

Exercise 3.2

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Question 1: Rationalise the denominators of each of the following (i – vii):

- (i) $3/\sqrt{5}$ (ii) $3/(2\sqrt{5})$ (iii) $1/\sqrt{12}$ (iv) $\sqrt{2}/\sqrt{5}$
 (v) $(\sqrt{3} + 1)/\sqrt{2}$ (vi) $(\sqrt{2} + \sqrt{5})/\sqrt{3}$ (vii) $3\sqrt{2}/\sqrt{5}$

Solution:

(i) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$= \frac{3 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$$

$$= \frac{3 \times \sqrt{5}}{5}$$

$$= 3\sqrt{5}/5$$

(ii) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{3}{2\sqrt{5}} = \frac{3 \times \sqrt{5}}{2 \times \sqrt{5} \times \sqrt{5}}$$

$$= \frac{3\sqrt{5}}{2 \times 5} = \frac{3\sqrt{5}}{10} = \frac{3}{10} \sqrt{5}$$

(iii) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{1}{\sqrt{12}} = \frac{1}{\sqrt{4 \times 3}} = \frac{1}{2\sqrt{3}}$$

$$= \frac{1 \times \sqrt{3}}{2\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{2 \times 3} = \frac{\sqrt{3}}{6}$$

(iv) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{\sqrt{10}}{5} = \frac{1}{5} \sqrt{10}$$

(v) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{\sqrt{3} + 1}{\sqrt{2}} = \frac{(\sqrt{3} + 1)\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{6} + \sqrt{2}}{2}$$

(vi) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{\sqrt{2} + \sqrt{5}}{\sqrt{3}} = \frac{(\sqrt{2} + \sqrt{5}) \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= \frac{\sqrt{6} + \sqrt{15}}{3}$$

(vii) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{3\sqrt{2}}{\sqrt{5}} = \frac{3\sqrt{2} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{3 \times \sqrt{10}}{5}$$

$$= \frac{3}{5} \sqrt{10}$$

Question 2: Find the value to three places of decimals of each of the following. It is given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$

(i) $\frac{2}{\sqrt{3}}$

(ii) $\frac{3}{\sqrt{10}}$

(iii) $\frac{\sqrt{5} + 1}{\sqrt{2}}$

(iv) $\frac{\sqrt{10} + \sqrt{15}}{\sqrt{2}}$

(v) $\frac{2 + \sqrt{3}}{3}$

(vi) $\frac{\sqrt{2} - 1}{\sqrt{5}}$

Solution:

(i) $\frac{2}{\sqrt{3}} = \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$$= \frac{2\sqrt{3}}{3} = \frac{2 \times 1.732}{3} = \frac{3.464}{3} = 1.154$$

(ii) $\frac{3}{\sqrt{10}} = \frac{3 \times \sqrt{10}}{\sqrt{10} \times \sqrt{10}} = \frac{3\sqrt{10}}{10}$

$$= \frac{3(3.162)}{10} = \frac{9.486}{10} = 0.9486$$

$$\begin{aligned}
 \text{(iii)} \quad \frac{\sqrt{5}+1}{\sqrt{2}} &= \frac{(\sqrt{5}+1) \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\
 &= \frac{\sqrt{10}+\sqrt{2}}{2} = \frac{3.162+1.414}{2} \\
 &= \frac{4.576}{2} = 2.288
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad \frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}} &= \frac{(\sqrt{10}+\sqrt{15})\sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\
 &= \frac{\sqrt{20}+\sqrt{30}}{2} = \frac{2\sqrt{5}+\sqrt{10} \times \sqrt{3}}{2} \\
 &= \frac{2(2.236)+3.162 \times 1.732}{2} = 4.974
 \end{aligned}$$

$$\text{(v)} \quad \frac{2+\sqrt{3}}{3} = \frac{2+1.732}{3} = \frac{3.732}{3} = 1.244$$

$$\begin{aligned}
 \text{(vi)} \quad \frac{\sqrt{2}-1}{\sqrt{5}} &= \frac{(\sqrt{2}-1) \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} \\
 &= \frac{\sqrt{10}-\sqrt{5}}{5} = \frac{3.162-2.236}{5} \\
 &= \frac{0.926}{5} = 0.185
 \end{aligned}$$

Question 3: Express each one of the following with rational denominator:

$$\text{(i)} \quad \frac{1}{3+\sqrt{2}} \quad \text{(ii)} \quad \frac{1}{\sqrt{6}-\sqrt{5}} \quad \text{(iii)} \quad \frac{16}{\sqrt{41}-5}$$

$$\text{(iv)} \quad \frac{30}{5\sqrt{3}-3\sqrt{5}} \quad \text{(v)} \quad \frac{1}{2\sqrt{5}-\sqrt{3}} \quad \text{(vi)} \quad \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$$

$$\text{(vii)} \quad \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \quad \text{(viii)} \quad \frac{3\sqrt{2}+1}{2\sqrt{5}-3} \quad \text{(ix)} \quad \frac{b^2}{\sqrt{a^2+b^2}+a}$$

Solution:Using identity: $(a + b)(a - b) = a^2 - b^2$ **(i)** Multiply and divide given number by $3 - \sqrt{2}$

$$\begin{aligned} & \frac{1}{3 + \sqrt{2}} \\ &= \frac{3 - \sqrt{2}}{(3 + \sqrt{2})(3 - \sqrt{2})} \\ &= \frac{3 - \sqrt{2}}{9 - 2} \\ &= \frac{3 - \sqrt{2}}{7} \end{aligned}$$

(ii) Multiply and divide given number by $\sqrt{6} + \sqrt{5}$

$$\begin{aligned} & \frac{1}{\sqrt{6} - \sqrt{5}} \\ &= \frac{\sqrt{6} + \sqrt{5}}{(\sqrt{6} - \sqrt{5})(\sqrt{6} + \sqrt{5})} \\ &= \frac{\sqrt{6} + \sqrt{5}}{6 - 5} \\ &= \sqrt{6} + \sqrt{5} \end{aligned}$$

(iii) Multiply and divide given number by $\sqrt{41} + 5$

$$\begin{aligned} & \frac{16}{\sqrt{41}-5} \\ &= \frac{16 \times (\sqrt{41}+5)}{(\sqrt{41}-5)(\sqrt{41}+5)} \\ &= \frac{16\sqrt{41}+80}{41-25} \\ &= \frac{16\sqrt{41}+80}{16} \\ &= \frac{16(\sqrt{41}+5)}{16} \\ &= \sqrt{41} + 5 \end{aligned}$$

(iv) Multiply and divide given number by $5\sqrt{3} + 3\sqrt{5}$

$$\begin{aligned} & \frac{30}{5\sqrt{3}-3\sqrt{5}} \\ &= \frac{30 \times (5\sqrt{3}+3\sqrt{5})}{(5\sqrt{3}-3\sqrt{5})(5\sqrt{3}+3\sqrt{5})} \\ &= \frac{30 \times (5\sqrt{3}+3\sqrt{5})}{75-45} \\ &= \frac{30 \times (5\sqrt{3}+3\sqrt{5})}{30} \\ &= 5\sqrt{3} + 3\sqrt{5} \end{aligned}$$

(v) Multiply and divide given number by $2\sqrt{5} + \sqrt{3}$

$$\begin{aligned} & \frac{1}{2\sqrt{5}-\sqrt{3}} \\ &= \frac{2\sqrt{5}+\sqrt{3}}{(2\sqrt{5}-\sqrt{3})(2\sqrt{5}+\sqrt{3})} \\ &= \frac{2\sqrt{5}+\sqrt{3}}{20-3} \\ &= \frac{2\sqrt{5}+\sqrt{3}}{17} \end{aligned}$$

(vi) Multiply and divide given number by $2\sqrt{2} + \sqrt{3}$

$$\begin{aligned} & \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}} \\ &= \frac{(\sqrt{3}+1)(2\sqrt{2}+\sqrt{3})}{(2\sqrt{2}+\sqrt{3})(2\sqrt{2}-\sqrt{3})} \\ &= \frac{(2\sqrt{6}+3+2\sqrt{2}+\sqrt{3})}{8-3} \\ &= \frac{(2\sqrt{6}+3+2\sqrt{2}+\sqrt{3})}{5} \end{aligned}$$

(vii) Multiply and divide given number by $6 - 4\sqrt{2}$

$$\begin{aligned} & \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \\ &= \frac{(6-4\sqrt{2})(6-4\sqrt{2})}{(6+4\sqrt{2})(6-4\sqrt{2})} \\ &= \frac{(6-4\sqrt{2})^2}{36-32} \\ &= \frac{36-48\sqrt{2}+32}{4} \\ &= \frac{68-48\sqrt{2}}{4} \\ &= \frac{4(17-12\sqrt{2})}{4} \\ &= 17 - 12\sqrt{2} \end{aligned}$$

(viii) Multiply and divide given number by $2\sqrt{5} + 3$

$$\begin{aligned} & \frac{3\sqrt{2}+1}{2\sqrt{5}-3} \\ &= \frac{(3\sqrt{2}+1) \times (2\sqrt{5}+3)}{(2\sqrt{5}-3)(2\sqrt{5}+3)} \\ &= \frac{6\sqrt{10}+9\sqrt{2}+2\sqrt{5}+3}{(20-9)} \\ &= \frac{6\sqrt{10}+9\sqrt{2}+2\sqrt{5}+3}{11} \end{aligned}$$

(ix) Multiply and divide given number by $\sqrt{a^2+b^2} - a$

$$\begin{aligned} & \frac{b^2}{\sqrt{a^2+b^2}+a} \\ &= \frac{b^2(\sqrt{a^2+b^2}-a)}{(\sqrt{a^2+b^2}+a)(\sqrt{a^2+b^2}-a)} \\ &= \frac{b^2(\sqrt{a^2+b^2}-a)}{(a^2+b^2)-a^2} \\ &= \frac{b^2(\sqrt{a^2+b^2}-a)}{b^2} \end{aligned}$$

Question 4: Rationalise the denominator and simplify:

(i) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (ii) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$ (iii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$

(iv) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$ (v) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$ (vi) $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

Solution:

[Use identities: $(a + b)(a - b) = a^2 - b^2$; $(a + b)^2 = (a^2 + 2ab + b^2)$ and $(a - b)^2 = (a^2 - 2ab + b^2)$]

(i) Multiply both numerator and denominator by $\sqrt{3}-\sqrt{2}$ to rationalise the denominator.

$$\begin{aligned} & \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \\ &= \frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} \\ &= \frac{(\sqrt{3}-\sqrt{2})^2}{3-2} \\ &= \frac{3-2\sqrt{3}\sqrt{2}+2}{1} \\ &= 5 - 2\sqrt{6} \end{aligned}$$

(ii) Multiply both numerator and denominator by $7-4\sqrt{3}$ to rationalise the denominator.

$$\begin{aligned} & \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \\ &= \frac{(5+2\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})} \\ &= \frac{(5+2\sqrt{3})(7-4\sqrt{3})}{49-48} \\ &= 35 - 20\sqrt{3} + 14\sqrt{3} - 24 \\ &= 11 - 6\sqrt{3} \end{aligned}$$

(iii) Multiply both numerator and denominator by $3+2\sqrt{2}$ to rationalise the denominator.

$$\begin{aligned} & \frac{1+\sqrt{2}}{3-2\sqrt{2}} \\ &= \frac{(1+\sqrt{2})(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})} \\ &= \frac{(1+\sqrt{2})(3+2\sqrt{2})}{9-8} \\ &= 3 + 2\sqrt{2} + 3\sqrt{2} + 4 \\ &= 7 + 5\sqrt{2} \end{aligned}$$

(iv) Multiply both numerator and denominator by $3\sqrt{5}+2\sqrt{6}$ to rationalise the denominator.

$$\begin{aligned} & \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} \\ &= \frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{(3\sqrt{5}-2\sqrt{6})(3\sqrt{5}+2\sqrt{6})} \\ &= \frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{45-24} \\ &= \frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{21} \\ &= \frac{6\sqrt{30}+24-15-2\sqrt{30}}{21} \\ &= \frac{4\sqrt{30}+9}{21} \end{aligned}$$

(v) Multiply both numerator and denominator by $\sqrt{48}-\sqrt{18}$ to rationalise the denominator.

$$\begin{aligned} & \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} \\ &= \frac{(4\sqrt{3}+5\sqrt{2})(\sqrt{48}-\sqrt{18})}{(\sqrt{48}+\sqrt{18})(\sqrt{48}-\sqrt{18})} \\ &= \frac{(4\sqrt{3}+5\sqrt{2})(\sqrt{48}-\sqrt{18})}{48-18} \\ &= \frac{48-12\sqrt{6}+20\sqrt{6}-30}{30} \\ &= \frac{18+8\sqrt{6}}{30} \\ &= \frac{9+4\sqrt{6}}{15} \end{aligned}$$

(vi) Multiply both numerator and denominator by $2\sqrt{2}-3\sqrt{3}$ to rationalise the denominator.

$$\begin{aligned} & \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} \\ &= \frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{3})} \\ &= \frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{8-27} \\ &= \frac{(4\sqrt{6}-2\sqrt{10})-18+3\sqrt{15}}{-19} \\ &= \frac{(18-4\sqrt{6}+2\sqrt{10}-3\sqrt{15})}{19} \end{aligned}$$

