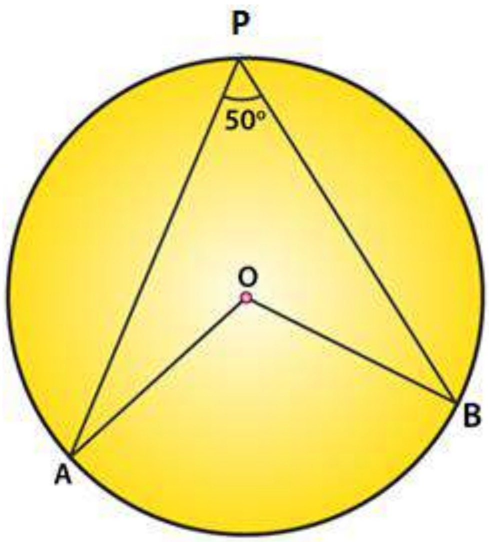
Exercise 16.4 Page No: 16.60

Question 1: In figure, O is the centre of the circle. If ∠APB = 50^0, find ∠AOB and ∠OAB.



Solution:

∠APB = 50^0 (Given)

By degree measure theorem: ∠AOB = 2∠APB

∠AOB = 2 × 50^0 = 100^0

Again, OA = OB [Radius of circle]
Then ∠OAB = ∠OBA [Angles opposite to equal sides]

Let ∠OAB = m

In ΔOAB,
By angle sum property: ∠OAB+∠OBA+∠AOB=180^0

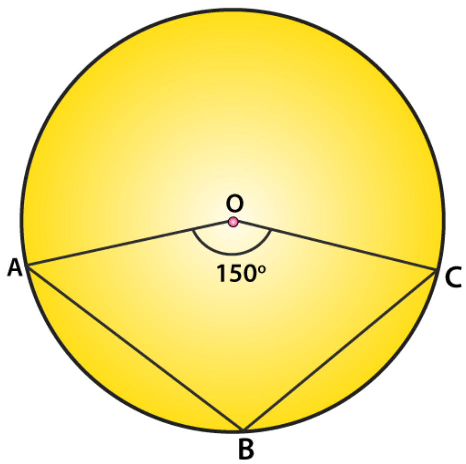
=> m + m + 100^0 = 180^0

=>2m = 180^0 – 100^0 = 80^0

=>m = 80^0/2 = 40^0

∠OAB = ∠OBA = 40^0

Question 2: In figure, it is given that O is the centre of the circle and ∠AOC = 150^0. Find ∠ABC.



Solution:

∠AOC = 150^0 (Given)

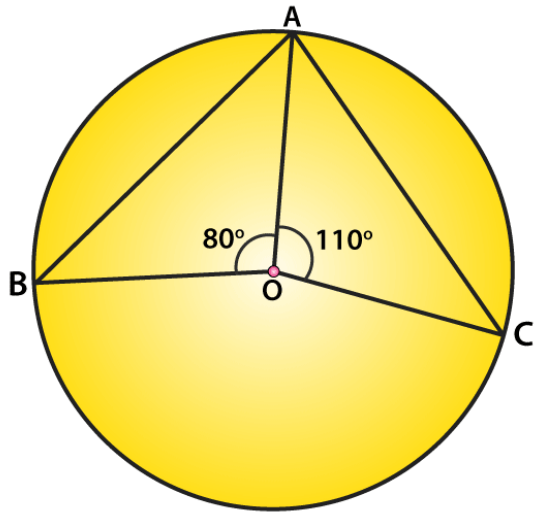
By degree measure theorem: ∠ABC = (reflex∠AOC)/2 …(1)

We know, ∠AOC + reflex(∠AOC) = 360^0 [Complex angle]
150^0 + reflex∠AOC = 360^0

or reflex ∠AOC = $360^0 - 150^0 = 210^0$

From (1) => ∠ABC = 210° /2 = 105°

Question 3: In figure, O is the centre of the circle. Find ∠BAC.



Solution:

Given: $\angle AOB = 80^0$ and $\angle AOC = 110^0$

Therefore, $\angle AOB + \angle AOC + \angle BOC = 360^0$ [Completeangle]

Substitute given values,

$80^0 + 100^0 + \angle BOC = 360^0$

$\angle BOC = 360^0 - 80^0 - 110^0 = 170^0$

or $\angle BOC = 170^0$

Now, by degree measure theorem

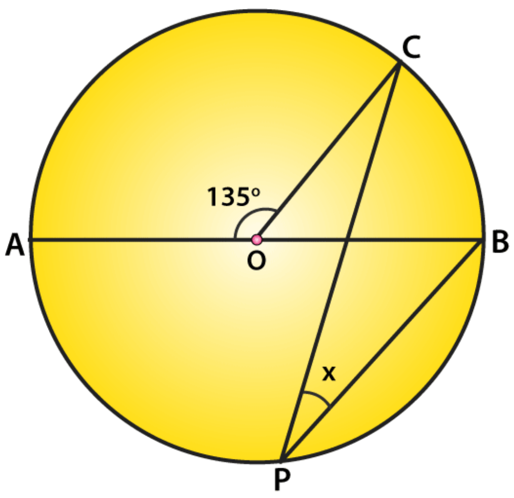
$\angle BOC = 2\angle BAC$

$170^0 = 2\angle BAC$

Or $\angle BAC = 170^0/2 = 85^0$

Question 4: If O is the centre of the circle, find the value of x in each of the following figures.

(i)



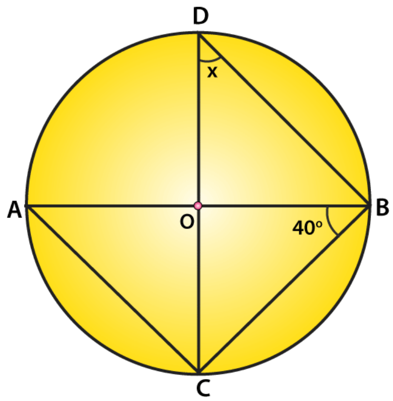
Solution:

$\angle AOC = 135^0$ (Given)

From figure, $\angle AOC + \angle BOC = 180^0$ [Linear pair of angles]
$135^0 + \angle BOC = 180^0$
or $\angle BOC = 180^0 - 135^0$
or $\angle BOC = 45^0$

Again, by degree measure theorem
$\angle BOC = 2\angle CPB$
$45^0 = 2x$
$x = 45^0/2$

(ii)

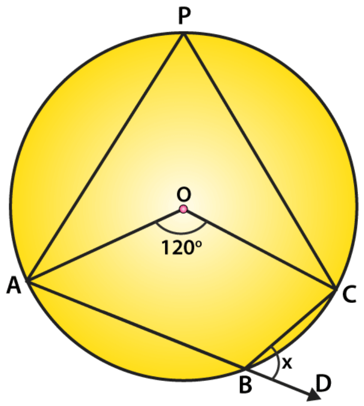


Solution:
$\angle ABC = 40^0$ (given)
$\angle ACB = 90^0$ [Angle in semicircle]
In ΔABC,
$\angle CAB + \angle ACB + \angle ABC = 180^0$ [angle sum property]
$\angle CAB + 90^0 + 40^0 = 180^0$
$\angle CAB = 180^0 - 90^0 - 40^0$
$\angle CAB = 50^0$

Now, $\angle CDB = \angle CAB$ [Angle is on same segment]

This implies, $x = 50^0$

(iii)



Solution:

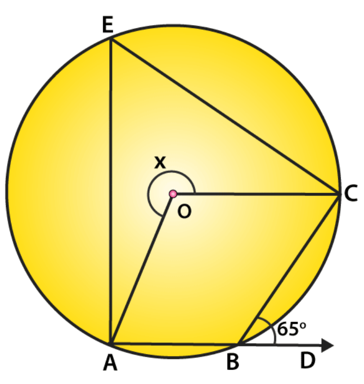
$\angle AOC = 120^0$ (given)

By degree measure theorem: $\angle AOC = 2\angle APC$
$120^0 = 2\angle APC$
$\angle APC = 120^0/2 = 60^0$

Again, $\angle APC + \angle ABC = 180^0$ [Sum of opposite angles of cyclic quadrilaterals = 180 °]
$60^0 + \angle ABC = 180^0$
$\angle ABC = 180^0 - 60^0$
$\angle ABC = 120^0$

$\angle ABC + \angle DBC = 180^0$ [Linear pair of angles]
$120^0 + x = 180^0$
$x = 180^0 - 120^0 = 60^0$
The value of x is 60^0

(iv)



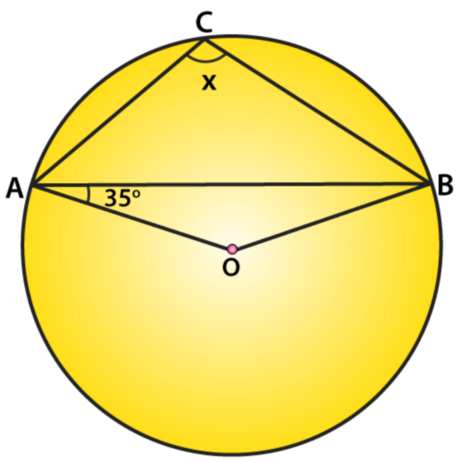
Solution:

$\angle CBD = 65^0$ (given)
From figure:
$\angle ABC + \angle CBD = 180^0$ [Linear pair of angles]
$\angle ABC + 65^0 = 180^0$
$\angle ABC = 180^0 - 65^0 = 115^0$

Again, reflex $\angle AOC = 2\angle ABC$ [Degree measure theorem]
$x = 2(115^0) = 230^0$

The value of x is 230^0

(v)



Solution:

$\angle OAB = 35^0$ (Given)
From figure:
$\angle OBA = \angle OAB = 35^0$ [Angles opposite to equal radii]

InΔAOB:

$\angle AOB + \angle OAB + \angle OBA = 180^0$ [angle sum property]

$\angle AOB + 35^0 + 35^0 = 180^0$

$\angle AOB = 180^0 - 35^0 - 35^0 = 110^0$

Now, $\angle AOB$ + reflex$\angle AOB = 360^0$ [Complex angle]

110^0 + reflex∠AOB = 360^0

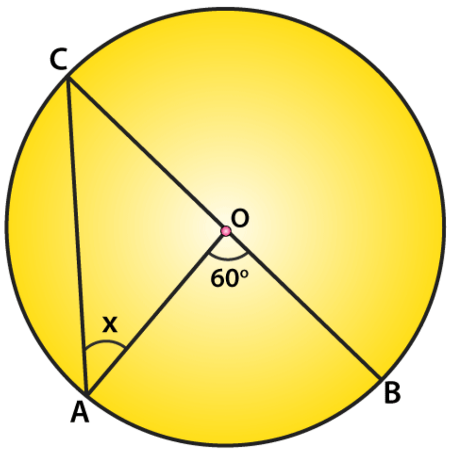
reflex∠AOB = $360^0 - 110^0 = 250^0$

By degree measure theorem: reflex ∠AOB = 2∠ACB

$250^0 = 2x$

$x = 250^0/2 = 125^0$

(vi)



Solution:

∠AOB = 60° (given)

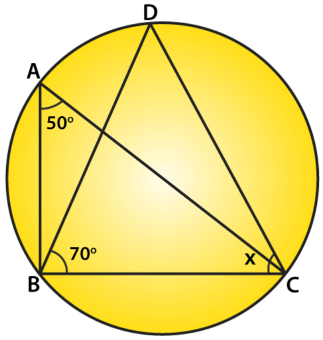
By degree measure theorem: reflex∠AOB = 2∠OAC

60° = 2∠ OAC

∠OAC = 60° / 2 = 30°   [Angles opposite to equal radii]

Or $x = 30^0$

(vii)



Solution:

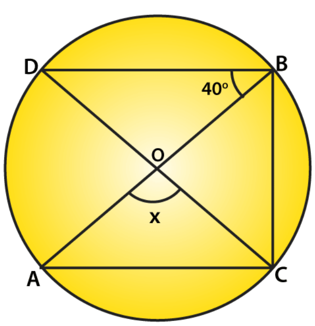
$\angle BAC = 50^0$ and $\angle DBC = 70^0$ (given)

From figure:
$\angle BDC = \angle BAC = 50^0$ [Angle on same segment]

Now,
In ΔBDC:
Using angle sum property, we have
$\angle BDC + \angle BCD + \angle DBC = 180^0$
Substituting given values, we get
$50^0 + x^0 + 70^0 = 180^0$
$x^0 = 180^0 - 50^0 - 70^0 = 60^0$

or $x = 60^o$.

(viii)



Solution:

$\angle DBO = 40^0$ (Given)

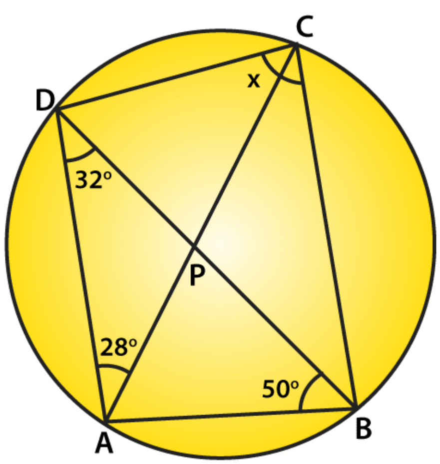
Form figure:

$\angle DBC = 90^0$ [Angle in a semicircle]
$\angle DBO + \angle OBC = 90^0$
$40^0+\angle OBC=90^0$
or $\angle OBC=90^0-40^0=50^0$

Again, By degree measure theorem: $\angle AOC = 2\angle OBC$

or $x = 2\times 50^0=100^0$

(ix)



Solution:
$\angle CAD = 28$, $\angle ADB = 32$ and $\angle ABC = 50$ (Given)

From figure:

In ΔDAB:
Angle sum property: $\angle ADB + \angle DAB + \angle ABD = 180^0$

By substituting the given values, we get

$32^0 + \angle DAB + 50^0 = 180^0$

$\angle DAB=180^0-32^0-50^0$

$\angle DAB = 98^0$

Now,

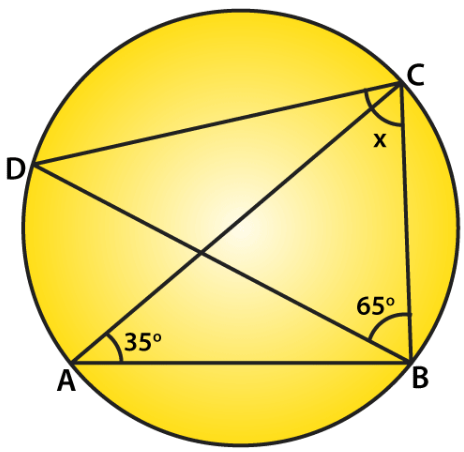
$\angle DAB+\angle DCB=180^0$ [Opposite angles of cyclic quadrilateral, their sum = 180 degrees]

$98^0+x=180^0$

or $x = 180^0-98^0=82^0$

The value of x is 82 degrees.

(x)



Solution:
$\angle BAC = 35^0$ and $\angle DBC = 65^0$

From figure:
$\angle BDC = \angle BAC = 35^0$ [Angle in same segment]

In ΔBCD:

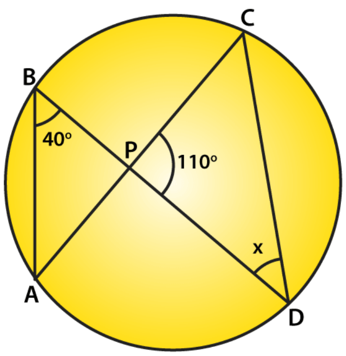
Angle sum property, we have

$\angle BDC + \angle BCD + \angle DBC = 180^0$

$35^0 + x + 65^0 = 180^0$

or $x = 180^0 - 35^0 - 65^0 = 80^0$

(xi)



Solution:

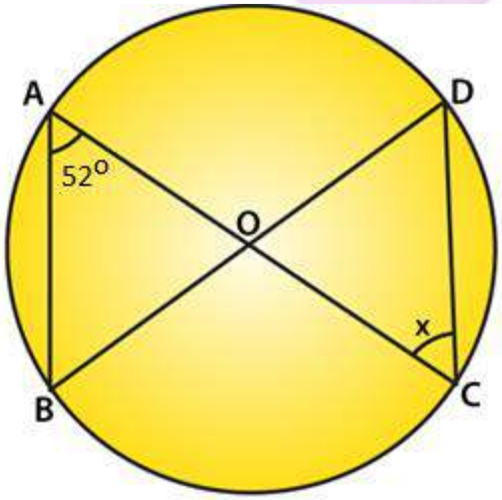
$\angle ABD = 40^0$, $\angle CPD = 110^0$ (Given)

Form figure:
$\angle ACD = \angle ABD = 40^0$ [Angle in same segment]

In ΔPCD,
Angle sum property: $\angle PCD + \angle CPO + \angle PDC = 180^0$
$400 + 110^0 + x = 180^0$
$x = 180^0 - 150^0 = 30^0$

The value of x is 30 degrees.

(xii)



Solution:

∠BAC = 52^0 (Given)

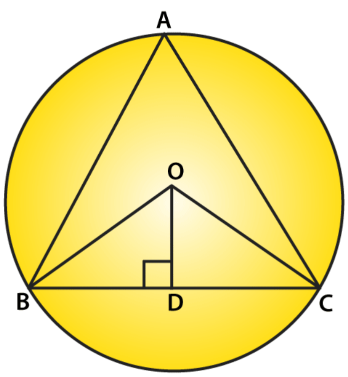
From figure:
∠BDC = ∠BAC = 52^0 [Angle in same segment]

Since OD = OC (radii), then ∠ODC = ∠OCD [Opposite angle to equal radii]

So, x = 52^0

Question 5: O is the circumcentre of the triangle ABC and OD is perpendicular on BC. Prove that ∠BOD = ∠A.

Solution:



In ΔOBD and ΔOCD:
OB = OC [Radius]
∠ODB = ∠ODC [Each 90^0]
OD = OD [Common]

Therefore, By RHS Condition
ΔOBD ≅ ΔOCD
So, ∠BOD = ∠COD…..(i)[By CPCT]

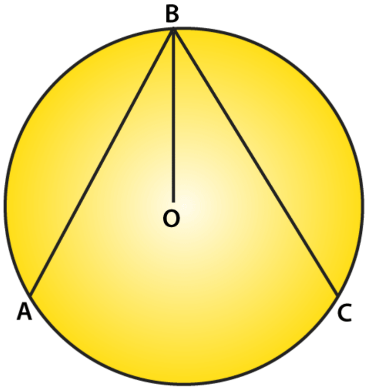
Again,
By degree measure theorem: ∠BOC = 2∠BAC

2∠BOD = 2∠BAC [Using(i)]

∠BOD = ∠BAC

Hence proved.

Question 6: In figure, O is the centre of the circle, BO is the bisector of ∠ABC. Show that AB = AC.



Solution:

Since, BO is the bisector of ∠ABC, then,
∠ABO = ∠CBO .....(i)

From figure:
Radius of circle = OB = OA = OB = OC
∠OAB = ∠OCB .....(ii) [opposite angles to equal sides]
∠ABO = ∠DAB .....(iii) [opposite angles to equal sides]

From equations (i), (ii) and (iii), we get

∠OAB = ∠OCB .....(iv)

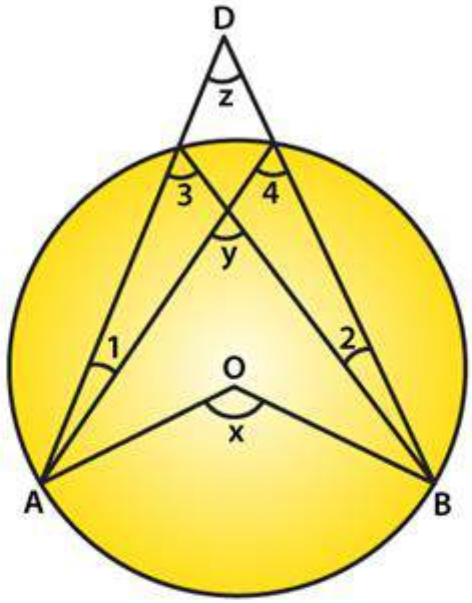
In ΔOAB and ΔOCB:

∠OAB = ∠OCB [From (iv)]
OB = OB [Common]

∠OBA = ∠OBC [Given]

Then, By AAS condition : ΔOAB ≅ ΔOCB
So, AB = BC [By CPCT]

Question 7: In figure, O is the centre of the circle, then prove that ∠x = ∠y + ∠z.



Solution:
From the figure:

∠3 = ∠4 ....(i) [Angles in same segment]

∠x = 2∠3 [By degree measure theorem]
∠x = ∠3 + ∠3
∠x = ∠3 + ∠4 (Using (i)) .....(ii)

Again, ∠y = ∠3 + ∠1 [By exterior angle property]
or ∠3 = ∠y − ∠1 .....(iii)

∠4 = ∠z + ∠1 .... (iv) [By exterior angle property]

Now, from equations (ii) , (iii) and (iv), we get

∠x = ∠y − ∠1 + ∠z + ∠1
or ∠x = ∠y + ∠z + ∠1 − ∠1
or x = ∠y + ∠z

Hence proved.