

Exercise VSAQs

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Question 1: Write the value of $(2 + \sqrt{3})(2 - \sqrt{3})$.**Solution:**

$$(2 + \sqrt{3})(2 - \sqrt{3})$$

$$= (2)^2 - (\sqrt{3})^2$$

$$[\text{Using identity : } (a + b)(a - b) = a^2 - b^2]$$

$$= 4 - 3$$

$$= 1$$

Question 2: Write the reciprocal of $5 + \sqrt{2}$.**Solution:**

$$\text{Reciprocal of } 5 + \sqrt{2} = \frac{1}{5 + \sqrt{2}}$$

Rationalisation of fraction

Multiply and divide given fraction by $5 - \sqrt{2}$

$$\begin{aligned} &= \frac{5 - \sqrt{2}}{(5 + \sqrt{2})(5 - \sqrt{2})} \\ &= \frac{5 - \sqrt{2}}{(5)^2 - (\sqrt{2})^2} \\ &= \frac{5 - \sqrt{2}}{25 - 2} \\ &= \frac{5 - \sqrt{2}}{23} \end{aligned}$$

Question 3: Write the rationalisation factor of $7 - 3\sqrt{5}$.**Solution:**Rationalisation factor of $7 - 3\sqrt{5}$ is $7 + 3\sqrt{5}$ **Question 4: If**

$$\frac{\sqrt{3} - 1}{\sqrt{3} + 1} = x + y\sqrt{3}$$

Find the values of x and y.

Solution:

[Using identities : $(a + b)(a - b) = a^2 - b^2$ and $(a - b)^2 = a^2 + b^2 - 2ab$]

Rationalising Denominator

$$\frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)} = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{3 + 1 - 2\sqrt{3}}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

Now,

$$2 - \sqrt{3} = x + y\sqrt{3}$$

On comparing,

$$x = 2, y = -1$$

Question 5: If $x = \sqrt{2} - 1$, then write the value of $1/x$.

Solution:

$$x = \sqrt{2} - 1$$

$$\text{or } 1/x = 1/(\sqrt{2} - 1)$$

Rationalising denominator, we have

$$= 1/(\sqrt{2} - 1) \times (\sqrt{2} + 1)/(\sqrt{2} + 1)$$

$$= (\sqrt{2} + 1)/(2 - 1)$$

$$= \sqrt{2} + 1$$

Question 6: Simplify

$$\sqrt{3 + 2\sqrt{2}}$$

Solution:

$$\sqrt{3 + 2\sqrt{2}}$$

$$= \sqrt{2 + 1 + 2\sqrt{2}}$$

$$= \sqrt{(\sqrt{2})^2 + (1)^2 + 2 \times \sqrt{2} \times 1}$$

$$= \sqrt{(\sqrt{2} + 1)^2} = \sqrt{2} + 1$$

[Because: $(a + b)^2 = a^2 + b^2 + 2ab$]

Question 7: Simplify

$$\sqrt{3 - 2\sqrt{2}}$$

Solution:

$$\begin{aligned} & \sqrt{3 - 2\sqrt{2}} \\ &= \sqrt{2 + 1 - 2\sqrt{2}} \\ &= \sqrt{(\sqrt{2})^2 + (1)^2 - 2 \times \sqrt{2} \times 1} \\ &= \sqrt{(\sqrt{2} - 1)^2} = \sqrt{2} - 1 \end{aligned}$$

[Because: $(a - b)^2 = a^2 + b^2 - 2ab$]**Question 8:** If $a = \sqrt{2} + 1$, then find the value of $a - 1/a$.**Solution:**

Given: $a = \sqrt{2} + 1$

$$1/a = 1/(\sqrt{2} + 1)$$

$$= 1/(\sqrt{2} + 1) \times (\sqrt{2} - 1)/(\sqrt{2} - 1)$$

$$= (\sqrt{2} - 1)/((\sqrt{2})^2 - (1)^2)$$

$$= (\sqrt{2} - 1)/1$$

$$= \sqrt{2} - 1$$

Now,

$$a - 1/a = (\sqrt{2} + 1) - (\sqrt{2} - 1)$$

$$= 2$$

Question 9: If $x = 2 + \sqrt{3}$, find the value of $x + 1/x$.**Solution:**

Given: $x = 2 + \sqrt{3}$

$$1/x = 1/(2 + \sqrt{3})$$

$$= 1/(2 + \sqrt{3}) \times (2 - \sqrt{3})/(2 - \sqrt{3})$$

$$= (2 - \sqrt{3}) / ((2)^2 - (\sqrt{3})^2)$$

$$= (2 - \sqrt{3}) / (4 - 3)$$

$$= (2 - \sqrt{3})$$

Now,

$$x + 1/x = (2 + \sqrt{3}) + (2 - \sqrt{3})$$

$$= 4$$

Question 10: Write the rationalisation factor of $\sqrt{5} - 2$.

Solution:

Rationalisation factor of $\sqrt{5} - 2$ is $\sqrt{5} + 2$

Question 11: If $x = 3 + 2\sqrt{2}$, then find the value of $\sqrt{x} - 1/\sqrt{x}$.

Solution:

$$x = 3 + 2\sqrt{2}$$

$$\begin{aligned} \frac{1}{x} &= \frac{1}{3 + 2\sqrt{2}} = \frac{(3 - 2\sqrt{2})}{(3 + 2\sqrt{2})(3 - 2\sqrt{2})} \\ &= \frac{3 - 2\sqrt{2}}{(3)^2 - (2\sqrt{2})^2} = \frac{3 - 2\sqrt{2}}{9 - 8} = \frac{3 - 2\sqrt{2}}{1} \end{aligned}$$

$$x + \frac{1}{x} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6$$

$$\begin{aligned} \text{Now, } \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 &= x + \frac{1}{x} - 2 \\ &= 6 - 2 = 4 = (2)^2 \end{aligned}$$

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) = 2$$