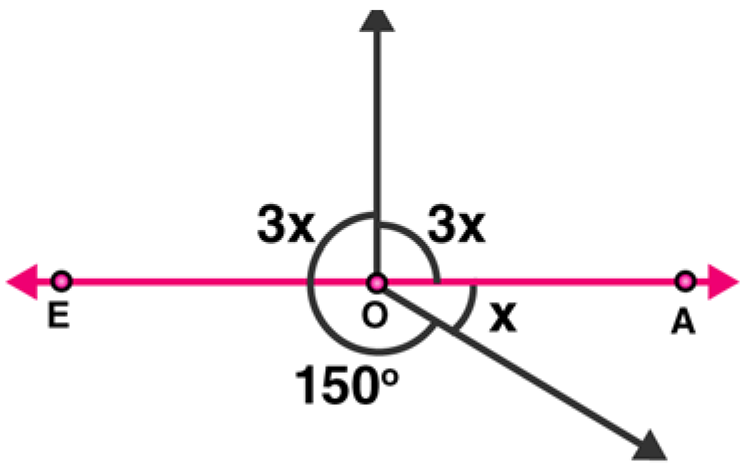
The 4 pair of adjacent angles are :
(∠AOD,∠DOB),(∠DOB,∠BOC),(∠COA, ∠AOD) and (∠BOC,∠COA).

Question 7: How many pairs of adjacent angles, in all, can you name in figure given?



Solution: Number of Pairs of adjacent angles, from the figure, are :
∠EOC and ∠DOC
∠EOD and ∠DOB
∠DOC and ∠COB
∠EOD and ∠DOA
∠DOC and ∠COA
∠BOC and ∠BOA
∠BOA and ∠BOD
∠BOA and ∠BOE
∠EOC and ∠COA
∠EOC and ∠COB
Hence, there are 10 pairs of adjacent angles.

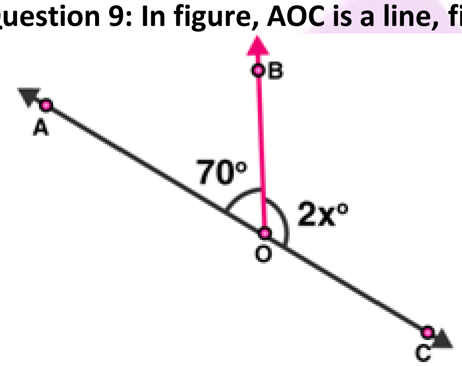
Question 8: In figure, determine the value of x.



Solution:

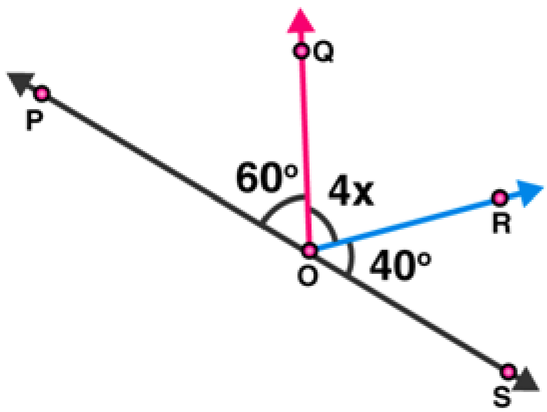
The sum of all the angles around a point O is equal to 360°.
Therefore,
$3x + 3x + 150 + x = 360^0$
$7x = 360^0 – 150^0$
$7x = 210^0$
$x = 210/7$
$x = 30^0$
Hence, the value of x is 30°.

Question 9: In figure, AOC is a line, find x.



Solution:
From the figure, ∠AOB and ∠BOC are linear pairs,
∠AOB +∠BOC =180°
$70 + 2x = 180$
$2x = 180 – 70$
$2x = 110$
$x = 110/2$
$x = 55$
Therefore, the value of x is 55^0.

Question 10: In figure, POS is a line, find x.



Solution:

From figure, ∠POQ and ∠QOS are linear pairs.

Therefore,

$\angle POQ + \angle QOS = 180^0$

$\angle POQ + \angle QOR + \angle SOR = 180^0$

$60^0 + 4x + 40^0 = 180^0$

$4x = 180^0 - 100^0$

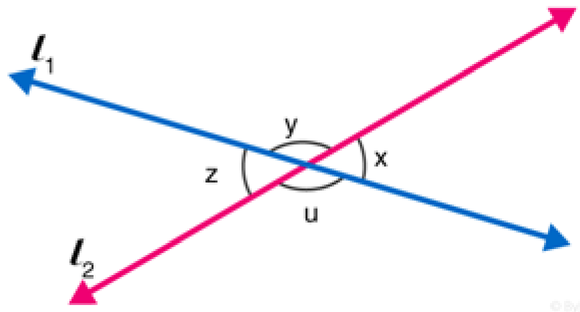
$4x = 80^0$

$x = 20^0$

Hence, the value of x is 20^0.

Exercise 8.3 Page No: 8.19

Question 1: In figure, lines l_1, and l_2 intersect at O, forming angles as shown in the figure. If x = 45. Find the values of y, z and u.



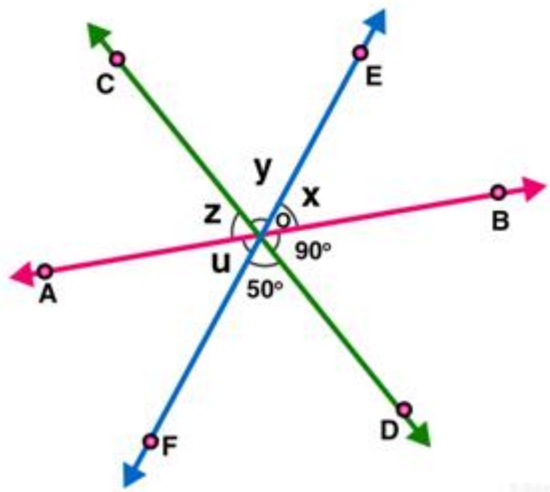
Solution:
Given: x = 45^0

Since vertically opposite angles are equal, therefore z = x = 45^0

z and u are angles that are a linear pair, therefore, z + u = 180^0
Solve, z + u = 180^0 , for u
u = 180^0 – z
u = 180^0 – 45
u = 135^0

Again, x and y angles are a linear pair.
x+ y = 180^0
y = 180^0 – x
y =180^0 – 45^0
y = 135^0
Hence, remaining angles are y = 135^0, u = 135^0 and z = 45^0.

Question 2 : In figure, three coplanar lines intersect at a point O, forming angles as shown in the figure. Find the values of x, y, z and u .



Solution:
(∠BOD, z); (∠DOF, y) are pair of vertically opposite angles.

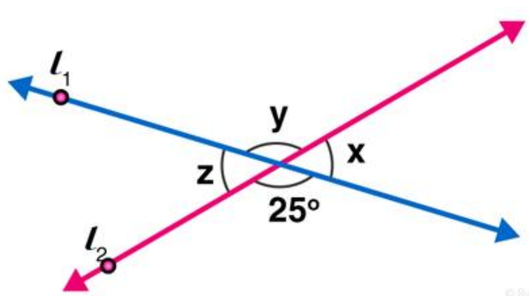
So, ∠BOD = z = 90^0

∠DOF = y = 50^0
[Vertically opposite angles are equal.]

Now, x + y + z = 180 [Linear pair]
[AB is a straight line]

x + y + z = 180
x + 50 + 90 = 180
x = 180 – 140
x = 40
Hence values of x, y, z and u are 40^0, 50^0, 90^0 and 40^0 respectively.

Question 3 : In figure, find the values of x, y and z.



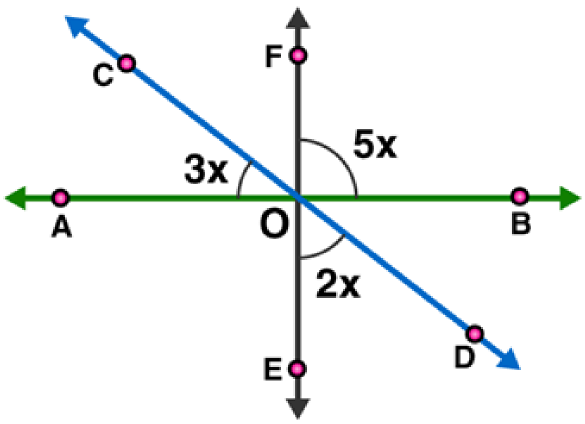
Solution:
From figure,

$y = 25^0$ [Vertically opposite angles are equal]
Now $\angle x + \angle y = 180^0$ [Linear pair of angles]

$x = 180 – 25$
$x = 155$

Also, $z = x = 155$ [Vertically opposite angles]
Answer: $y = 25^0$ and $z = 155^0$

Question 4 : In figure, find the value of x.



Solution:

$\angle AOE = \angle BOF = 5x$ [Vertically opposite angles]

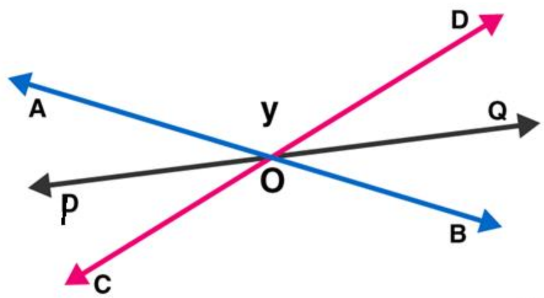
$\angle COA+\angle AOE+\angle EOD = 180^0$ [Linear pair]

$3x + 5x + 2x = 180$
$10x = 180$
$x = 180/10$
$x = 18$

The value of $x = 18^0$

Question 5 : Prove that bisectors of a pair of vertically opposite angles are in the same straight line.

Solution:



Lines AB and CD intersect at point O, such that

∠AOC = ∠BOD (vertically angles) …(1)

Also OP is the bisector of AOC and OQ is the bisector of BOD

To Prove: POQ is a straight line.

OP is the bisector of ∠AOC:
∠AOP = ∠COP …(2)
OQ is the bisector of ∠BOD:
∠BOQ = ∠QOD …(3)

Now,
Sum of the angles around a point is 360^o.

∠AOC + ∠BOD + ∠AOP + ∠COP + ∠BOQ + ∠QOD = 360^0

∠BOQ + ∠QOD + ∠DOA + ∠AOP + ∠POC + ∠COB = 360^0

2∠QOD + 2∠DOA + 2∠AOP = 360^0 (Using (1), (2) and (3))

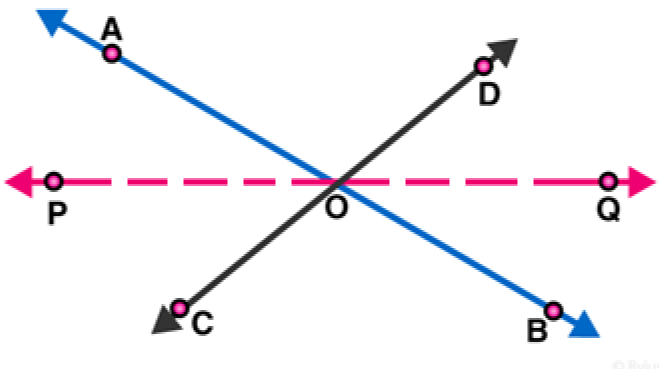
∠QOD + ∠DOA + ∠AOP = 180^0
POQ = 180^0

Which shows that, the bisectors of pair of vertically opposite angles are on the same straight line.

Hence Proved.

Question 6 : If two straight lines intersect each other, prove that the ray opposite to the bisector of one of the angles thus formed bisects the vertically opposite angle.

Solution: Given AB and CD are straight lines which intersect at O.
OP is the bisector of ∠ AOC.
To Prove : OQ is the bisector of ∠BOD
Proof :



AB, CD and PQ are straight lines which intersect in O.

Vertically opposite angles: ∠ AOP = ∠ BOQ

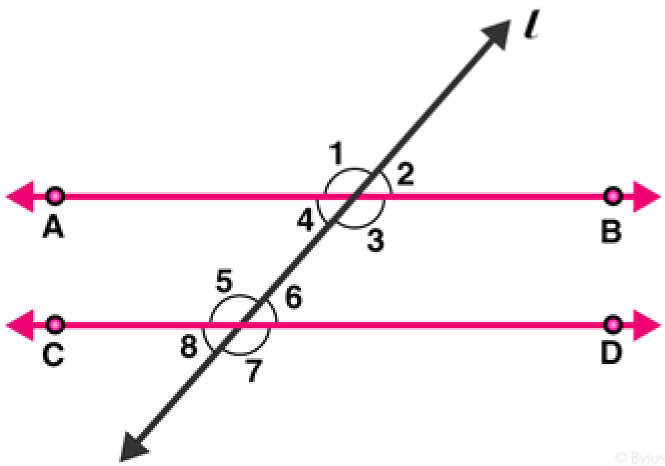
Vertically opposite angles: ∠ COP = ∠ DOQ

OP is the bisector of ∠ AOC : ∠ AOP = ∠ COP

Therefore, ∠BOQ = ∠ DOQ
Hence, OQ is the bisector of ∠BOD.

Exercise 8.4 Page No: 8.38

Question 1: In figure, AB, CD and ∠1 and ∠2 are in the ratio 3 : 2. Determine all angles from 1 to 8.



Solution:

Let ∠1 = 3x and ∠2 = 2x

From figure: ∠1 and ∠2 are linear pair of angles

Therefore, ∠1 + ∠2 = 180
3x + 2x = 180
5x = 180
x = 180 / 5
=> x = 36

So, ∠1 = 3x = 108^0 and ∠2 = 2x = 72^0

As we know, vertically opposite angles are equal.

Pairs of vertically opposite angles are:
(∠1 = ∠3); (∠2 = ∠4) ; (∠5, ∠7) and (∠6 , ∠8)

∠1 = ∠3 = 108°

∠2 = ∠4 = 72°

∠5 = ∠7

∠6 = ∠8

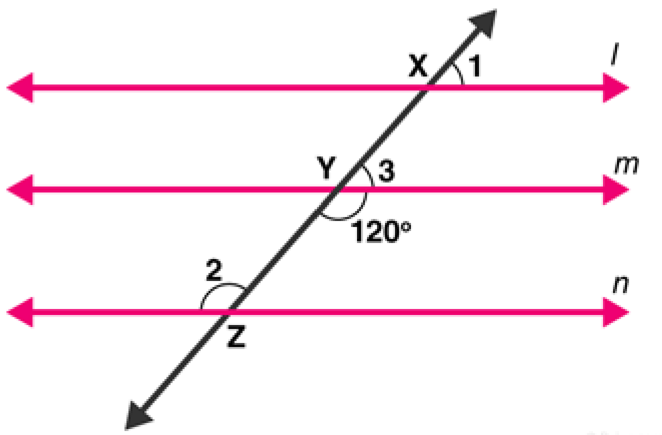
We also know, if a transversal intersects any parallel lines, then the corresponding angles are equal

∠1 = ∠5 = ∠7 = 108°

∠2 = ∠6 = ∠8 = 72°

Answer: ∠1 = 108°, ∠2 = 72°, ∠3 = 108°, ∠4 = 72°, ∠5 = 108°, ∠6 = 72°, ∠7 = 108° and ∠8 = 72°

Question 2: In figure, l, m and n are parallel lines intersected by transversal p at X, Y and Z respectively. Find ∠1, ∠2 and ∠3.



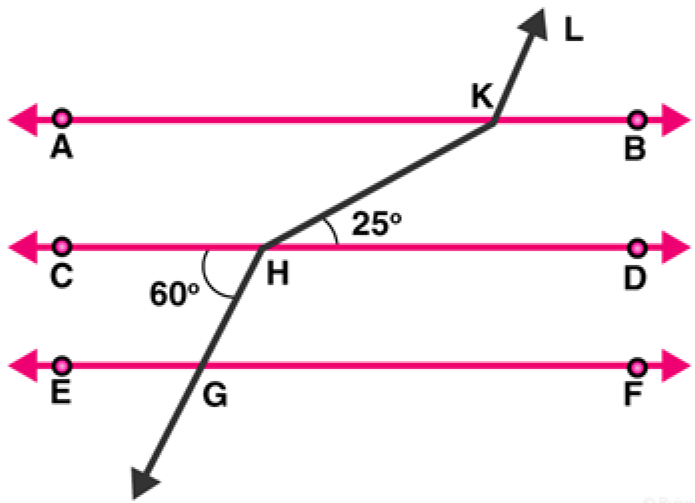
Solution: From figure:
∠Y = 120° [Vertical opposite angles]
∠3 + ∠Y = 180° [Linear pair angles theorem]
=> ∠3= 180 – 120
=> ∠3= 60°

Line l is parallel to line m,
∠1 = ∠3 [Corresponding angles]
∠1 = 60°

Also, line m is parallel to line n,
∠2 = ∠Y [Alternate interior angles are equal]
∠2 = 120°

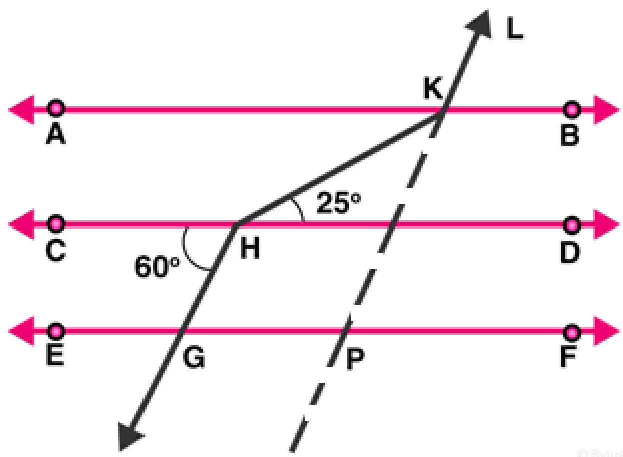
Answer: ∠1 = 60°, ∠2 = 120° and ∠3 = 60°.

Question 3: In figure, AB || CD || EF and GH || KL. Find ∠HKL.



Solution:

Extend LK to meet line GF at point P.



From figure, CD || GF, so, alternate angles are equal.

∠CHG =∠HGP = 60°

∠HGP =∠KPF = 60° [Corresponding angles of parallel lines are equal]

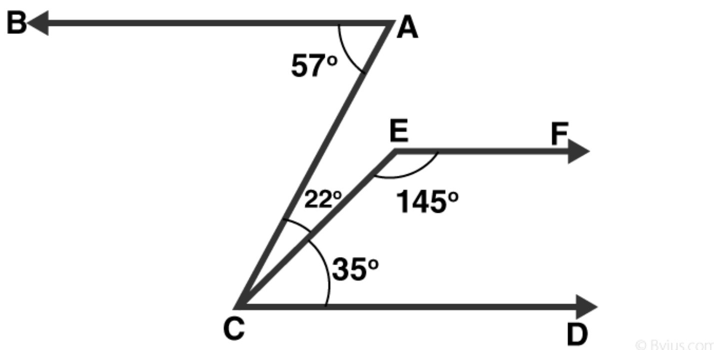
Hence, ∠KPG =180 – 60 = 120°

=> ∠GPK = ∠AKL= 120° [Corresponding angles of parallel lines are equal]

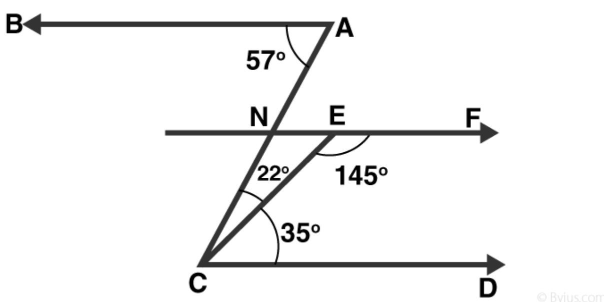
∠AKH = ∠KHD = 25° [alternate angles of parallel lines]

Therefore, ∠HKL = ∠AKH + ∠AKL = 25 + 120 = 145°

Question 4: In figure, show that AB || EF.



Solution: Produce EF to intersect AC at point N.



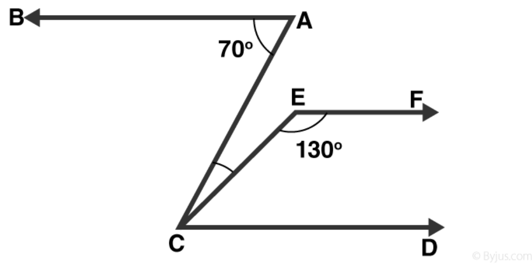
From figure, ∠BAC = 57° and
∠ACD = 22°+35° = 57°

Alternative angles of parallel lines are equal
=> BA || EF …..(1)

Sum of Co-interior angles of parallel lines is 180°
EF || CD
∠DCE + ∠CEF = 35 + 145 = 180° …(2)
From (1) and (2)
AB || EF
[Since, Lines parallel to the same line are parallel to each other]

Hence Proved.

Question 5 : In figure, if AB || CD and CD || EF, find ∠ACE.



Solution:

Given: CD || EF

∠ FEC + ∠ECD = 180°

[Sum of co-interior angles is supplementary to each other]

=> ∠ECD = 180° – 130° = 50°

Also, BA || CD

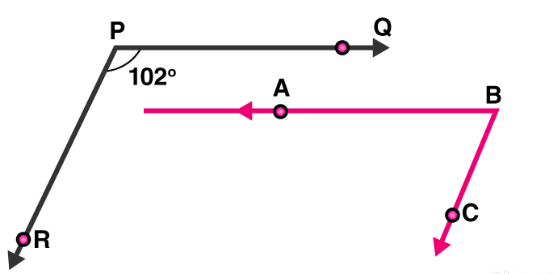
=> ∠BAC = ∠ACD = 70°

[Alternative angles of parallel lines are equal]

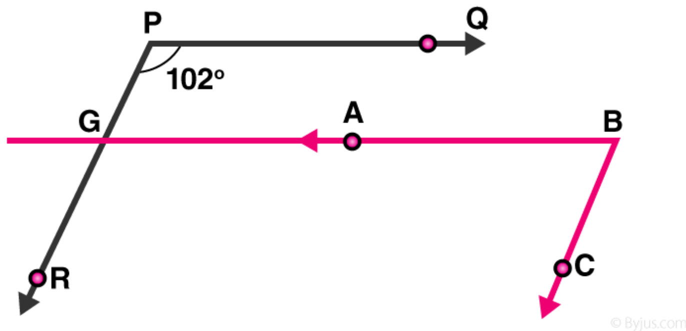
But, ∠ACE + ∠ECD =70°

=> ∠ACE = 70° — 50° = 20°

Question 6: In figure, PQ || AB and PR || BC. If ∠QPR = 102°, determine ∠ABC. Give reasons.



Solution: Extend line AB to meet line PR at point G.



Given: PQ || AB,

∠QPR = ∠BGR =102°
[Corresponding angles of parallel lines are equal]

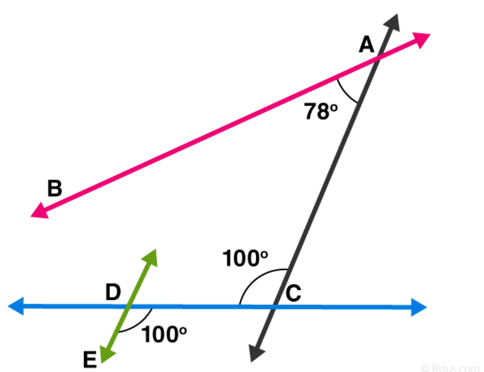
And PR || BC,
∠RGB+ ∠CBG =180°
[Corresponding angles are supplementary]

∠CBG = 180° – 102° = 78°

Since, ∠CBG = ∠ABC

=>∠ABC = 78°

Question 7 : In figure, state which lines are parallel and why?



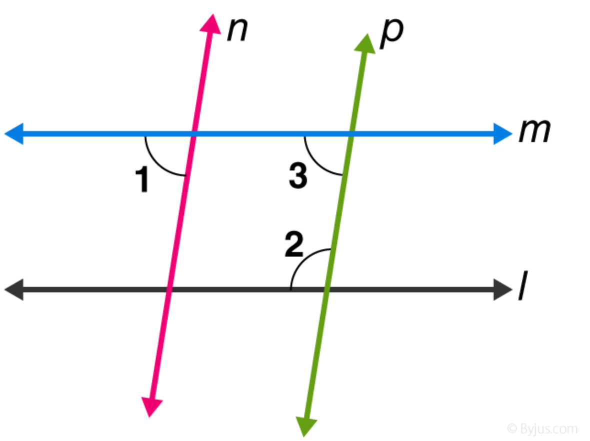
Solution:
We know, If a transversal intersects two lines such that a pair of alternate interior angles are equal, then the two lines are parallel

From figure:
=> ∠EDC = ∠DCA = 100°

Lines DE and AC are intersected by a transversal DC such that the pair of alternate angles are equal.

So, DE || AC

Question 8: In figure, if l||m, n || p and ∠1 = 85°, find ∠2.



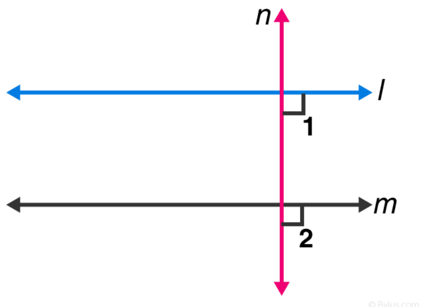
Solution:
Given: ∠1 = 85°
As we know, when a line cuts the parallel lines, the pair of alternate interior angles are equal.

=> ∠1 = ∠3 = 85°

Again, co-interior angles are supplementary, so
∠2 + ∠3 = 180°
∠2 + 55° =180°
∠2 = 180° – 85°
∠2 = 95°

Question 9 : If two straight lines are perpendicular to the same line, prove that they are parallel to each other.
Solution:
Let lines l and m are perpendicular to n, then

∠1= ∠2=90°



Since, lines l and m cut by a transversal line n and the corresponding angles are equal, which shows that, line l is parallel to line m.

Question 10: Prove that if the two arms of an angle are perpendicular to the two arms of another angle, then the angles are either equal or supplementary.

Solution: Let the angles be ∠ACB and ∠ABD

Let AC perpendicular to AB, and CD is perpendicular to BD.

To Prove : ∠ACD = ∠ABD OR ∠ACD + ∠ABD =180°

Proof :
In a quadrilateral,
∠A+ ∠C+ ∠D+ ∠B = 360°
[Sum of angles of quadrilateral is 360°]

=> 180° + ∠C + ∠B = 360°

=> ∠C + ∠B = 360° –180°

Therefore, ∠ACD + ∠ABD = 180°
And ∠ABD = ∠ACD = 90°

Hence, angles are equal as well as supplementary.

Exercise VSAQs Page No: 8.42

Question 1: Define complementary angles.

Solution: When the sum of two angles is 90 degrees, then the angles are known as complementary angles.

Question 2: Define supplementary angles.

Solution: When the sum of two angles is 180°, then the angles are known as supplementary angles.

Question 3: Define adjacent angles.

Solution: Two angles are Adjacent when they have a common side and a common vertex.

Question 4: The complement of an acute angle is ______.

Solution: An acute angle

Question 5: The supplement of an acute angle is ______.

Solution: An obtuse angle

Question 6: The supplement of a right angle is _____.

Solution: A right angle