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Question 1: In a ΔABC, D, E and F are, respectively, the mid points of BC, CA and AB. If the lengths of sides AB, BC and CA are 7 cm, 8 cm and 9 cm, respectively, find the perimeter of ΔDEF.

Solution:

Given: AB = 7 cm, BC = 8 cm, AC = 9 cm

In ΔABC,

In a ΔABC, D, E and F are, respectively, the mid points of BC, CA and AB.

According to Midpoint Theorem:

EF = 1/2BC, DF = 1/2 AC and DE = 1/2 AB

Now, Perimeter of ΔDEF = DE + EF + DF

= 1/2 (AB + BC + AC)

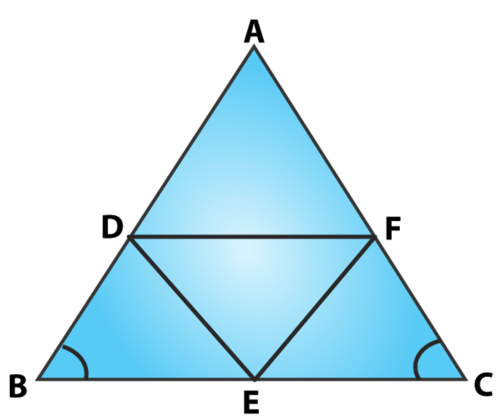
= 1/2 (7 + 8 + 9)

= 12

Perimeter of ΔDEF = 12cm

Question 2: In a ΔABC, ∠A = 50^0, ∠B = 60^0 and ∠C = 70^0. Find the measures of the angles of the triangle formed by joining the mid-points of the sides of this triangle.

Solution:



In ΔABC,

D, E and F are mid points of AB,BC and AC respectively.

In a Quadrilateral DECF:

By Mid-point theorem,

DE || AC => DE = AC/2

And CF = AC/2
=> DE = CF

Therefore, DECF is a parallelogram.

$\angle C = \angle D = 70^0$
[Opposite sides of a parallelogram]

Similarly,

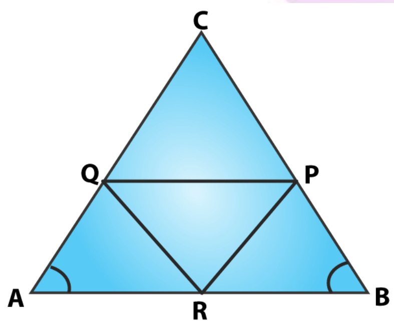
ADEF is a parallelogram, $\angle A = \angle E = 50^0$

BEFD is a parallelogram, $\angle B = \angle F = 60^0$

Hence, Angles of ΔDEF are: $\angle D = 70^0$, $\angle E = 50^0$, $\angle F = 60^0$.

Question 3: In a triangle, P, Q and R are the mid points of sides BC, CA and AB respectively. If AC = 21 cm, BC = 29 cm and AB = 30 cm, find the perimeter of the quadrilateral ARPQ.

Solution:



In ΔABC,

R and P are mid points of AB and BC

By Mid-point Theorem

RP ∥ AC => RP = AC/2

In a quadrilateral, ARPQ

RP ∥ AQ => RP = AQ
[A pair of side is parallel and equal]

Therefore, ARPQ is a parallelogram.

Now, AR = AB/2 = 30/2 = 15 cm
[AB = 30 cm (Given)]

AR = QP = 15 cm
[Opposite sides are equal]

And RP = AC/2 = 21/2 = 10.5 cm
[AC = 21 cm (Given)]

RP = AQ = 10.5cm
[Opposite sides are equal]

Now,

Perimeter of ARPQ = AR + QP + RP +AQ

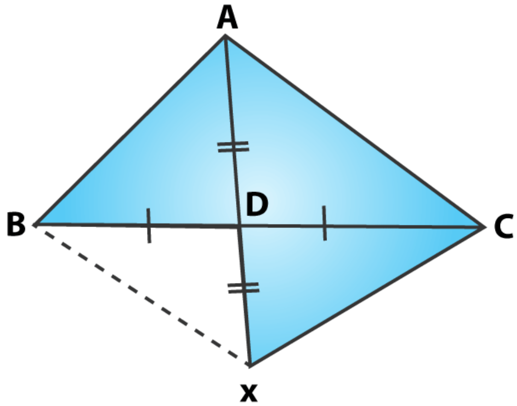
= 15 +15 +10.5 +10.5

= 51

Perimeter of quadrilateral ARPQ is 51 cm.

Question 4: In a ΔABC median AD is produced to X such that AD = DX. Prove that ABXC is a parallelogram.

Solution:



In a quadrilateral ABXC,

AD = DX [Given]

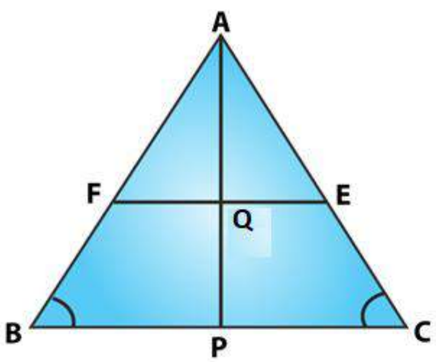
BD = DC [Given]

From figure, Diagonals AX and BC bisect each other.

ABXC is a parallelogram.
Hence Proved.

Question 5: In a ΔABC, E and F are the mid-points of AC and AB respectively. The altitude AP to BC intersects FE at Q. Prove that AQ = QP.

Solution:

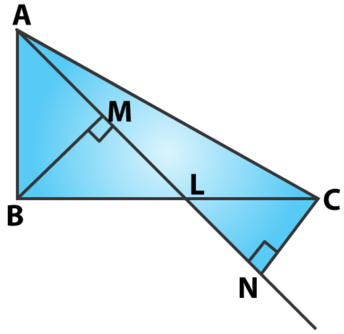


In a ΔABC
E and F are mid points of AC and AB (Given)
EF ǁ BC => EF = BC/2 and
[By mid-point theorem]
In ΔABP
F is the mid-point of AB, again by mid-point theorem
FQ ǁ BP

Q is the mid-point of AP
AQ = QP
Hence Proved.

Question 6: In a ΔABC, BM and CN are perpendiculars from B and C respectively on any line passing through A. If L is the mid-point of BC, prove that ML = NL.

Solution:



Given that,

In ΔBLM and ΔCLN

∠BML = ∠CNL = 90^0

BL = CL [L is the mid-point of BC]

∠MLB = ∠NLC [Vertically opposite angle]

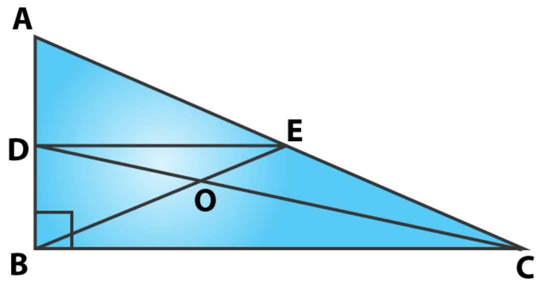
By ASA criterion:

ΔBLM ≅ ΔCLN

So, LM = LN [By CPCT]

Question 7: In figure, triangle ABC is a right-angled triangle at B. Given that AB = 9 cm, AC = 15 cm and D, E are the mid-points of the sides AB and AC respectively, calculate

(i) The length of BC **(ii) The area of ΔADE.**



Solution:

In ΔABC,
∠B=90^0 (Given)
AB = 9 cm, AC = 15 cm (Given)

By using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$=>15^2 = 9^2 + BC^2$

$=>BC^2 = 225 – 81 = 144$

or BC = 12

Again,

AD = DB = AB/2 = 9/2 = 4.5 cm [D is the mid−point of AB

D and E are mid-points of AB and AC

DE ∥ BC => DE = BC/2 [By mid−point theorem]

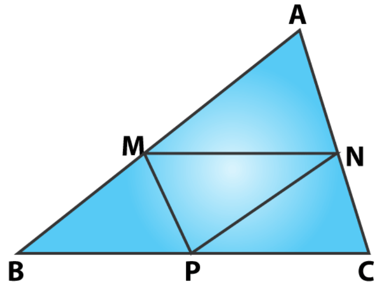
Now,
Area of ΔADE = 1/2 x AD x DE

= 1/2 x 4.5 x 6

=13.5

Area of ΔADE is 13.5 cm^2

Question 8: In figure, M, N and P are mid-points of AB, AC and BC respectively. If MN = 3 cm, NP = 3.5 cm and MP = 2.5 cm, calculate BC, AB and AC.



Solution:

Given: MN = 3 cm, NP = 3.5 cm and MP = 2.5 cm.

M and N are mid-points of AB and AC

By mid−point theorem, we have

MN∥BC => MN = BC/2

or BC = 2MN

BC = 6 cm

[MN = 3 cm given)

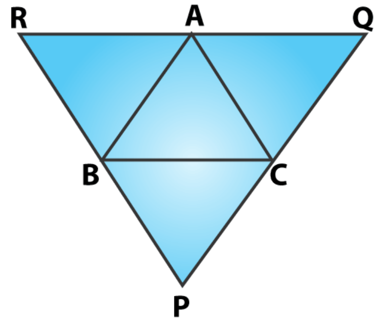
Similarly,

AC = 2MP = 2 (2.5) = 5 cm

AB = 2 NP = 2 (3.5) = 7 cm

Question 9: ABC is a triangle and through A, B, C lines are drawn parallel to BC, CA and AB respectively intersecting at P, Q and R. Prove that the perimeter of ΔPQR is double the perimeter of ΔABC.

Solution:



ABCQ and ARBC are parallelograms.

Therefore, BC = AQ and BC = AR

=>AQ = AR

=>A is the mid-point of QR

Similarly B and C are the mid points of PR and PQ respectively.

By mid−point theorem, we have

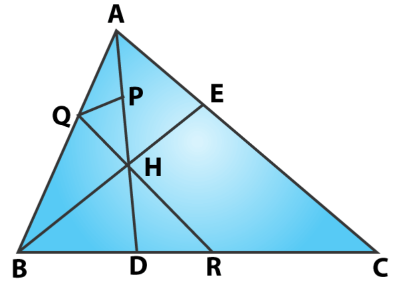
AB = PQ/2, BC = QR/2 and CA = PR/2

or PQ = 2AB, QR = 2BC and PR = 2CA

=>PQ + QR + RP = 2 (AB + BC + CA)

=> Perimeter of ΔPQR = 2 (Perimeter of ΔABC)
Hence proved.

Question 10: In figure, BE ⊥ AC, AD is any line from A to BC intersecting BE in H. P, Q and R are respectively the mid-points of AH, AB and BC. Prove that ∠PQR = 90^0.



Solution:

BE⊥AC and P, Q and R are respectively mid-point of AH, AB and BC. (Given)

In ΔABC, Q and R are mid-points of AB and BC respectively.

By Mid-point theorem:

QR ∥ AC .....(i)

In ΔABH, Q and P are the mid-points of AB and AH respectively

QP ∥ BH ....(ii)

But, BE⊥AC

From (i) and (ii) we have,

QP⊥QR

=>∠PQR = 90^0

Hence Proved.