Exercise 21.2

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Question 1: Find the volume of a sphere whose radius is:
(i) 2 cm (ii) 3.5 cm (iii) 10.5 cm.

Solution:
Volume of a sphere = $4/3\pi r^3$ Cubic Units
Where, r = radius of a sphere

(i) Radius = 2 cm

Volume = $4/3 \times 22/7 \times (2)^3$

= 33.52

Volume = 33.52 cm^3

(ii) Radius = 3.5cm

Therefore volume = $4/3 \times 22/7 \times (3.5)^3$
= 179.666

Volume = 179.666 cm^3

(iii) Radius = 10.5 cm

Volume = $4/3 \times 22/7 \times (10.5)^3$

= 4851

Volume = 4851 cm^3

Question 2: Find the volume of a sphere whose diameter is:
(i) 14 cm (ii) 3.5 dm (iii) 2.1 m

Solution:
Volume of a sphere = $4/3\pi r^3$ Cubic Units
Where, r = radius of a sphere

(i) diameter =14 cm
So, radius = diameter/2 = 14/2 = 7cm

Volume = $4/3 \times 22/7 \times (7)^3$

= 1437.33

Volume = 1437.33 cm^3

(ii) diameter = 3.5 dm

So, radius = diameter/2 = 3.5/2 = 1.75 dm

Volume = $4/3 \times 22/7 \times (1.75)^3$

= 22.46

Volume = 22.46 dm^3

(iii) diameter = 2.1 m

So, radius = diameter/2 = 2.1/2 = 1.05 m

Volume = $4/3 \times 22/7 \times (1.05)^3$
= 4.851

Volume = 4.851 m^3

Question 3: A hemispherical tank has the inner radius of 2.8 m. Find its capacity in liters.

Solution:

Radius of hemispherical tank = 2.8 m

Capacity of hemispherical tank = $2/3\ \pi r^3$

$=2/3 \times 22/7 \times (2.8)^3\ m^3$

= 45.997 m^3

[Using $1m^3$ = 1000 liters]

Therefore, capacity in litres = 45997 litres

Question 4: A hemispherical bowl is made of steel 0.25 cm thick. The inside radius of the bowl is 5 cm. Find the volume of steel used in making the bowl.

Solution:

Inner radius of a hemispherical bowl = 5 cm

Outer radius of a hemispherical bowl = 5 cm + 0.25 cm = 5.25 cm

Volume of steel used = Outer volume - Inner volume

$= 2/3 \times \pi \times ((5.25)^3 - (5)^3)$

$= 2/3 \times 22/7 \times ((5.25)^3 - (5)^3)$

$= 41.282$

Volume of steel used is 41.282 cm^3

Question 5: How many bullets can be made out of a cube of lead, whose edge measures 22 cm, each bullet being 2 cm in diameter?

Solution:

Edge of a cube = 22 cm
Diameter of bullet = 2 cm
So, radius of bullet (r) = 1 cm

Volume of the cube = $(side)^3 = (22)^3\ cm^3 = 10648\ cm^3$

And,

Volume of each bullet which will be spherical in shape = $4/3\pi r^3$

$= 4/3 \times 22/7 \times (1)^3\ cm^3$

$= 4/3 \times 22/7\ cm^3$

$= 88/21\ cm^3$

Number of bullets = (Volume of cube) / (Volume of bullet)

= 10648/88/21
= 2541

Therefore, 2541 bullets can be made.

Question 6: A shopkeeper has one laddoo of radius 5 cm. With the same material, how many laddoos of radius 2.5 cm can be made?

Solution:
Volume of laddoo having radius 5 cm (V1) = $4/3\times22/7\times(5)^3$

= $11000/21\ cm^3$

Also, Volume of laddoo having radius 2.5 cm (V2) = $4/3\pi r^3$

= $4/3\times22/7\times(2.5)^3\ cm^3$

= $1375/21\ cm^3$

Therefore,

Number of laddoos of radius 2.5 cm that can be made = V1/V2 = 11000/1375 = 8

Question 7: A spherical ball of lead 3 cm in diameter is melted and recast into three spherical balls. If the diameters of two balls be 3/2cm and 2 cm, find the diameter of the third ball.

Solution:
Volume of lead ball with radius 3/2 cm = $4/3\pi r^3$

= $4/3\times\pi\times(3/2)^3$

Let, Diameter of first ball (d1) = 3/2cm

Radius of first ball (r1) = 3/4 cm

Diameter of second ball (d2) = 2 cm

Radius of second ball (r2) = 2/2 cm = 1 cm

Diameter of third ball (d3) = d

Radius of third ball (r3) = d/2 cm

Now,

Volume of lead ball= $\frac{4}{3} \times \pi \times \left(\frac{3}{4}\right)^3 + \frac{4}{3} \times \pi \times (1)^3 + \frac{4}{3} \times \pi \times \left(\frac{d}{2}\right)^3$

$\frac{4}{3} \times \pi \times \left(\frac{3}{2}\right)^3 = \frac{4}{3} \times \pi \times \left(\frac{3}{4}\right)^3 + \frac{4}{3} \times \pi \times (1)^3 + \frac{4}{3} \times \pi \times \left(\frac{d}{2}\right)^3$

$\frac{4}{3}\pi\left[\left(\frac{3}{2}\right)^3\right] = \frac{4}{3}\pi\left[\left(\frac{3}{4}\right)^3 + (1)^3 + \left(\frac{d}{2}\right)^3\right]$

$\frac{27}{8} = \frac{27}{64} + 1 + \frac{d^3}{8}$

$d^3 = 8\left[\frac{27}{8} - \frac{27}{64} - 1\right]$

$\frac{d^3}{8} = \frac{125}{64}$

$\frac{d}{2} = \frac{5}{4}$

$d = \frac{10}{4}$

d= 2.5

So, diameter of third ball is 2.5 cm.

Question 8: A sphere of radius 5 cm is immersed in water filled in a cylinder, the level of water rises 5/3 cm. Find the radius of the cylinder.

Solution:
Radius of sphere = 5 cm (Given)
Let 'r' be the radius of cylinder.
We know, Volume of sphere = $4/3\pi r^3$

By putting values, we get

= $4/3\times\pi\times(5)^3$

Height (h) of water rises is 5/3 cm (Given)

Volume of water rises in cylinder = $\pi r^2 h$

Therefore, Volume of water rises in cylinder = Volume of sphere

So, $\pi r^2 h = 4/3\pi r^3$

$\pi\, r^2 \times 5/3 = 4/3 \times \pi \times (5)^3$

or $r^2 = 100$

or $r = 10$

Therefore, radius of the cylinder is 10 cm.

Question 9: If the radius of a sphere is doubled, what is the ratio of the volume of the first sphere to that of the second sphere?

Solution:

Let r be the radius of the first sphere then 2r be the radius of the second sphere.

Now,

$$\frac{\text{Volume of first sphere}}{\text{Volume of second sphere}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(2r)^3} = \frac{1}{8}$$

Ratio of volume of the first sphere to the second sphere is 1:8.

Question 10: A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

Solution:

Volume of the cone = Volume of the hemisphere (Given)

$1/3\pi r^2 h = 2/3\ \pi r^3$
(Using respective formulas)

$r^2 h = 2r^3$

or $h = 2r$

Since, cone and a hemisphere have equal bases which implies they have the same radius.
$h/r = 2$

or h : r = 2 : 1

Therefore, Ratio of their heights is 2:1

Question 11: A vessel in the form of a hemispherical bowl is full of water. Its contents are emptied in a right circular cylinder. The internal radii of the bowl and the cylinder are 3.5 cm and 7 cm respectively. Find the height to which the water will rise in the cylinder.

Solution:

Volume of water in the hemispherical bowl = Volume of water in the cylinder … (Given)

Inner radius of the bowl (r_1) = 3.5cm

Inner radius of cylinder (r_2) = 7cm

Volume of water in the hemispherical bowl = Volume of water in the cylinder

$2/3\pi r_1^3 = \pi r_2^2 h$
[Using respective formulas]

Where h be the height to which water rises in the cylinder.

$2/3\pi(3.5)^3 = \pi(7)^2 h$

or h = 7/12

Therefore, 7/12 cm be the height to which water rises in the cylinder.

Question 12: A cylinder whose height is two thirds of its diameter, has the same volume as a sphere of radius 4 cm. Calculate the radius of the base of the cylinder.

Solution:
Radius of a sphere (R)= 4 cm (Given)

Height of the cylinder = 2/3 diameter (given)

We know, Diameter = 2(Radius)

Let h be the height and r be the base radius of a cylinder, then

h = 2/3× (2r) = 4r/3

Volume of the cylinder = Volume of the sphere

$\pi r^2 h = 4/3\pi R^3$

$\pi \times r^2 \times (4r/3) = 4/3\ \pi\ (4)^3$

$(r)^3 = (4)^3$

or r = 4

Therefore, radius of the base of the cylinder is 4 cm.

Question 13: A vessel in the form of a hemispherical bowl is full of water. The contents are emptied into a cylinder. The internal radii of the bowl and cylinder are respectively 6 cm and 4 cm. Find the height of water in the cylinder.

Solution:
Radius of a bowl (R)= 6 cm (Given)

Radius of a cylinder (r) = 4 cm (given)

Let h be the height of a cylinder.
Now,
Volume of water in hemispherical bowl = Volume of cylinder

$2/3\ \pi\ R^3 = \pi r^2\ h$

$2/3\ \pi\ (6)^3 = \pi(4)^2\ h$

or h = 9

Therefore, height of water in the cylinder 9 cm.

Question 14: A cylindrical tub of radius 16 cm contains water to a depth of 30 cm. A spherical iron ball is dropped into the tub and thus level of water is raised by 9 cm. What is the radius of the ball?

Solution:

Let r be the radius of the iron ball.

Radius of the cylinder (R) = 16 cm (Given)

A spherical iron ball is dropped into the cylinder and thus the level of water is raised by 9 cm. So, height (h) = 9 cm

From statement,

Volume of iron ball = Volume of water raised in the hub

$4/3\pi r^3 = \pi R^2 h$

$4/3\ r^3 = (16)^2 \times 9$

or $r^3 = 1728$

or $r = 12$

Therefore, radius of the ball = 12cm.