

Exercise 1.1

Page No: 1.5

Question 1: Is zero a rational number? Can you write it in the form p/q, where p and q are integers and $q \neq 0$?

Solution:

Yes, zero is a rational number.

It can be written in p/q form provided that $q \neq 0$.

For Example: 0/1 or 0/3 or 0/4 etc.

Question 2: Find five rational numbers between 1 and 2.

Solution:

We know, one rational number between two numbers m and n = (m+n)/2

To find: 5 rational numbers between 1 and 2

Step 1: Rational number between 1 and 2

=(1+2)/2

= 3/2

Step 2: Rational number between 1 and 3/2

=(1+3/2)/2

= 5/4

Step 3: Rational number between 1 and 5/4

=(1+5/4)/2

= 9/8

Step 4: Rational number between 3/2 and 2

= 1/2 [(3/2) + 2)]

= 7/4



Step 5: Rational number between 7/4 and 2

= 1/2 [7/4 + 2]

= 15/8

Arrange all the results: 1 < 9/8 < 5/4 < 3/2 < 7/4 < 15/8 < 2

Therefore required integers are, 9/8, 5/4, 3/2, 7/4, 15/8

Question 3: Find six rational numbers between 3 and 4.

Solution:

Steps to find n rational numbers between any two numbers:

Step 1: Multiply and divide both the numbers by n+1.

In this example, we have to find 6 rational numbers between 3 and 4. Here n = 6

Multiply 3 and 4 by 7

 $3 \times 7/7 = 21/7$ and

 $4 \times 7/7 = 28/7$

Step 2: Choose 6 numbers between 21/7 and 28/7

3 = 21/7 < 22/7 < 23/7 < 24/7 < 25/7 < 26/7 < 27/7 < 28/7 = 4

Therefore, 6 rational numbers between 3 and 4 are

22/7, 23/7, 24/7, 25/7, 26/7, 27/7

Question 4: Find five rational numbers between 3/5 and 4/5.

Solution:

Steps to find n rational numbers between any two numbers:

Step 1: Multiply and divide both the numbers by n+1.



In this example, we have to find 5 rational numbers between 3/5 and 4/5. Here n = 5

Multiply 3/5 and 4/5 by 6

 $3/5 \times 6/6 = 18/30$ and

 $4/5 \times 6/6 = 24/30$

Step 2: Choose 5 numbers between 18/30 and 24/30

3/5 = 18/30 < 19/30 < 20/30 < 21/30 < 22/30 < 23/30 < 24/30 = 4/5

Therefore, 5 rational numbers between 3/5 and 4/5 are

19/30, 20/30, 21/30, 22/30, 23/30

Question 5: Are the following statements true or false? Give reason for your answer.

- (i) Every whole number is a natural number.
- (ii) Every integer is a rational number.
- (iii) Every rational number is an integer.
- (iv) Every natural number is a whole number,
- (v) Every integer is a whole number.
- (vi) Every rational number is a whole number.

Solution:

(i) False.

Reason: As 0 is not a natural number.

- (ii) True.
- (iii) False.

Reason: Numbers such as 1/2, 3/2, 5/3 are rational numbers but not integers.



(iv) True.

(v) False.

Reason: Negative numbers are not whole numbers.

(vi) False.

Reason: Proper fractions are not whole numbers





Exercise 1.2

Page No: 1.12

Question 1: Express the following rational numbers as decimals. (i) 42/100 (ii) 327/500 (iii) 15/4

Solution:

(i) By long division method

100) 42 (0.42

400

200

200

 $\bar{0}$

Therefore, $\frac{42}{100} = 0.42$

(ii) By long division method

500) 327.000 (0.654

3000

2700

2500

2000

2000

 $\overline{0}$

Therefore, $\frac{327}{500} = 0.654$

(iii) By long division method

12

30

28

20

20

 $\overline{0}$

Therefore, $\frac{15}{4}$ = 3.75

Question 2: Express the following rational numbers as decimals. (i) 2/3 (ii) -4/9 (iii) -2/15 (iv) -22/13 (v) 437/999 (vi) 33/26 Solution:

(i) Divide 2/3 using long division:

$$\frac{2}{3} = 0.666... = 0.\overline{6}$$

(ii) Divide using long division: -4/9

- 9) 4.000 (0.444
- 3600
- 4000
- 3600
- 4000
- 3600
- 400

$$-\frac{4}{9} = -0.4444... = -0.\overline{4}$$

(iii) Divide using long division: -2/15

$$-\frac{2}{15} = -0.133 = -0.1\overline{3}$$

(iv) Divide using long division: -22/13

 $\begin{array}{c}
1.69230769 \\
13) \overline{22.000} \\
\underline{13} \\
\underline{90} \\
78 \\
\overline{120} \\
\underline{117} \\
30 \\
\underline{26} \\
40 \\
\underline{39} \\
\overline{100} \\
\underline{91} \\
\underline{90} \\
78 \\
\overline{120} \\
\underline{117} \\
\overline{3}
\end{array}$

 $-\frac{22}{13} = -1.6923076923... = -1.\overline{692307}$

(v) Divide using long division: 437/999

999) 437.0000 (0.43743

3996

3740

2997

7430

6993

4370

3996

3740

2997

743

 $\frac{437}{999} = 0.43743... = 0.\overline{437}$



(vi) Divide using long division: 33/26

```
1.2692307692
26)33.000000000
     70
     52
     180
     156
      240
      234
       60
        52
          80
          78
           200
             180
             156
              24
      \frac{33}{26} = 1.269230769... = 1.\overline{2692307}
```

Question 3: Look at several examples of rational numbers in the form p/q ($q \ne 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations. Can you guess what property q must satisfy?

Solution:

The decimal representation will be terminating, if the denominators have factors 2 or 5 or both. Therefore, p/q is a terminating decimal, when prime factorization of q must have only powers of 2 or 5 or both.

Exercise 1.3 Page No: 1.22

Question 1: Express each of the following decimals in the form p/q:

- (i) 0.39
- (ii) 0.750
- (iii) 2.15
- (iv) 7.010
- (v) 9.90
- (vi) 1.0001

Solution:

- 0.39 = 39/100
- (ii)
- 0.750 = 750/1000 = 3/4
- (iii)
- 2.15 = 215/100 = 43/20
- (iv)
- 7.010 = 7010/1000 = 701/100
- 9.90 = 990/100 = 99/10
- (vi)
- 1.0001 = 10001/10000

Question 2: Express each of the following decimals in the form p/q:

- (i) $0.\overline{4}$
- (ii) $0.\overline{37}$
- (iii) 0.54 (iv) 0.621
- (v) 125. $\bar{3}$
 - $(vi) \ 4.7$
- (vii) 0.47

Solution:

- (i) Let x = 0.4
- or $x = 0.4 = 0.444 \dots (1)$

Multiplying both sides by 10

- 10x = 4.444(2)
- Subtract (1) by (2), we get
- 10x x = 4.444... 0.444...
- 9x = 4
- x = 4/9
- => 0.4 = 4.9

- (ii) Let x = 0.3737.. (1) Multiplying both sides by 100 100x = 37.37...(2)
- Subtract (1) from (2), we get
- 100x x = 37.37... 0.3737...
- 100x x = 37
- 99x = 37
- x = 37/99
- (iii) Let x = 0.5454... (1)
- Multiplying both sides by 100
- 100x = 54.5454... (2)
- Subtract (1) from (2), we get
- 100x x = 54.5454.... 0.5454....
- 99x = 54
- x = 54/99
- (iv) Let x = 0.621621... (1)
- Multiplying both sides by 1000
- 1000x = 621.621621.... (2)
- Subtract (1) from (2), we get
- 1000x x = 621.621621.... 0.621621....
- 999x = 621
- x = 621/999
- or x = 23/37
- (v) Let x = 125.3333....(1)
- Multiplying both sides by 10
- 10x = 1253.3333.... (2)
- Subtract (1) from (2), we get
- 10x x = 1253.3333.... 125.3333....
- 9x = 1128
- or x = 1128/9
- or x = 376/3
- (vi) Let x = 4.7777.... (1)
- Multiplying both sides by 10
- 10x = 47.7777... (2)



Subtract (1) from (2), we get

10x - x = 47.7777.... - 4.7777.... 9x = 43x = 43/9

(vii) Let x = 0.47777...Multiplying both sides by 10 10x = 4.7777... ...(1) Multiplying both sides by 100 100x = 47.7777... (2) Subtract (1) from (2), we get 100x - 10x = 47.7777... - 4.7777... 90x = 43x = 43/90



Page No: 1.30 Exercise 1.4

Question 1: Define an irrational number.

Solution:

A number which cannot be expressed in the form of p/q, where p and q are integers and $q \neq 0$. It is non-terminating or non-repeating decimal.

Question 2: Explain, how irrational numbers differ from rational numbers? **Solution:**

An irrational number is a real number which can be written as a decimal but not as a fraction i.e. it cannot be expressed as a ratio of integers.

It cannot be expressed as terminating or repeating decimal.

For example, $\sqrt{2}$ is an irrational number

A rational number is a real number which can be written as a fraction and as a decimal i.e. it can be expressed as a ratio of integers.

It can be expressed as terminating or repeating decimal.

For examples: 0.10 and 5/3 are rational numbers

Question 3: Examine, whether the following numbers are rational or irrational:

(i) √7

(iv)
$$\sqrt{3} + \sqrt{2}$$

(v) $\sqrt{3} + \sqrt{5}$

(vi)
$$(\sqrt{2} - 2)^2$$

(vii)
$$(2 - \sqrt{2})(2 + \sqrt{2})$$

(viii) $(\sqrt{3} + \sqrt{2})^2$ (ix) $\sqrt{5} - 2$

(xi) √225

(xiv) 1.101001000100001......

Solution:

(i) √7

Not a perfect square root, so it is an irrational number.

(ii) √4

A perfect square root of 2.

We can express 2 in the form of 2/1, so it is a rational number.



(iii) 2 +
$$\sqrt{3}$$

Here, 2 is a rational number but $\sqrt{3}$ is an irrational number

Therefore, the sum of a rational and irrational number is an irrational number.

(iv)
$$\sqrt{3} + \sqrt{2}$$

V3 is not a perfect square thus an irrational number.

V2 is not a perfect square, thus an irrational number.

Therefore, sum of V2 and V3 gives an irrational number.

(v)
$$\sqrt{3} + \sqrt{5}$$

V3 is not a perfect square and hence, it is an irrational number

Similarly, V5 is not a perfect square and also an irrational number.

Since, sum of two irrational number, is an irrational number, therefore $\sqrt{3} + \sqrt{5}$ is an irrational number.

(vi)
$$(\sqrt{2} - 2)^2$$

$$(\sqrt{2}-2)^2 = 2 + 4 - 4\sqrt{2}$$

$$= 6 - 4 \sqrt{2}$$

Here, 6 is a rational number but 4V2 is an irrational number.

Since, the sum of a rational and an irrational number is an irrational number, therefore, $(\sqrt{2} - 2)2$ is an irrational number.

(vii)
$$(2 - \sqrt{2})(2 + \sqrt{2})$$

We can write the given expression as;

$$(2 - \sqrt{2})(2 + \sqrt{2}) = ((2)^2 - (\sqrt{2})^2)$$

[Since,
$$(a + b)(a - b) = a^2 - b^2$$
]



$$= 4 - 2 = 2 \text{ or } 2/1$$

Since, 2 is a rational number, therefore, $(2 - \sqrt{2})(2 + \sqrt{2})$ is a rational number.

(viii) $(\sqrt{3} + \sqrt{2})^2$

We can write the given expression as;

 $(\sqrt{3} + \sqrt{2})^2 = (\sqrt{3})^2 + (\sqrt{2})^2 + 2\sqrt{3} \times \sqrt{2}$

 $= 3 + 2 + 2\sqrt{6}$

 $= 5 + 2\sqrt{6}$

[using identity, $(a+b)^2 = a^2 + 2ab + b^2$]

Since, the sum of a rational number and an irrational number is an irrational number, therefore, $(\sqrt{3} + \sqrt{2})^2$ is an irrational number.

(ix) $\sqrt{5} - 2$

√5 is an irrational number whereas 2 is a rational number.

The difference of an irrational number and a rational number is an irrational number.

Therefore, $\sqrt{5} - 2$ is an irrational number.

(x) √23

Since, $\sqrt{23} = 4.795831352331...$

As decimal expansion of this number is non-terminating and non-recurring therefore, it is an irrational number.

(xi) √225

 $\sqrt{225} = 15 \text{ or } 15/1$

 $\sqrt{225}$ is rational number as it can be represented in the form of p/q and q not equal to zero.



(xii) 0.3796

As the decimal expansion of the given number is terminating, therefore, it is a rational number.

(xiii) 7.478478......

As the decimal expansion of this number is non-terminating recurring decimal, therefore, it is a rational number.

(xiv) 1.101001000100001......

As the decimal expansion of given number is non-terminating and non-recurring, therefore, it is an irrational number

Question 4: Identify the following as rational or irrational numbers. Give the decimal representation of rational numbers:

(iv)
$$\sqrt{\frac{9}{27}}$$
 (v) - $\sqrt{64}$ (vi) $\sqrt{100}$

Solution:

(i) √4

 $\sqrt{4}$ = 2, which can be written in the form of a/b. Therefore, it is a rational number.

Its decimal representation is 2.0.

(ii) 3V18

 $3\sqrt{18} = 9\sqrt{2}$

Since, the product of a rational and an irrational number is an irrational number.

Therefore, 3V18 is an irrational.

Or $3 \times \sqrt{18}$ is an irrational number.

(iii) √1.44

 $\sqrt{1.44} = 1.2$

Since, every terminating decimal is a rational number, Therefore, V1.44 is a rational number.



And, its decimal representation is 1.2.

$$\sqrt{9/27} = 1/\sqrt{3}$$

Since, we know, quotient of a rational and an irrational number is irrational numbers, therefore, $\sqrt{9/27}$ is an irrational number.

$$-\sqrt{64} = -8 \text{ or } -8/1$$

Therefore, $-\sqrt{64}$ is a rational number.

Its decimal representation is -8.0.

(vi) √100

 $\sqrt{100} = 10$

Since, 10 can be expressed in the form of a/b, such as 10/1,

Therefore, V100 is a rational number.

And it's decimal representation is 10.0.

Question 5: In the following equation, find which variables x, y, z etc. represent rational or irrational numbers:

(i)
$$x^2 = 5$$

(ii)
$$y^2 = 9$$

(iii)
$$z^2 = 0.04$$

(iv)
$$u^2 = 17/4$$

$$(v) v^2 = 3$$

$$(vi) w^2 = 27$$

(vii)
$$t^2 = 0.4$$

Solution:



(i)
$$x^2 = 5$$

Taking square root both the sides,

x = √5

√5 is not a perfect square root, so it is an irrational number.

(ii)
$$y^2 = 9$$

$$y^2 = 9$$

or
$$y = 3$$

3 can be expressed in the form of a/b, such as 3/1, so it a rational number.

(iii)
$$z^2 = 0.04$$

$$z^2 = 0.04$$

Taking square root both the sides, we get

$$z = 0.2$$

0.2 can be expressed in the form of a/b such as 2/10, so it is a rational number.

(iv)
$$u^2 = 17/4$$

Taking square root both the sides, we get

$$u = \sqrt{17/2}$$

Since, quotient of an irrational and a rational number is irrational, therefore, u is an Irrational number.

Taking square root both the sides, we get

$$v = \sqrt{3}$$

Since, $\sqrt{3}$ is not a perfect square root, so v is irrational number.



(vi)
$$w^2 = 27$$

Taking square root both the sides, we get

 $w = 3\sqrt{3}$

Since, the product of a rational and irrational is an irrational number. Therefore, w is an irrational number.

(vii) $t^2 = 0.4$

Taking square root both the sides, we get

 $t = \sqrt{(4/10)}$

 $t = 2/\sqrt{10}$

Since, quotient of a rational and an irrational number is irrational number. Therefore, t is an irrational number.

Exercise 1.5 Page No: 1.35

Question 1: Complete the following sentences:

- (i) Every point on the number line corresponds to a number which many be either or
- (ii) The decimal form of an irrational number is neither nor
- (iii) The decimal representation of a rational number is either or
- (iv) Every real number is either ... number or ... number.

Solution:

- (i) Every point on the number line corresponds to a <u>real</u> number which many be either <u>rational</u> or irrational.
- (ii) The decimal form of an irrational number is neither terminating nor repeating.
- (iii) The decimal representation of a rational number is either <u>terminating</u> or <u>non-terminating</u> <u>recurring</u>.
- (iv) Every real number is either rational number or an irrational number.

Question 2: Represent $\sqrt{6}, \sqrt{7}, \sqrt{8}$ on the number line.

Solution:

Find the equivalent values of $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$

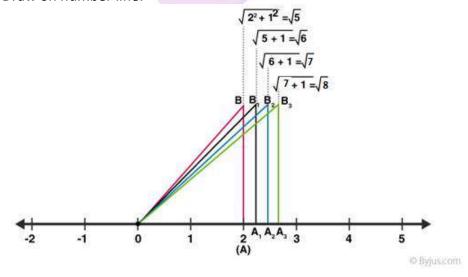
 $\sqrt{6} = 2.449$

 $\sqrt{7} = 2.645$

 $\sqrt{8} = 2.828$

We can see that, all the given numbers lie between 2 and 3.

Draw on number line:



Question 3: Represent $\sqrt{3.5}, \sqrt{9.4}, \sqrt{10.5}$ and on the real number line.

Solution:

Represent **V3.5** on number line

Step 1: Draw a line segment AB = 3.5 units

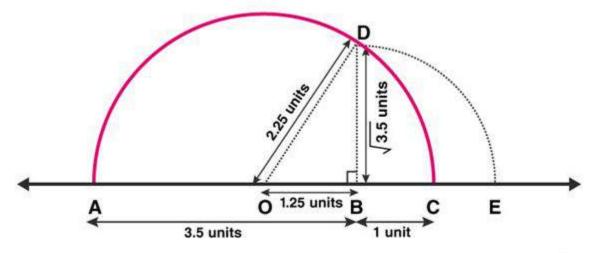
Step 2: Produce B till point C, such that BC = 1 unit

Step 3: Find the mid-point of AC, say O.

Step 4: Taking O as the centre draw a semi circle, passing through A and C.

Step 5: Draw a line passing through B perpendicular to OB, and cut semicircle at D.

Step 6: Consider B as a centre and BD as radius draw an arc cutting OC produced at E.



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Now, from right triangle OBD,

$$BD^2 = OD^2 - OB^2$$

$$= OC^2 - (OC - BC)^2$$

$$(As, OD = OC)$$

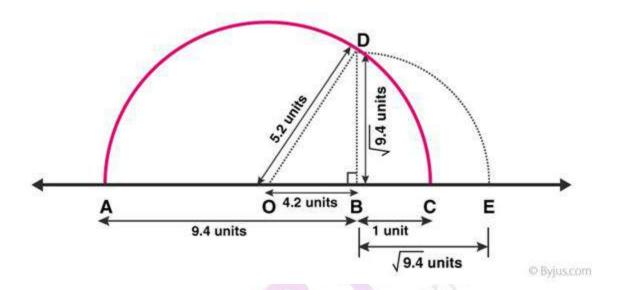
$$BD^2 = 2OC \times BC - (BC)^2$$

$$=> BD = \sqrt{3.5}$$

Represent v9.4 on number line

Step 1: Draw a line segment AB = 9.4 units

Follow step 2 to Step 6 mentioned above.



 $BD^2 = 2OC \times BC - (BC)^2$

= 2 x 5.2 x 1 - 1

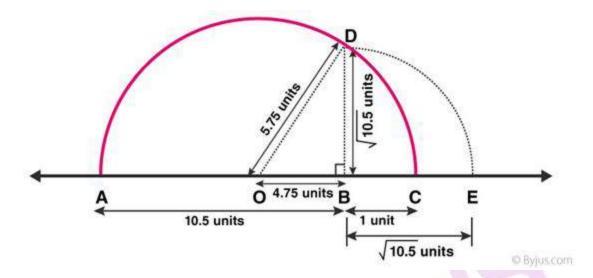
= 9.4

 $=> BD = \sqrt{9.4}$

Represent √10.5 on number line

Step 1: Draw a line segment AB = 10.5 units

Follow step 2 to Step 6 mentioned above, we get



 $BD^2 = 2OC \times BC - (BC)^2$

= 10.5

 $=> BD = \sqrt{10.5}$

Question 4: Find whether the following statements are true or false:

- (i) Every real number is either rational or irrational.
- (ii) π is an irrational number.
- (iii) Irrational numbers cannot be represented by points on the number line.

Solution:

- (i) True.
- (ii) True.
- (ii) False.



Exercise 1.6 Page No: 1.39

Question 1: Visualise 2.665 on the number line, using successive magnification.

Solution:

2.665 is lies between 2 and 3 on the number line.

Divide selected segment into 10 equal parts and mark each point of division as 2.1, 2.2,, 2.9, 2.10

2.665 is lies between 2.6 and 2.7

Divide line segment between 2.6 and 2.7 in 10 equal parts such as 2.661, 2.662, and so on.

Here we can see that 5th point will represent 2.665.

Question 2: Visualise the representation of 5.37 on the number line upto 5 decimal places, that is upto 5.37777.

Solution:

Clearly 5.37 is located between 5 and 6.

Again by successive magnification, and successively decrease 5.37 located between 5.3 and 5.4.

For more clarity, divide 5.3 and 5.4 portion of the number line into 10 equal parts and we can see 5.37 lies between 5.37 and 5.38.

To visualize 5.37 more accurately, divide line segment between 5.37 and 5.38 in ten equal parts. 5.37 lies between 5.377 and 5.378.

Again divide above portion between 5.377 and 5.378 into 10 equal parts, which shows 5.37 is located closer to 5.3778 than to 5.3777



