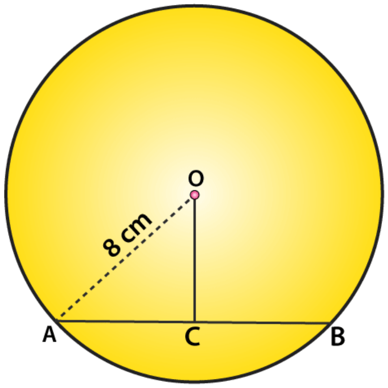
Exercise 16.2

Question 1: The radius of a circle is 8 cm and the length of one of its chords is 12 cm. Find the distance of the chord from the centre.

Solution:



Radius of circle (OA) = 8 cm (Given)

Chord (AB) = 12cm (Given)

Draw a perpendicular OC on AB.

We know, perpendicular from centre to chord bisects the chord

Which implies, AC = BC = 12/2 = 6 cm

In right ΔOCA:

Using Pythagoras theorem,

$OA^2 = AC^2 + OC^2$

$64 = 36 + OC^2$

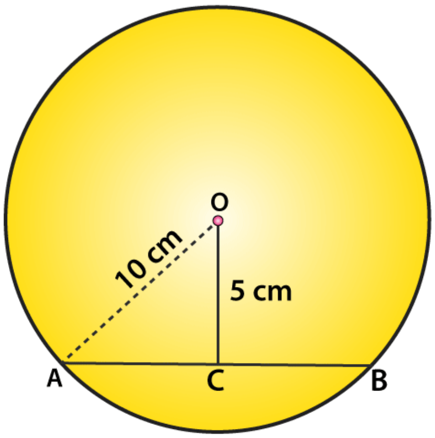
$OC^2 = 64 – 36 = 28$

or OC = √28 = 5.291 (approx.)

The distance of the chord from the centre is 5.291 cm.

Question 2: Find the length of a chord which is at a distance of 5 cm from the centre of a circle of radius 10 cm.

Solution:



Distance of the chord from the centre = OC = 5 cm (Given)

Radius of the circle = OA = 10 cm (Given)

In ΔOCA:
Using Pythagoras theorem,

$OA^2 = AC^2 + OC^2$

$100 = AC^2 + 25$

$AC^2 = 100 - 25 = 75$

$AC = \sqrt{75} = 8.66$

As, perpendicular from the centre to chord bisects the chord.

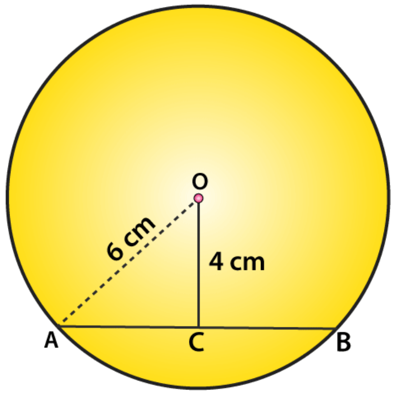
Therefore, AC = BC = 8.66 cm

=> AB = AC + BC = 8.66 + 8.66 = 17.32

Answer: AB = 17.32 cm

Question 3: Find the length of a chord which is at a distance of 4 cm from the centre of a circle of radius 6 cm.

Solution:



Distance of the chord from the centre = OC = 4 cm (Given)

Radius of the circle = OA = 6 cm (Given)

In ΔOCA:
Using Pythagoras theorem,

$OA^2 = AC^2 + OC^2$

$36 = AC^2 + 16$

$AC^2 = 36 – 16 = 20$

$AC = \sqrt{20} = 4.47$

Or AC = 4.47cm

As, perpendicular from the centre to chord bisects the chord.

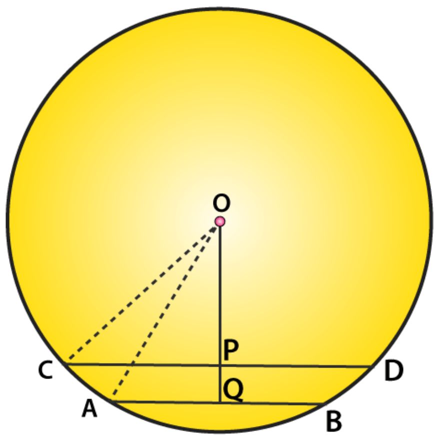
Therefore, AC = BC = 4.47 cm

=> AB = AC + BC = 4.47 + 4.47 = 8.94

Answer: AB = 8.94 cm

Question 4: Two chords AB, CD of lengths 5 cm, 11 cm respectively of a circle are parallel. If the distance between AB and CD is 3 cm, find the radius of the circle.

Solution:



Given: AB = 5 cm, CD = 11 cm, PQ = 3 cm

Draw perpendiculars OP on CD and OQ on AB

Let OP = x cm and OC = OA = r cm

We know, perpendicular from centre to chord bisects it.
Since OP⊥CD, we have
CP = PD = 11/2 cm

And OQ⊥AB
AQ = BQ = 5/2 cm

In ΔOCP:
By Pythagoras theorem,

$OC^2 = OP^2 + CP^2$

$r^2 = x^2 + (11/2)^2$ .....(1)

In ΔOQA:
By Pythagoras theorem,

$OA^2=OQ^2+AQ^2$

$r^2 = (x+3)^2 + (5/2)^2$ .....(2)

From equations (1) and (2), we get

$(x+3)^2 + (5/2)^2 = x^2 + (11/2)^2$

Solve above equation and find the value of x.

$x^2 + 6x + 9 + 25/4 = x^2 + 121/4$
(using identity, $(a+b)^2 = a^2 + b^2 + 2ab$)

$6x = 121/4 – 25/4 − 9$

$6x = 15$

or $x = 15/6 = 5/2$

Substitute the value of x in equation (1), and find the length of radius,

$r^2 = (5/2)^2 + (11/2)^2$

$= 25/4 + 121/4$

$= 146/4$

or $r = \sqrt{146/4}$ cm

Question 5: Give a method to find the centre of a given circle.

Solution:

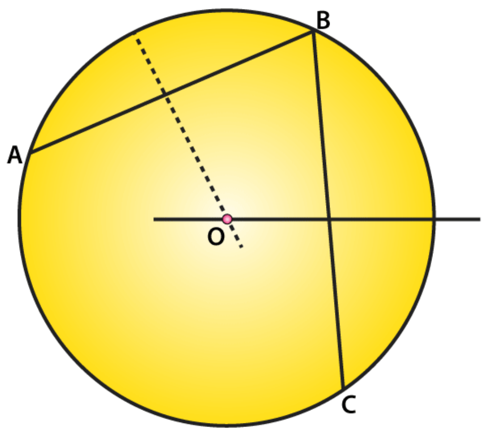
Steps of Construction:

Step 1: Consider three points A, B and C on a circle.

Step 2: Join AB and BC.

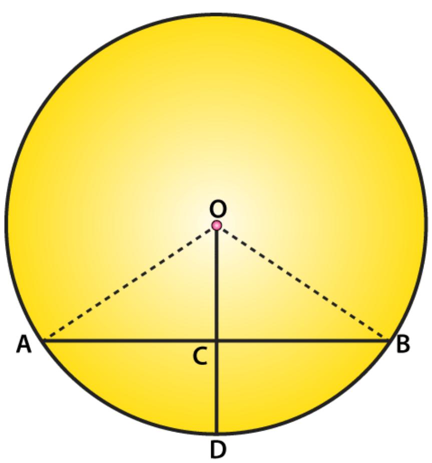
Step 3: Draw perpendicular bisectors of chord AB and BC which intersect each other at a point, say O.

Step 4: This point O is a centre of the circle, because we know that, the Perpendicular bisectors of chord always pass through the centre.



Question 6: Prove that the line joining the mid-point of a chord to the centre of the circle passes through the mid-point of the corresponding minor arc.

Solution:



From figure, Let C is the mid-point of chord AB.
To prove: D is the mid-point of arc AB.
Now, In ΔOAC and ΔOBC
OA = OB [Radius of circle]
OC = OC [Common]
AC = BC [C is the mid-point of chord AB (given)]

So, by SSS condition: ΔOAC ≅ ΔOBC
So, ∠AOC = ∠BOC (BY CPCT)

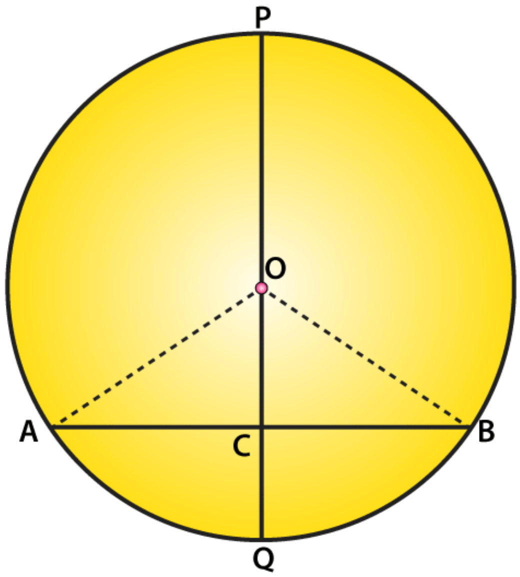
$$\Rightarrow m\bar{AD} \cong m\bar{BD}$$

$$\Rightarrow \bar{AD} \cong \bar{BD}$$

Therefore, D is the mid-point of arc AB. Hence Proved.

Question 7: Prove that a diameter of a circle which bisects a chord of the circle also bisects the angle subtended by the chord at the centre of the circle.

Solution:



Form figure: PQ is a diameter of circle which bisects the chord AB at C. (Given)

To Prove: PQ bisects ∠AOB

Now,

In ΔBOC and ΔAOC

OA = OB [Radius]

OC = OC [Common side]

AC = BC [Given]

Then, by SSS condition: ΔAOC ≅ ΔBOC

So, ∠AOC = ∠BOC [By c.p.c.t.]

Therefore, PQ bisects ∠AOB. Hence proved.