Exercise 1.4 Page No: 1.30

Question 1: Define an irrational number.
Solution:
A number which cannot be expressed in the form of p/q, where p and q are integers and q ≠ 0. It is non-terminating or non-repeating decimal.

Question 2: Explain, how irrational numbers differ from rational numbers?
Solution:
An irrational number is a real number which can be written as a decimal but not as a fraction i.e. it cannot be expressed as a ratio of integers.
It cannot be expressed as terminating or repeating decimal.

For example, √2 is an irrational number

A rational number is a real number which can be written as a fraction and as a decimal i.e. it can be expressed as a ratio of integers.
It can be expressed as terminating or repeating decimal.

For examples: 0.10 and 5/3 are rational numbers

Question 3: Examine, whether the following numbers are rational or irrational:

(i) $\sqrt{7}$ (ii) $\sqrt{4}$ (iii) $2 + \sqrt{3}$ (iv) $\sqrt{3} + \sqrt{2}$

(v) $\sqrt{3} + \sqrt{5}$ (vi) $(\sqrt{2} - 2)^2$ (vii) $(2 - \sqrt{2})(2 + \sqrt{2})$

(viii) $(\sqrt{3} + \sqrt{2})^2$ (ix) $\sqrt{5} - 2$ (x) $\sqrt{23}$

(xi) $\sqrt{225}$ (xii) 0.3796 (xiii) 7.478478……

(xiv) 1.101001000100001……

Solution:
(i) √7
Not a perfect square root, so it is an irrational number.

(ii) √4
A perfect square root of 2.

We can express 2 in the form of 2/1, so it is a rational number.

(iii) $2 + \sqrt{3}$

Here, 2 is a rational number but $\sqrt{3}$ is an irrational number

Therefore, the sum of a rational and irrational number is an irrational number.

(iv) $\sqrt{3} + \sqrt{2}$

$\sqrt{3}$ is not a perfect square thus an irrational number.

$\sqrt{2}$ is not a perfect square, thus an irrational number.

Therefore, sum of $\sqrt{2}$ and $\sqrt{3}$ gives an irrational number.

(v) $\sqrt{3} + \sqrt{5}$

$\sqrt{3}$ is not a perfect square and hence, it is an irrational number

Similarly, $\sqrt{5}$ is not a perfect square and also an irrational number.

Since, sum of two irrational number, is an irrational number, therefore $\sqrt{3} + \sqrt{5}$ is an irrational number.

(vi) $(\sqrt{2} – 2)^2$

$(\sqrt{2} – 2)^2 = 2 + 4 – 4\sqrt{2}$

$= 6 - 4\sqrt{2}$

Here, 6 is a rational number but $4\sqrt{2}$ is an irrational number.

Since, the sum of a rational and an irrational number is an irrational number, therefore, $(\sqrt{2} – 2)2$ is an irrational number.

(vii) $(2 – \sqrt{2})(2 + \sqrt{2})$

We can write the given expression as;

$(2 – \sqrt{2})(2 + \sqrt{2}) = ((2)^2 – (\sqrt{2})^2)$

[Since, $(a + b)(a – b) = a^2 – b^2$]

= 4 – 2 = 2 or 2/1

Since, 2 is a rational number, therefore, $(2 - \sqrt{2})(2 + \sqrt{2})$ is a rational number.

(viii) $(\sqrt{3} + \sqrt{2})^2$

We can write the given expression as;

$(\sqrt{3} + \sqrt{2})^2 = (\sqrt{3})^2 + (\sqrt{2})^2 + 2\sqrt{3} \times \sqrt{2}$

$= 3 + 2 + 2\sqrt{6}$

$= 5 + 2\sqrt{6}$

[using identity, $(a+b)^2 = a^2 + 2ab + b^2$]

Since, the sum of a rational number and an irrational number is an irrational number, therefore, $(\sqrt{3} + \sqrt{2})^2$ is an irrational number.

(ix) $\sqrt{5} - 2$

$\sqrt{5}$ is an irrational number whereas 2 is a rational number.

The difference of an irrational number and a rational number is an irrational number.

Therefore, $\sqrt{5} - 2$ is an irrational number.

(x) $\sqrt{23}$

Since, $\sqrt{23} = 4.795831352331\ldots$

As decimal expansion of this number is non-terminating and non-recurring therefore, it is an irrational number.

(xi) $\sqrt{225}$

$\sqrt{225} = 15$ or 15/1

$\sqrt{225}$ is rational number as it can be represented in the form of p/q and q not equal to zero.

(xii) 0.3796

As the decimal expansion of the given number is terminating, therefore, it is a rational number.

(xiii) 7.478478……

As the decimal expansion of this number is non-terminating recurring decimal, therefore, it is a rational number.

(xiv) 1.101001000100001……

As the decimal expansion of given number is non-terminating and non-recurring, therefore, it is an irrational number

Question 4: Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers:

(i) $\sqrt{4}$ (ii) $3\sqrt{18}$ (iii) $\sqrt{1.44}$

(iv) $\sqrt{\frac{9}{27}}$ (v) $-\sqrt{64}$ (vi) $\sqrt{100}$

Solution:
(i) √4
√4 = 2, which can be written in the form of a/b. Therefore, it is a rational number.

Its decimal representation is 2.0.

(ii) 3√18

3√18 = 9√2

Since, the product of a rational and an irrational number is an irrational number.

Therefore, 3√18 is an irrational.

Or 3 × √18 is an irrational number.

(iii) √1.44

√1.44 = 1.2

Since, every terminating decimal is a rational number, Therefore, √1.44 is a rational number.

And, its decimal representation is 1.2.

(iv) √9/27

√9/27 = 1/√3

Since, we know, quotient of a rational and an irrational number is irrational numbers, therefore, √9/27 is an irrational number.

(v) – √64

– √64 = – 8 or – 8/1

Therefore, – √64 is a rational number.

Its decimal representation is –8.0.

(vi) √100

√100 = 10

Since, 10 can be expressed in the form of a/b, such as 10/1,

Therefore, √100 is a rational number.

And it's decimal representation is 10.0.

Question 5: In the following equation, find which variables x, y, z etc. represent rational or irrational numbers:

(i) $x^2 = 5$

(ii) $y^2 = 9$

(iii) $z^2 = 0.04$

(iv) $u^2 = 17/4$

(v) $v^2 = 3$

(vi) $w^2 = 27$

(vii) $t^2 = 0.4$

Solution:

(i) $x^2 = 5$

Taking square root both the sides,

$x = \sqrt{5}$

$\sqrt{5}$ is not a perfect square root, so it is an irrational number.

(ii) $y^2 = 9$

$y^2 = 9$
or $y = 3$

3 can be expressed in the form of a/b, such as 3/1, so it a rational number.

(iii) $z^2 = 0.04$

$z^2 = 0.04$

Taking square root both the sides, we get

$z = 0.2$

0.2 can be expressed in the form of a/b such as 2/10, so it is a rational number.

(iv) $u^2 = 17/4$

Taking square root both the sides, we get

$u = \sqrt{17}/2$

Since, quotient of an irrational and a rational number is irrational, therefore, u is an Irrational number.

(v) $v^2 = 3$

Taking square root both the sides, we get

$v = \sqrt{3}$

Since, $\sqrt{3}$ is not a perfect square root, so v is irrational number.

(vi) $w^2 = 27$

Taking square root both the sides, we get

$w = 3\sqrt{3}$

Since, the product of a rational and irrational is an irrational number. Therefore, w is an irrational number.

(vii) $t^2 = 0.4$

Taking square root both the sides, we get

$t = \sqrt{(4/10)}$

$t = 2/\sqrt{10}$

Since, quotient of a rational and an irrational number is irrational number. Therefore, t is an irrational number.