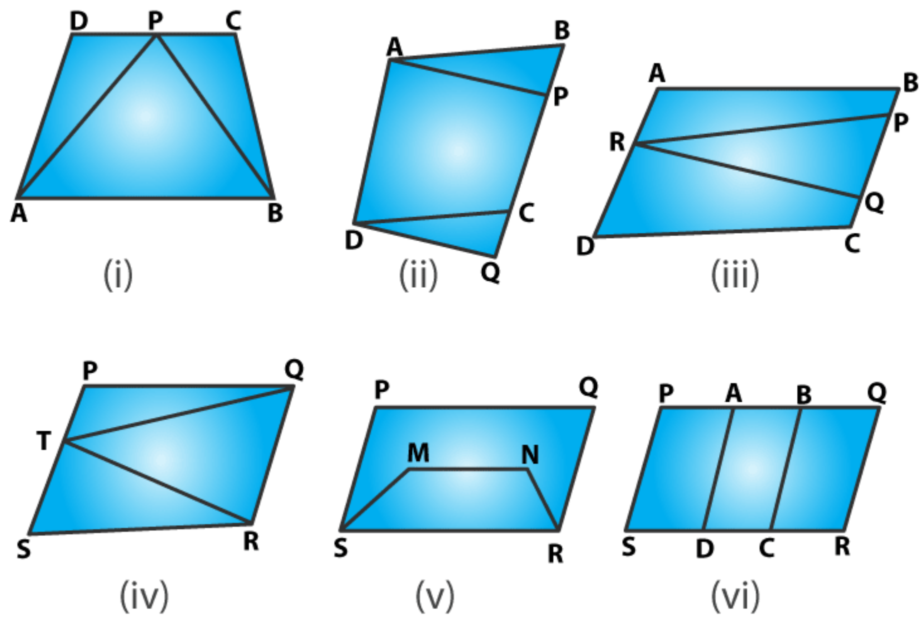
Exercise 15.1 Page No: 15.3

Question 1: Which of the following figures lie on the same base and between the same parallel. In such a case, write the common base and two parallels:



Solution:

(i) Triangle APB and trapezium ABCD are on the common base AB and between the same parallels AB and DC.
So,
Common base = AB
Parallel lines: AB and DC

(ii) Parallelograms ABCD and APQD are on the same base AD and between the same parallels AD and BQ.

Common base = AD
Parallel lines: AD and BQ

(iii) Consider, parallelogram ABCD and ΔPQR, lies between the same parallels AD and BC. But not sharing common base.

(iv) ΔQRT and parallelogram PQRS are on the same base QR and lies between same parallels QR and PS.

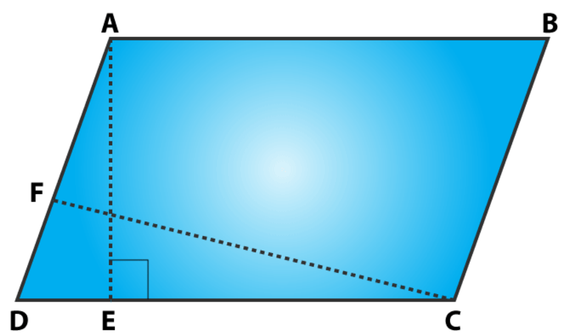
Common base = QR
Parallel lines: QR and PS

(v) Parallelograms PQRS and trapezium SMNR share common base SR, but not between the same parallels.

(vi) Parallelograms: PQRS, AQRD, BCQR are between the same parallels. Also,
Parallelograms: PQRS, BPSC, APSD are between the same parallels.

Exercise 15.2 Page No: 15.14

Question 1: If figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.



Solution:

In parallelogram ABCD, AB = 16 cm, AE = 8 cm and CF = 10 cm

Since, opposite sides of a parallelogram are equal, then

AB = CD = 16 cm

We know, Area of parallelogram = Base x Corresponding height

Area of parallelogram ABCD:

CD x AE = AD x CF

16 x 18 = AD x 10

AD = 12.8

Measure of AD = 12.8 cm

Question 2: In Q.No. 1, if AD = 6 cm, CF = 10 cm and AE = 8 cm, find AB.

Solution: Area of a parallelogram ABCD:

From figure:
AD × CF = CD × AE

6 x 10 = CD x 8

CD = 7.5

Since, opposite sides of a parallelogram are equal.

=> AB = DC = 7.5 cm

Question 3: Let ABCD be a parallelogram of area 124 cm^2. If E and F are the mid-points of sides AB and CD respectively, then find the area of parallelogram AEFD.

Solution:
ABCD be a parallelogram.
Area of parallelogram = 124 cm^2 (Given)

Consider a point P and join AP which is perpendicular to DC.

Now, Area of parallelogram EBCF = FC x AP and

Area of parallelogram AFED = DF x AP

Since F is the mid-point of DC, so DF = FC

From above results, we have

Area of parallelogram AEFD = Area of parallelogram EBCF = 1/2 (Area of parallelogram ABCD)

= 124/2
= 62

Area of parallelogram AEFD is 62 cm^2.

Question 4: If ABCD is a parallelogram, then prove that

ar(Δ ABD) = ar(Δ BCD) = ar(Δ ABC)=ar(Δ ACD) = 1/2 ar($||^{gm}$ ABCD)

Solution:
ABCD is a parallelogram.

When we join the diagonal of parallelogram, it divides it into two quadrilaterals.
Step 1: Let AC is the diagonal, then, Area (ΔABC) = Area (ΔACD) = 1/2(Area of ll^{gm} ABCD)

Step 2: Let BD be another diagonal

Area (ΔABD) = Area (ΔBCD) = 1/2(Area of ll^{gm} ABCD)

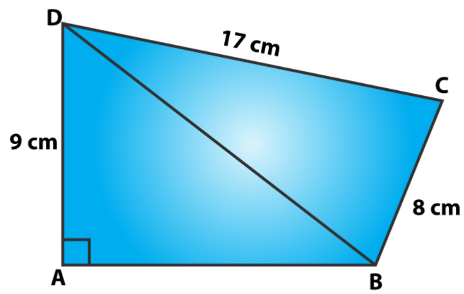
Now,
From Step 1 and step 2, we have

Area (ΔABC) = Area (ΔACD) = Area (ΔABD) = Area (ΔBCD) = 1/2(Area of ll^{gm} ABCD)

Hence Proved.

Exercise 15.3 Page No: 15.40

Question 1: In figure, compute the area of quadrilateral ABCD.



Solution:

A quadrilateral ABCD with DC = 17 cm, AD = 9 cm and BC = 8 cm (Given)

In right ΔABD,
Using Pythagorean Theorem,

$AB^2 + AD^2 = BD^2$

$15^2 = AB^2 + 9^2$

$AB^2 = 225-81=144$

AB = 12

Area of ΔABD = 1/2(12×9) cm^2 = 54 cm^2

In right ΔBCD:
Using Pythagorean Theorem,

$CD^2 = BD^2 + BC^2$

$17^2 = BD^2 + 8^2$

$BD^2 = 289 – 64 = 225$

or BD = 15

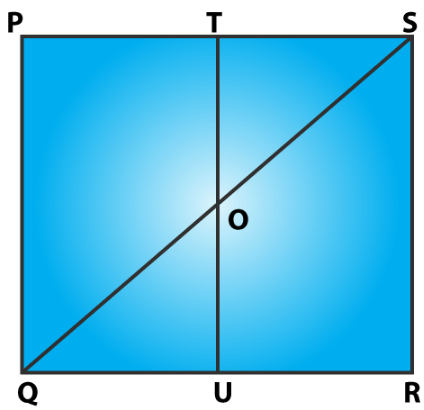
Area of ΔBCD = 1/2(8x17) cm^2 = 68 cm^2

Now, area of quadrilateral ABCD = Area of ΔABD + Area of ΔBCD

= 54 cm^2 + 68 cm^2

= 112 cm^2

Question 2: In figure, PQRS is a square and T and U are, respectively, the mid-points of PS and QR . Find the area of ΔOTS if PQ = 8 cm.



Solution:

T and U are mid points of PS and QR respectively (Given)

Therefore, TU||PQ => TO||PQ

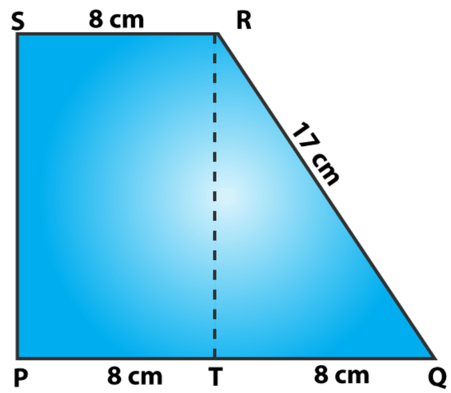
In ΔPQS ,
T is the mid-point of PS and TO||PQ

So, TO = 1/2 PQ = 4 cm
(PQ = 8 cm given)

Also, TS = 1/2 PS = 4 cm
[PQ = PS, As PQRS is a square)

Now,
Area of ΔOTS = 1/2(TO×TS)
= 1/2(4×4) cm^2
= 8cm^2
Area of ΔOTS is 8 cm^2.

Question 3: Compute the area of trapezium PQRS in figure.



Solution:

From figure,

Area of trapezium PQRS = Area of rectangle PSRT + Area of ΔQRT

= PT × RT + 1/2 (QT×RT)

= 8 × RT + 1/2(8×RT)

= 12 RT

In right ΔQRT,
Using Pythagorean Theorem,

$QR^2 = QT^2 + RT^2$

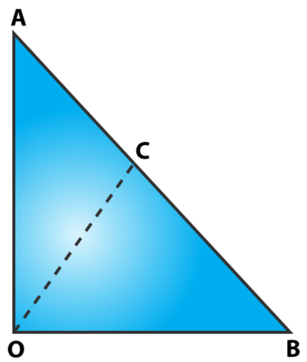
$RT^2 = QR^2 - QT^2$

$RT^2 = 17^2 - 8^2 = 225$

or RT = 15

Therefore, Area of trapezium = 12×15 cm^2 = 180 cm^2

Question 4: In figure, ∠AOB = 90°, AC = BC, OA = 12 cm and OC = 6.5 cm. Find the area of ΔAOB.



Solution:

Given: A triangle AOB, with ∠AOB = 90°, AC = BC, OA = 12 cm and OC = 6.5 cm

As we know, the midpoint of the hypotenuse of a right triangle is equidistant from the vertices.

So, CB = CA = OC = 6.5 cm

AB = 2 CB = 2 x 6.5 cm = 13 cm

In right ΔOAB:
Using Pythagorean Theorem, we get

$AB^2 = OB^2 + OA^2$

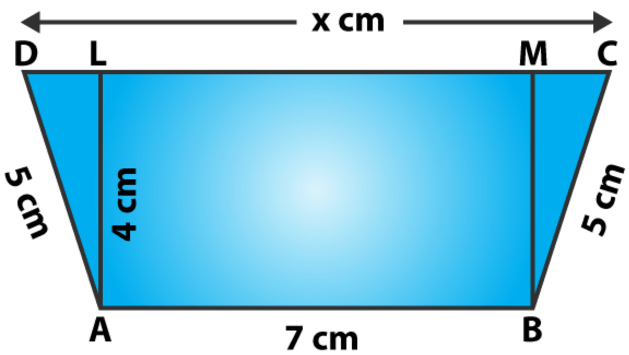
$13^2 = OB^2 + 12^2$

$OB^2 = 169 – 144 = 25$

or OB = 5 cm

Now, Area of ΔAOB = ½(Base x height) cm^2 = 1/2(12 x 5) cm^2 = 30cm^2

Question 5: In figure, ABCD is a trapezium in which AB = 7 cm, AD = BC = 5 cm, DC = x cm, and distance between AB and DC is 4 cm. Find the value of x and area of trapezium ABCD.



Solution:

Given: ABCD is a trapezium, where AB = 7 cm, AD = BC = 5 cm, DC = x cm, and
Distance between AB and DC = 4 cm

Consider AL and BM are perpendiculars on DC, then

AL = BM = 4 cm and LM = 7 cm.

In right ΔBMC :
Using Pythagorean Theorem, we get

$BC^2 = BM^2 + MC^2$

$25 = 16 + MC^2$

$MC^2 = 25 – 16 = 9$

or MC = 3

Again,
In right Δ ADL :
Using Pythagorean Theorem, we get

$AD^2 = AL^2 + DL^2$

$25 = 16 + DL^2$

$DL^2 = 25 – 16 = 9$

or DL = 3

Therefore, x = DC = DL + LM + MC = 3 + 7 + 3 = 13
=> x = 13 cm

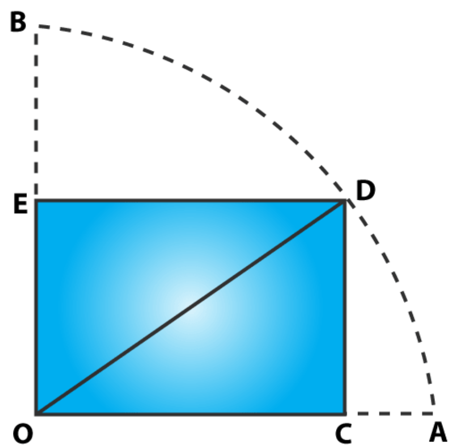
Now,

Area of trapezium ABCD = 1/2(AB + CD) AL

= 1/2(7+13)4
= 40

Area of trapezium ABCD is 40 cm^2.

Question 6: In figure, OCDE is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If OE = 2√5 cm, find the area of the rectangle.



Solution:

From given:
Radius = OD = 10 cm and OE = 2√5 cm

In right ΔDEO,
By Pythagoras theorem

$OD^2 = OE^2 + DE^2$

$(10)^2 = (2\sqrt{5})^2 + DE^2$

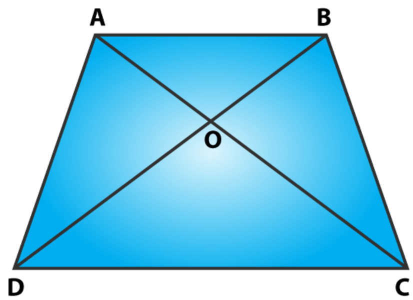
$100 – 20 = DE^2$

DE = 4√5

Now,
Area of rectangle OCDE = Length x Breadth = OE x DE = $2\sqrt{5} \times 4\sqrt{5} = 40$

Area of rectangle is 40 cm^2.

Question 7: In figure, ABCD is a trapezium in which AB || DC. Prove that ar(ΔAOD) = ar(ΔBOC)



Solution:

ABCD is a trapezium in which AB || DC (Given)

To Prove: ar(ΔAOD) = ar(ΔBOC)

Proof:

From figure, we can observe that ΔADC and ΔBDC are sharing common base i.e. DC and between same parallels AB and DC.

Then, ar(ΔADC) = ar(ΔBDC) ……(1)

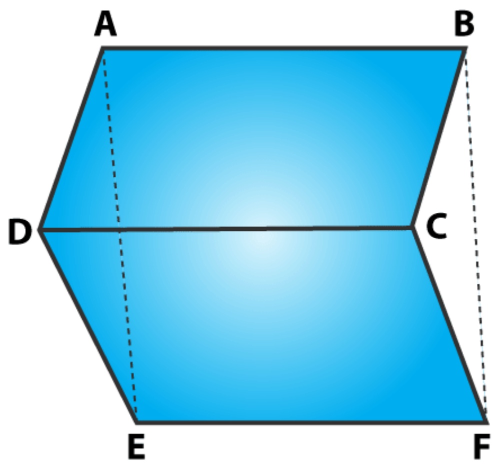
ΔADC is the combination of triangles, ΔAOD and ΔDOC. Similarly, ΔBDC is the combination of ΔBOC and ΔDOC triangles.

Equation (1) => ar(ΔAOD) + ar(ΔDOC) = ar(ΔBOC) + ar(ΔDOC)

or ar(ΔAOD) = ar(ΔBOC)

Hence Proved.

Question 8: In figure, ABCD, ABFE and CDEF are parallelograms. Prove that ar(ΔADE) = ar(ΔBCF).



Solution:

Here, ABCD, CDEF and ABFE are parallelograms:

Which implies:
AD = BC
DE = CF and
AE = BF

Again, from triangles ADE and BCF:

AD = BC, DE = CF and AE = BF

By SSS criterion of congruence, we have

$\Delta ADE \cong \Delta BCF$

Since both the triangles are congruent, then ar(ΔADE) = ar(ΔBCF).
Hence Proved,

Question 9: Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that: ar(ΔAPB) x ar(ΔCPD) = ar(ΔAPD) x ar(ΔBPC).

Solution:

Consider: BQ and DR are two perpendiculars on AC.

To prove: ar(ΔAPB) x ar(ΔCPD) = ar(ΔAPD) x ar(ΔBPC).

Now,

L.H.S. = ar(ΔAPB) x ar(ΔCDP)

= (1/2 x AP × BQ) × (1/2 × PC × DR)

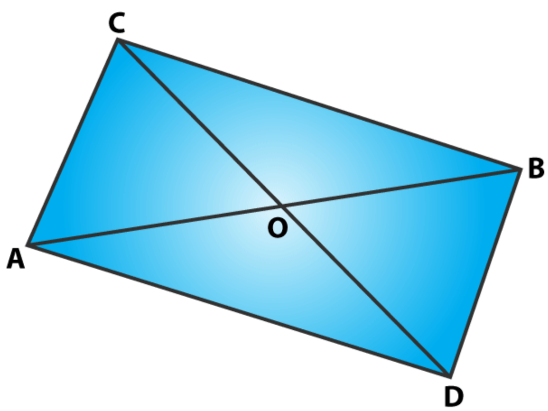
= (1/2 x PC × BQ) × (1/2 × AP × DR)

= ar(ΔAPD) x ar(ΔBPC)

= R.H.S.

Hence proved.

Question 10: In figure, ABC and ABD are two triangles on the base AB. If line segment CD is bisected by AB at O, show that ar(ΔABC) = ar(ΔABD).



Solution:

Draw two perpendiculars CP and DQ on AB.

Now,

ar(ΔABC) = 1/2×AB×CP ·······(1)

ar(ΔABD) = 1/2×AB×DQ ·······(2)

To prove the result, ar(ΔABC) = ar(ΔABD), we have to show that CP = DQ.

In right angled triangles, ΔCPO and ΔDQO

∠CPO = ∠DQO = 90^o

CO = OD (Given)

∠COP = ∠DOQ (Vertically opposite angles)

By AAS condition: ΔCP0 ≅ ΔDQO
So, CP = DQ ………….(3)
(By CPCT)

From equations (1), (2) and (3), we have

ar(ΔABC) = ar(ΔABD)

Hence proved.

Exercise VSAQs

Page No: 15.55

Question 1: If ABC and BDE are two equilateral triangles such that D is the mid-point of BC, then find ar(△ABC) : ar(△BDE).

Solution:

Given: ABC and BDE are two equilateral triangles.
We know, area of an equilateral triangle = $\sqrt{3}/4\ (\text{side})^2$

Let "a" be the side measure of given triangle.

Find ar(△ABC):

ar(△ABC) = $\sqrt{3}/4\ (a)^2$

Find ar(△BDE):

ar(△BDE) = $\sqrt{3}/4\ (a/2)^2$

(D is the mid-point of BC)

Now,
ar(△ABC) : ar(△BDE)

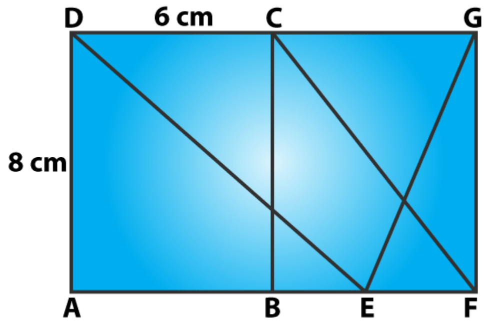
or $\sqrt{3}/4\ (a)^2 : \sqrt{3}/4\ (a/2)^2$

or 1 : 1/4

or 4:1

This implies, ar(△ABC) : ar(△BDE) = 4:1

Question 2: In figure, ABCD is a rectangle in which CD = 6 cm, AD = 8 cm. Find the area of parallelogram CDEF.



Solution:

ABCD is a rectangle, where CD = 6 cm and AD = 8 cm (Given)

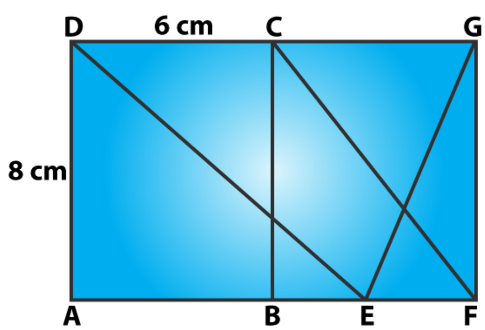
From figure: Parallelogram CDEF and rectangle ABCD on the same base and between the same parallels, which means both have equal areas.

Area of parallelogram CDEF = Area of rectangle ABCD ....(1)

Area of rectangle ABCD = CD x AD = 6 x 8 cm^2 = 48 cm^2

Equation (1) => Area of parallelogram CDEF = 48 cm^2.

Question 3: In figure, find the area of ΔGEF.



Solution:

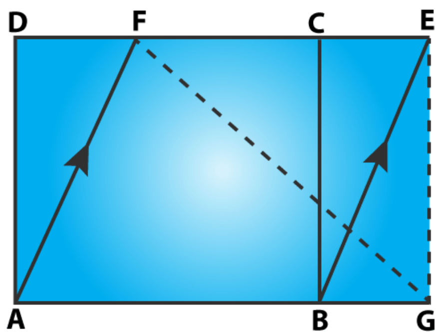
From figure:
Parallelogram CDEF and rectangle ABCD on the same base and between the same parallels, which means both have equal areas.
Area of CDEF = Area of ABCD = 8 x 6 cm^2= 48 cm^2

Again,
Parallelogram CDEF and triangle EFG are on the same base and between the same parallels, then
Area of a triangle = ½(Area of parallelogram)

In this case,

Area of a triangle EFG = ½(Area of parallelogram CDEF) = 1/2(48 cm^2) = 24 cm^2.

Question 4: In figure, ABCD is a rectangle with sides AB = 10 cm and AD = 5 cm. Find the area of ΔEFG.



Solution:

From figure:
Parallelogram ABEF and rectangle ABCD on the same base and between the same parallels, which means both have equal areas.
Area of ABEF = Area of ABCD = 10 x 5 cm^2= 50 cm^2

Again,
Parallelogram ABEF and triangle EFG are on the same base and between the same parallels, then
Area of a triangle = ½(Area of parallelogram)

In this case,

Area of a triangle EFG = ½(Area of parallelogram ABEF) = 1/2(50 cm^2) = 25 cm^2.

Question 5: PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on PQ. If PS = 5 cm, then find ar(△RAS).

Solution:

PQRS is a rectangle with PS = 5 cm and PR = 13 cm (Given)

In △PSR:
Using Pythagoras theorem,

$SR^2 = PR^2 – PS^2 = (13)^2 – (5)^2 = 169 – 25 = 114$

or SR = 12

Now,
Area of △RAS = 1/2 x SR x PS

= 1/2 x 12 x 5

= 30

Therefore, Area of △RAS is 30 cm^2.