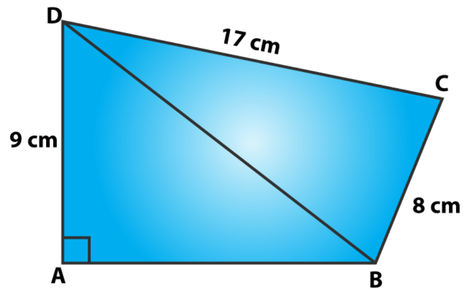
Exercise 15.3

Question 1: In figure, compute the area of quadrilateral ABCD.



Solution:

A quadrilateral ABCD with DC = 17 cm, AD = 9 cm and BC = 8 cm (Given)

In right ΔABD,
Using Pythagorean Theorem,

$AB^2 + AD^2 = BD^2$

$15^2 = AB^2 + 9^2$

$AB^2 = 225-81=144$

AB = 12

Area of ΔABD = 1/2(12×9) cm^2 = 54 cm^2

In right ΔBCD:
Using Pythagorean Theorem,

$CD^2 = BD^2 + BC^2$

$17^2 = BD^2 + 8^2$

$BD^2 = 289 – 64 = 225$

or BD = 15

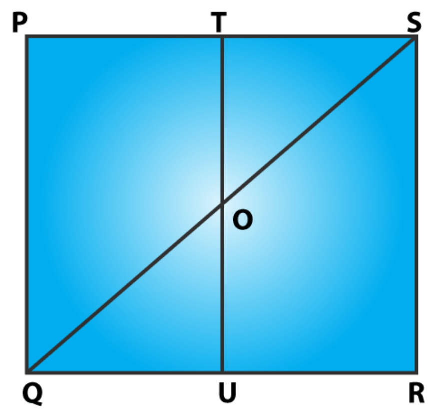
Area of ΔBCD = 1/2(8x17) cm^2 = 68 cm^2

Now, area of quadrilateral ABCD = Area of ΔABD + Area of ΔBCD

= 54 cm^2 + 68 cm^2

= 112 cm^2

Question 2: In figure, PQRS is a square and T and U are, respectively, the mid-points of PS and QR . Find the area of ΔOTS if PQ = 8 cm.



Solution:

T and U are mid points of PS and QR respectively (Given)

Therefore, TU||PQ => TO||PQ

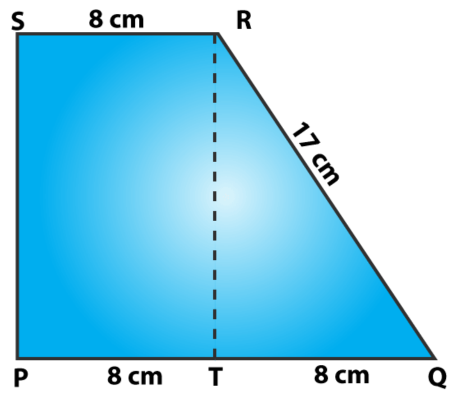
In ΔPQS ,
T is the mid-point of PS and TO||PQ

So, TO = 1/2 PQ = 4 cm
(PQ = 8 cm given)

Also, TS = 1/2 PS = 4 cm
[PQ = PS, As PQRS is a square)

Now,
Area of ΔOTS = 1/2(TO×TS)
= 1/2(4×4) cm^2
= 8cm^2
Area of ΔOTS is 8 cm^2.

Question 3: Compute the area of trapezium PQRS in figure.



Solution:

From figure,

Area of trapezium PQRS = Area of rectangle PSRT + Area of ΔQRT

= PT × RT + 1/2 (QT×RT)

= 8 × RT + 1/2(8×RT)

= 12 RT

In right ΔQRT,
Using Pythagorean Theorem,

$QR^2 = QT^2 + RT^2$

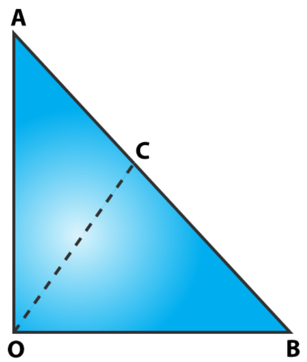
$RT^2 = QR^2 - QT^2$

$RT^2 = 17^2 - 8^2 = 225$

or RT = 15

Therefore, Area of trapezium = 12×15 cm^2 = 180 cm^2

Question 4: In figure, $\angle AOB = 90^o$, AC = BC, OA = 12 cm and OC = 6.5 cm. Find the area of ΔAOB.



Solution:

Given: A triangle AOB, with $\angle AOB = 90^o$, AC = BC, OA = 12 cm and OC = 6.5 cm

As we know, the midpoint of the hypotenuse of a right triangle is equidistant from the vertices.

So, CB = CA = OC = 6.5 cm

AB = 2 CB = 2 x 6.5 cm = 13 cm

In right ΔOAB:
Using Pythagorean Theorem, we get

$AB^2 = OB^2 + OA^2$

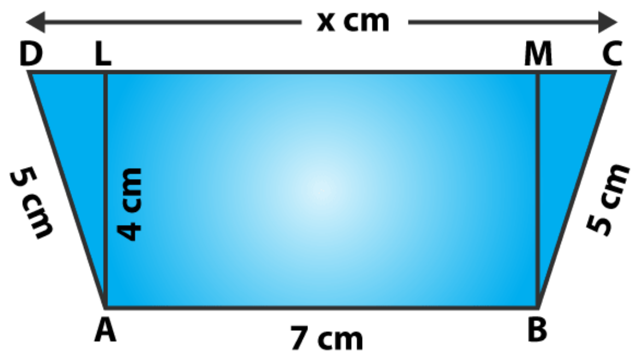
$13^2 = OB^2 + 12^2$

$OB^2 = 169 - 144 = 25$

or OB = 5 cm

Now, Area of ΔAOB = ½(Base x height) cm^2 = 1/2(12 x 5) cm^2 = 30cm^2

Question 5: In figure, ABCD is a trapezium in which AB = 7 cm, AD = BC = 5 cm, DC = x cm, and distance between AB and DC is 4 cm. Find the value of x and area of trapezium ABCD.



Solution:

Given: ABCD is a trapezium, where AB = 7 cm, AD = BC = 5 cm, DC = x cm, and Distance between AB and DC = 4 cm

Consider AL and BM are perpendiculars on DC, then

AL = BM = 4 cm and LM = 7 cm.

In right ΔBMC :
Using Pythagorean Theorem, we get

$BC^2 = BM^2 + MC^2$

$25 = 16 + MC^2$

$MC^2 = 25 – 16 = 9$

or MC = 3

Again,
In right Δ ADL :
Using Pythagorean Theorem, we get

$AD^2 = AL^2 + DL^2$

$25 = 16 + DL^2$

$DL^2 = 25 – 16 = 9$

or DL = 3

Therefore, x = DC = DL + LM + MC = 3 + 7 + 3 = 13
=> x = 13 cm

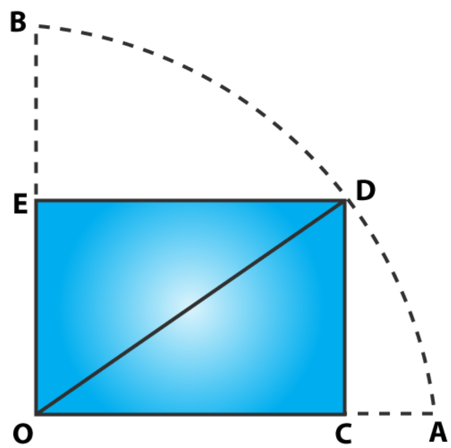
Now,

Area of trapezium ABCD = 1/2(AB + CD) AL

= 1/2(7+13)4
= 40

Area of trapezium ABCD is 40 cm^2.

Question 6: In figure, OCDE is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If OE = 2√5 cm, find the area of the rectangle.



Solution:

From given:
Radius = OD = 10 cm and OE = 2√5 cm

In right ΔDEO,
By Pythagoras theorem

$OD^2 = OE^2 + DE^2$

$(10)^2 = (2\sqrt{5})^2 + DE^2$

$100 - 20 = DE^2$

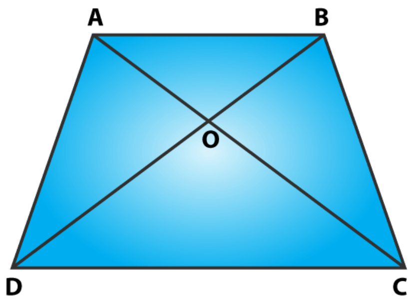
DE = 4√5

Now,

Area of rectangle OCDE = Length x Breadth = OE x DE = $2\sqrt{5} \times 4\sqrt{5} = 40$

Area of rectangle is 40 cm^2.

Question 7: In figure, ABCD is a trapezium in which AB || DC. Prove that ar(ΔAOD) = ar(ΔBOC)



Solution:

ABCD is a trapezium in which AB || DC (Given)

To Prove: ar(ΔAOD) = ar(ΔBOC)

Proof:

From figure, we can observe that ΔADC and ΔBDC are sharing common base i.e. DC and between same parallels AB and DC.

Then, ar(ΔADC) = ar(ΔBDC) ……(1)

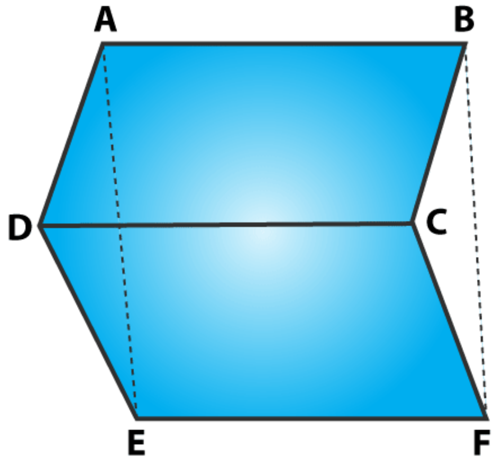
ΔADC is the combination of triangles, ΔAOD and ΔDOC. Similarly, ΔBDC is the combination of ΔBOC and ΔDOC triangles.

Equation (1) => ar(ΔAOD) + ar(ΔDOC) = ar(ΔBOC) + ar(ΔDOC)

or ar(ΔAOD) = ar(ΔBOC)

Hence Proved.

Question 8: In figure, ABCD, ABFE and CDEF are parallelograms. Prove that ar(ΔADE) = ar(ΔBCF).



Solution:

Here, ABCD, CDEF and ABFE are parallelograms:

Which implies:
AD = BC
DE = CF and
AE = BF

Again, from triangles ADE and BCF:

AD = BC, DE = CF and AE = BF

By SSS criterion of congruence, we have

ΔADE ≅ ΔBCF

Since both the triangles are congruent, then ar(ΔADE) = ar(ΔBCF).
Hence Proved,

Question 9: Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that: ar(ΔAPB) x ar(ΔCPD) = ar(ΔAPD) x ar(ΔBPC).

Solution:

Consider: BQ and DR are two perpendiculars on AC.

To prove: ar(ΔAPB) x ar(ΔCPD) = ar(ΔAPD) x ar(ΔBPC).

Now,

L.H.S. = ar(ΔAPB) x ar(ΔCDP)

= (1/2 x AP × BQ) × (1/2 × PC × DR)

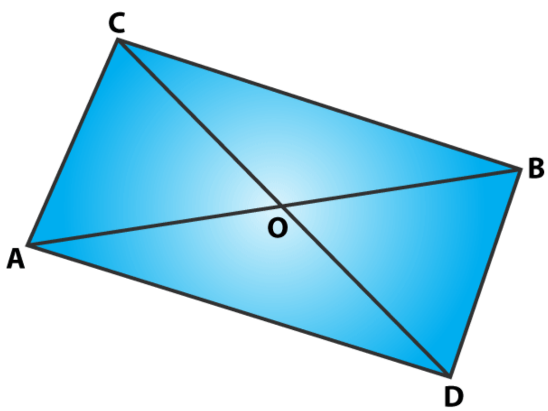
= (1/2 x PC × BQ) × (1/2 × AP × DR)

= ar(ΔAPD) x ar(ΔBPC)

= R.H.S.

Hence proved.

Question 10: In figure, ABC and ABD are two triangles on the base AB. If line segment CD is bisected by AB at O, show that ar(ΔABC) = ar(ΔABD).



Solution:

Draw two perpendiculars CP and DQ on AB.

Now,

ar(ΔABC) = 1/2×AB×CP ·······(1)

ar(ΔABD) = 1/2×AB×DQ ·······(2)

To prove the result, ar(ΔABC) = ar(ΔABD), we have to show that CP = DQ.

In right angled triangles, ΔCPO and ΔDQO

∠CPO = ∠DQO = 90^o

CO = OD (Given)

∠COP = ∠DOQ (Vertically opposite angles)

By AAS condition: ΔCP0 ≅ ΔDQO

So, CP = DQ ………….(3)

(By CPCT)

From equations (1), (2) and (3), we have

ar(ΔABC) = ar(ΔABD)

Hence proved.