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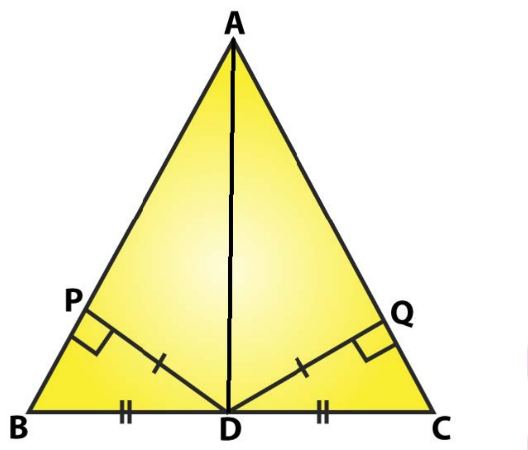
Question 1: ABC is a triangle and D is the mid-point of BC. The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.

Solution:

Given: D is the midpoint of BC and PD = DQ in a triangle ABC.

To prove: ABC is isosceles triangle.



In △BDP and △CDQ
PD = QD (Given)
BD = DC (D is mid-point)
$\angle BPD = \angle CQD = 90^{o}$

By RHS Criterion: △BDP ≅ △CDQ

BP = CQ … (i) (By CPCT)

In △APD and △AQD

PD = QD (given)
AD = AD (common)
APD = AQD = 90^{o}

By RHS Criterion: △APD ≅ △AQD
So, PA = QA … (ii) (By CPCT)

Adding (i) and (ii)

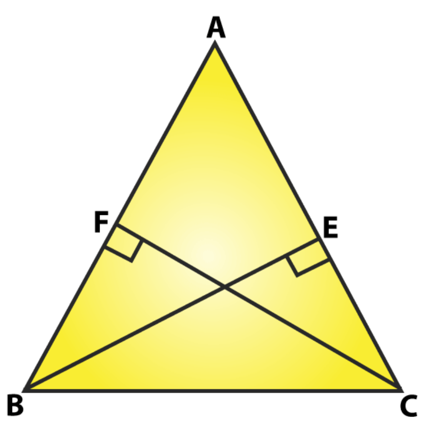
BP + PA = CQ + QA

AB = AC
Two sides of the triangle are equal, so ABC is an isosceles.

Question 2: ABC is a triangle in which BE and CF are, respectively, the perpendiculars to the sides AC and AB. If BE = CF, prove that Δ ABC is isosceles

Solution:
ABC is a triangle in which BE and CF are perpendicular to the sides AC and AB respectively s.t. BE = CF.

To prove: Δ ABC is isosceles



In Δ BCF and Δ CBE,
∠ BFC = CEB = 90° [Given]

BC = CB [Common side]

And CF = BE [Given]

By RHS congruence criterion: ΔBFC ≅ ΔCEB

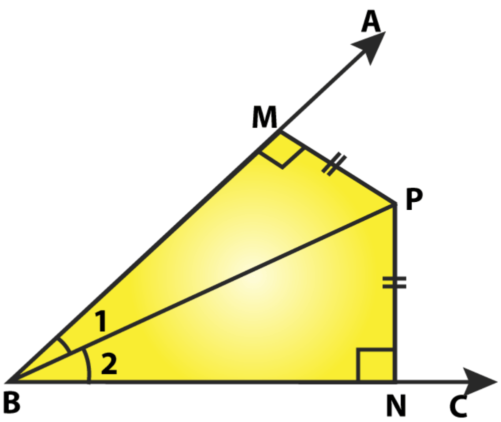
So, ∠ FBC = ∠ EBC [By CPCT]

=>∠ ABC = ∠ ACB

AC = AB [Opposite sides to equal angles are equal in a triangle]
Two sides of triangle ABC are equal.
Therefore, Δ ABC is isosceles. Hence Proved.

Question 3: If perpendiculars from any point within an angle on its arms are congruent. Prove that it lies on the bisector of that angle.

Solution:



Consider an angle ABC and BP be one of the arm within the angle.

Draw perpendiculars PN and PM on the arms BC and BA.

In Δ BPM and Δ BPN,

∠ BMP = ∠ BNP = 90° [given]

BP = BP [Common side]

MP = NP [given]

By RHS congruence criterion: ΔBPM≅ΔBPN

So, ∠ MBP = ∠ NBP [By CPCT]

BP is the angular bisector of ∠ABC.
Hence proved