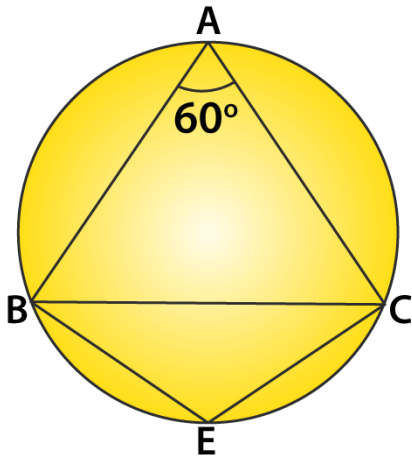


Exercise 16.5

Question 1: In figure, ΔABC is an equilateral triangle. Find $m\angle BEC$.



Solution:

ΔABC is an equilateral triangle. (Given)

Each angle of an equilateral triangle is 60 degrees.

In quadrilateral ABEC:

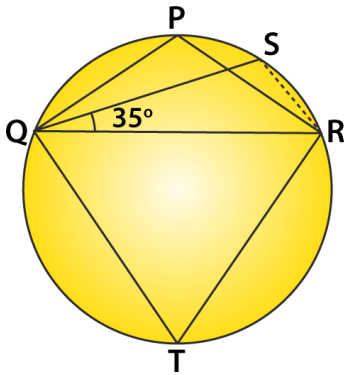
$\angle BAC + \angle BEC = 180^\circ$ (Opposite angles of quadrilateral)

$$60^\circ + \angle BEC = 180^\circ$$

$$\angle BEC = 180^\circ - 60^\circ$$

$$\angle BEC = 120^\circ$$

Question 2: In figure, ΔPQR is an isosceles triangle with $PQ = PR$ and $m\angle PQR = 35^\circ$. Find $m\angle QSR$ and $m\angle QTR$.



Solution:

Given: ΔPQR is an isosceles triangle with $PQ = PR$ and $m\angle PQR = 35^\circ$

In ΔPQR :

$\angle PQR = \angle PRQ = 35^\circ$ (Angle opposite to equal sides)

Again, by angle sum property

$$\angle P + \angle Q + \angle R = 180^\circ$$

$$\angle P + 35^\circ + 35^\circ = 180^\circ$$

$$\angle P + 70^\circ = 180^\circ$$

$$\angle P = 180^\circ - 70^\circ$$

$$\angle P = 110^\circ$$

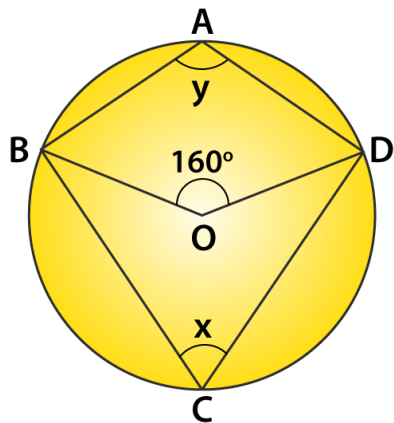
Now, in quadrilateral SQTR,

$\angle QSR + \angle QTR = 180^\circ$ (Opposite angles of quadrilateral)

$$110^\circ + \angle QTR = 180^\circ$$

$$\angle QTR = 70^\circ$$

Question 3: In figure, O is the centre of the circle. If $\angle BOD = 160^\circ$, find the values of x and y.



Solution:

From figure: $\angle BOD = 160^\circ$

By degree measure theorem: $\angle BOD = 2 \angle BCD$

$$160^\circ = 2x$$

$$\text{or } x = 80^\circ$$

Now, in quadrilateral ABCD,

$\angle BAD + \angle BCD = 180^\circ$ (Opposite angles of Cyclic quadrilateral)

$$y + x = 180^\circ$$

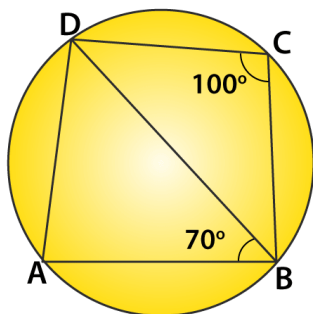
Putting value of x,

$$y + 80^\circ = 180^\circ$$

$$y = 100^\circ$$

Answer: $x = 80^\circ$ and $y = 100^\circ$.

Question 4: In figure, ABCD is a cyclic quadrilateral. If $\angle BCD = 100^\circ$ and $\angle ABD = 70^\circ$, find $\angle ADB$.



Solution:

From figure:

In quadrilateral ABCD,

$\angle DCB + \angle BAD = 180^\circ$ (Opposite angles of Cyclic quadrilateral)

$$100^\circ + \angle BAD = 180^\circ$$

$$\angle BAD = 80^\circ$$

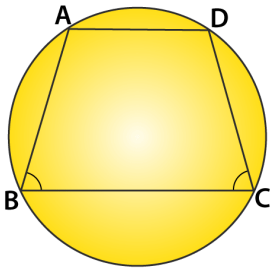
In $\triangle BAD$:

By angle sum property: $\angle ADB + \angle DAB + \angle ABD = 180^\circ$

$$\angle ADB + 80^\circ + 70^\circ = 180^\circ$$

$$\angle ADB = 30^\circ$$

Question 5: If ABCD is a cyclic quadrilateral in which $AD \parallel BC$ (figure). Prove that $\angle B = \angle C$.



Solution:

Given: ABCD is a cyclic quadrilateral with $AD \parallel BC$

$$\Rightarrow \angle A + \angle C = 180^\circ \dots\dots(1)$$

[Opposite angles of cyclic quadrilateral]

$$\text{and } \angle A + \angle B = 180^\circ \dots\dots(2)$$

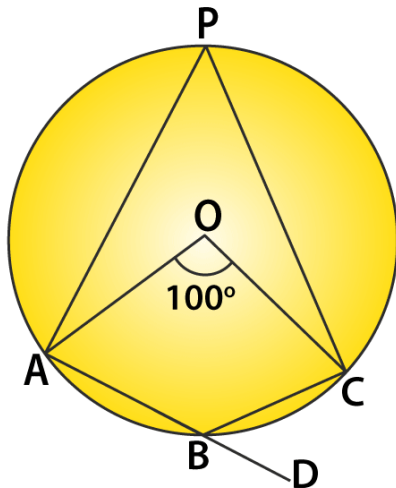
[Co-interior angles]

From (1) and (2), we have

$$\angle B = \angle C$$

Hence proved.

Question 6: In figure, O is the centre of the circle. Find $\angle CBD$.



Solution:

Given: $\angle BOC = 100^\circ$

By degree measure theorem: $\angle AOC = 2 \angle APC$

$$100^\circ = 2 \angle APC$$

$$\text{or } \angle APC = 50^\circ$$

Again,

$\angle APC + \angle ABC = 180^\circ$ (Opposite angles of a cyclic quadrilateral)

$$50^\circ + \angle ABC = 180^\circ$$

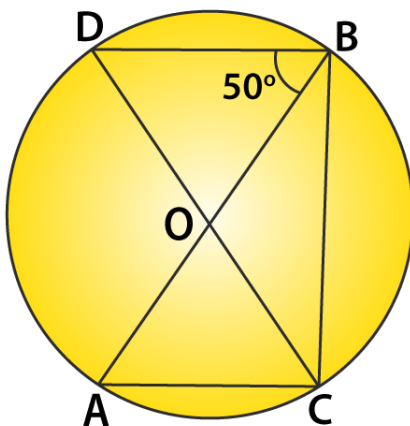
$$\text{or } \angle ABC = 130^\circ$$

Now, $\angle ABC + \angle CBD = 180^\circ$ (Linear pair)

$$130^\circ + \angle CBD = 180^\circ$$

$$\text{or } \angle CBD = 50^\circ$$

Question 7: In figure, AB and CD are diameters of a circle with centre O. If $\angle OBD = 50^\circ$, find $\angle AOC$.



Solution:

Given: $\angle OBD = 50^\circ$

Here, AB and CD are the diameters of the circles with centre O.

$\angle DBC = 90^\circ$ (i)

[Angle in the semi-circle]

Also, $\angle DBC = 50^\circ + \angle OBC$

$90^\circ = 50^\circ + \angle OBC$

or $\angle OBC = 40^\circ$

Again, By degree measure theorem: $\angle AOC = 2 \angle ABC$

$\angle AOC = 2\angle OBC = 2 \times 40^\circ = 80^\circ$

Question 8: On a semi-circle with AB as diameter, a point C is taken, so that $m(\angle CAB) = 30^\circ$. Find $m(\angle ACB)$ and $m(\angle ABC)$.

Solution:

Given: $m(\angle CAB) = 30^\circ$

To Find: $m(\angle ACB)$ and $m(\angle ABC)$.

Now,

$\angle ACB = 90^\circ$ (Angle in semi-circle)

Now,

In $\triangle ABC$, by angle sum property: $\angle CAB + \angle ACB + \angle ABC = 180^\circ$

$30^\circ + 90^\circ + \angle ABC = 180^\circ$

$\angle ABC = 60^\circ$

Answer: $\angle ACB = 90^\circ$ and $\angle ABC = 60^\circ$

Question 9: In a cyclic quadrilateral ABCD if $AB \parallel CD$ and $\angle B = 70^\circ$, find the remaining angles.

Solution:

A cyclic quadrilateral ABCD with $AB \parallel CD$ and $B = 70^\circ$.

$\angle B + \angle C = 180^\circ$ (Co-interior angle)

$70^\circ + \angle C = 180^\circ$

$\angle C = 110^\circ$

And,

$\Rightarrow \angle B + \angle D = 180^\circ$ (Opposite angles of Cyclic quadrilateral)

$70^\circ + \angle D = 180^\circ$

$\angle D = 110^\circ$

Again, $\angle A + \angle C = 180^\circ$ (Opposite angles of cyclic quadrilateral)

$$\angle A + 110^\circ = 180^\circ$$

$$\angle A = 70^\circ$$

Answer: $\angle A = 70^\circ$, $\angle C = 110^\circ$ and $\angle D = 110^\circ$

Question 10: In a cyclic quadrilateral ABCD, if $m \angle A = 3(m \angle C)$. Find $m \angle A$.

Solution:

$$\angle A + \angle C = 180^\circ \dots(1)$$

[Opposite angles of cyclic quadrilateral]

Since $m \angle A = 3(m \angle C)$ (given)

$$\Rightarrow \angle A = 3\angle C \dots(2)$$

$$\text{Equation (1)} \Rightarrow 3\angle C + \angle C = 180^\circ$$

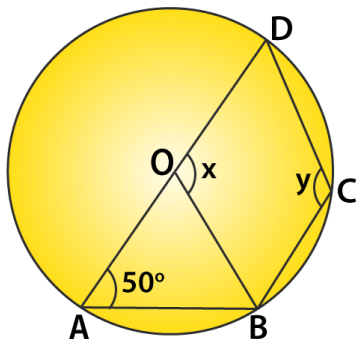
$$\text{or } 4\angle C = 180^\circ$$

$$\text{or } \angle C = 45^\circ$$

From equation (2)

$$\angle A = 3 \times 45^\circ = 135^\circ$$

Question 11: In figure, O is the centre of the circle $\angle DAB = 50^\circ$. Calculate the values of x and y.



Solution:

Given : $\angle DAB = 50^\circ$

By degree measure theorem: $\angle BOD = 2 \angle BAD$

so, $x = 2(50^\circ) = 100^\circ$

Since, ABCD is a cyclic quadrilateral, we have

$$\angle A + \angle C = 180^\circ$$

$$50^\circ + y = 180^\circ$$

$$y = 130^\circ$$