## Exercise 3.1

Question 1: Simplify each of the following:
(i) $\sqrt[3]{4} \times \sqrt[3]{16}$
(ii) $\frac{\sqrt[4]{1250}}{\sqrt[4]{2}}$

## Solution:

(i)

Using: $\sqrt[n]{a} \times \sqrt[n]{b}=\sqrt[n]{a \times b}$
$=\sqrt[3]{4 \times 16}$
$=\sqrt[3]{64}$
$=\sqrt[3]{4^{3}}$
$=\left(4^{3}\right)^{\frac{1}{3}}$
$=4$
(ii)

$$
\text { (Note: } \frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}} \text { ) }
$$

$=\sqrt[4]{\frac{1250}{2}}$
$=\sqrt[4]{\frac{2 \times 625}{2}}$
$=\sqrt[4]{625}$
$=\sqrt[4]{5^{4}}$
$=5^{\left(4 \times \frac{1}{4}\right)}$
$=5$

Question 2: Simplify the following expressions:
(i) $(4+\sqrt{ } 7)(3+\sqrt{ } 2)$
(ii) $(3+\sqrt{ } 3)(5-\sqrt{ } 2)$
(iii) $(\sqrt{ } 5-2)(\sqrt{ } 3-\sqrt{ })$

Solution:
(i) $(4+\sqrt{ } 7)(3+\sqrt{ } 2)$
$=12+4 \sqrt{ } 2+3 \sqrt{ } 7+\sqrt{ } 14$
(ii) $(3+\sqrt{ } 3)(5-\sqrt{ } 2)$
$=15-3 \sqrt{ } 2+5 \sqrt{ } 3-\sqrt{ } 6$
(iii) $(\sqrt{ } 5-2)(\sqrt{ } 3-\sqrt{ } 5)$
$=\mathrm{V} 15-\mathrm{V} 25-2 \mathrm{~V} 3+2 \mathrm{~V} 5$
$=\sqrt{ } 15-5-2 \sqrt{ } 3+2 \sqrt{ } 5$

Question 3: Simplify the following expressions:
(i) $(11+\mathrm{V} 11)(11-\mathrm{V} 11)$
(ii) $(5+\sqrt{ } 7)(5-\sqrt{ })$
(iii) $(\mathbf{V} \mathbf{8}-\mathbf{V} \mathbf{2})(\mathbf{V 8}+\mathbf{V} \mathbf{2})$
(iv) $(3+\sqrt{ } 3)(3-\sqrt{ } 3)$
(v) $(\sqrt{ } 5-\sqrt{ } 2)(\sqrt{ } 5+\sqrt{ } 2)$

## Solution:

Using Identity: $(a-b)(a+b)=a^{\wedge} 2-b^{\wedge} 2$
(i) $(11+\mathrm{V} 11)(11-\mathrm{V} 11)$
$=11^{\wedge} 2-(\mathrm{V} 11)^{\wedge} 2$
= 121 - 11
= 110
(ii) $(5+\sqrt{ })(5-\mathrm{V} 7)$
$=\left(5^{\wedge} 2-(\sqrt{ } 7)^{\wedge} 2\right)$
$=25-7=18$
(iii) $(\sqrt{ } 8-\sqrt{ } 2)(\sqrt{ } 8+\sqrt{ } 2)$
$=(\mathrm{V} 8)^{\wedge} 2-(\mathrm{V} 2)^{\wedge} 2$
= 8-2
$=6$
(iv) $(3+\sqrt{ } 3)(3-\sqrt{ } 3)$
$=(3)^{\wedge} 2-(\sqrt{ } 3)^{\wedge} 2$
$=9-3$
$=6$
(v) $(\sqrt{ } 5-\sqrt{ } 2)(\sqrt{ } 5+\sqrt{ } 2)$
$=(\sqrt{ } 5)^{\wedge} 2-(\sqrt{ })^{\wedge}{ }^{\wedge}$
$=5-2$
$=3$

Question 4: Simplify the following expressions:
(i) $(\sqrt{ } 3+\sqrt{ } 7)^{2}$
(ii) $(\sqrt{ } 5-\sqrt{ })^{2}$
(iii) $(2 \sqrt{ } 5+3 \sqrt{ } 2)^{2}$

Solution:
Using identities: $(a-b)^{\wedge} 2=a^{\wedge} 2+b^{\wedge} 2-2 a b$ and $(a+b)^{\wedge} 2=a^{\wedge} 2+b^{\wedge} 2+2 a b$
(i) $(\sqrt{ } 3+\sqrt{ } 7)^{2}$
$=(\sqrt{ } 3)^{\wedge} 2+(v 7)^{2}+2(\sqrt{ } 3)(\sqrt{ } 7)$
$=3+7+2 \mathrm{~V} 21$
$=10+2 \mathrm{~V} 21$
(ii) $(\sqrt{ } 5-\sqrt{ } 3)^{2}$
$=(\sqrt{ } 5)^{\wedge} 2+(\sqrt{ } 3)^{2}-2(V 5)(\sqrt{ } 3)$
$=5+3-2 \mathrm{~V} 15$
$=8-2 \mathrm{~V} 15$
(iii) $(2 \sqrt{ } 5+3 \sqrt{ } 2)^{2}$
$=(2 \sqrt{ } 5)^{\wedge} 2+(3 \sqrt{ } 2)^{2}+2(2 \sqrt{ } 5)(3 \sqrt{ } 2)$
$=20+18+12 \mathrm{~V} 10$
$=38+12 \mathrm{~V} 10$

## Exercise 3.2

Question 1: Rationalise the denominators of each of the following ( $\mathbf{i}-\mathrm{vii}$ ):
$\begin{array}{llll}\text { (i) } 3 / \sqrt{ } 5 \text { (ii) } 3 /(2 \mathrm{~V} 5) \text { (iii) } 1 / \sqrt{ } 12 \text { (iv) } \sqrt{ } 2 / \sqrt{ } 5 \\ \text { (v) }(\sqrt{ } 3+1) / \sqrt{ } 2 \quad \text { (vi) }(\sqrt{ } 2+\sqrt{ } 5) / \sqrt{ } 3 & \text { (vii) } 3 \sqrt{ } 2 / \sqrt{ } 5\end{array}$
Solution:
(i) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$
\begin{aligned}
& =\frac{3 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} \\
& =\frac{3 \times \sqrt{5}}{5} \\
& =3 \sqrt{ } 5 / 5
\end{aligned}
$$

(ii) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$
\begin{aligned}
& \frac{3}{2 \sqrt{5}}=\frac{3 \times \sqrt{5}}{2 \times \sqrt{5} \times \sqrt{5}} \\
& =\frac{3 \sqrt{5}}{2 \times 5}=\frac{3 \sqrt{5}}{10}=\frac{3}{10} \sqrt{5}
\end{aligned}
$$

(iii) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$
\begin{aligned}
& \frac{1}{\sqrt{12}}=\frac{1}{\sqrt{4 \times 3}}=\frac{1}{2 \sqrt{3}} \\
& =\frac{1 \times \sqrt{3}}{2 \sqrt{3} \times \sqrt{3}}=\frac{\sqrt{3}}{2 \times 3}=\frac{\sqrt{3}}{6}
\end{aligned}
$$

(iv) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$
\frac{\sqrt{2}}{\sqrt{5}}=\frac{\sqrt{2} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}=\frac{\sqrt{10}}{5}=\frac{1}{5} \sqrt{10}
$$

(v) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$
\frac{\sqrt{3}+1}{\sqrt{2}}=\frac{(\sqrt{3}+1) \sqrt{2}}{\sqrt{2} \times \sqrt{2}}=\frac{\sqrt{6}+\sqrt{2}}{2}
$$

(vi) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$
\begin{aligned}
& \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}=\frac{(\sqrt{2}+\sqrt{5}) \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\
& =\frac{\sqrt{6}+\sqrt{15}}{3}
\end{aligned}
$$

(vii) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$
\begin{aligned}
& \frac{3 \sqrt{2}}{\sqrt{5}}=\frac{3 \sqrt{2} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}=\frac{3 \times \sqrt{10}}{5} \\
& =\frac{3}{5} \sqrt{10}
\end{aligned}
$$

Question 2: Find the value to three places of decimals of each of the following. It is given that $\mathrm{V} 2=1.414, \mathrm{~V} 3=1.732, \mathrm{~V} 5=2.236$ and $\mathrm{V} 10=3.162$
(i) $\frac{2}{\sqrt{3}}$
(ii) $\frac{3}{\sqrt{10}}$
(iii) $\frac{\sqrt{5}+1}{\sqrt{2}}$
(iv) $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$
(v) $\frac{2+\sqrt{3}}{3}$
(vi) $\frac{\sqrt{2}-1}{\sqrt{5}}$

Solution:
(i) $\frac{2}{\sqrt{3}}=\frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$$
=\frac{2 \sqrt{3}}{3}=\frac{2 \times 1.732}{3}=\frac{3.464}{3}=1.154
$$

(ii) $\frac{3}{\sqrt{10}}=\frac{3 \times \sqrt{10}}{\sqrt{10} \times \sqrt{10}}=\frac{3 \sqrt{10}}{10}$

$$
=\frac{3(3.162)}{10}=\frac{9.486}{10}=0.9486
$$

(iii) $\frac{\sqrt{5}+1}{\sqrt{2}}=\frac{(\sqrt{5}+1) \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

$$
\begin{aligned}
& =\frac{\sqrt{10}+\sqrt{2}}{2}=\frac{3.162+1.414}{2} \\
& =\frac{4.576}{2}=2.288
\end{aligned}
$$

(iv) $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}=\frac{(\sqrt{10}+\sqrt{15}) \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

$$
=\frac{\sqrt{20}+\sqrt{30}}{2}=\frac{2 \sqrt{5}+\sqrt{10} \times \sqrt{3}}{2}
$$

$$
=\frac{2(2.236)+3.162 \times 1.732}{2}=4.974
$$

(v) $\frac{2+\sqrt{3}}{3}=\frac{2+1.732}{3}=\frac{3.732}{3}=1.244$
(vi) $\frac{\sqrt{2}-1}{\sqrt{5}}=\frac{(\sqrt{2}-1) \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$

$$
\begin{aligned}
& =\frac{\sqrt{10}-\sqrt{5}}{5}=\frac{3.162-2.236}{5} \\
& =\frac{0.926}{5}=0.185
\end{aligned}
$$

Question 3: Express each one of the following with rational denominator:
(i) $\frac{1}{3+\sqrt{2}}$
(ii) $\frac{1}{\sqrt{6}-\sqrt{5}}$
(iii) $\frac{16}{\sqrt{41}-5}$
(iv) $\frac{30}{5 \sqrt{3}-3 \sqrt{5}}$
(v) $\frac{1}{2 \sqrt{5}-\sqrt{3}}$
(vi) $\frac{\sqrt{3}+1}{2 \sqrt{2}-\sqrt{3}}$
(vii) $\frac{6-4 \sqrt{2}}{6+4 \sqrt{2}}$
(viii) $\frac{3 \sqrt{2}+1}{2 \sqrt{5}-3}$
(ix) $\frac{b^{2}}{\sqrt{a^{2}+b^{2}}+a}$

## Solution:

Using identity: $(a+b)(a-b)=a^{\wedge} 2-b^{\wedge} 2$
(i) Multiply and divide given number by 3-12

$$
\begin{aligned}
& \frac{1}{3+\sqrt{2}} \\
= & \frac{3-\sqrt{2}}{(3+\sqrt{2})(3-\sqrt{2})} \\
= & \frac{3-\sqrt{2}}{9-2} \\
= & \frac{3-\sqrt{2}}{7}
\end{aligned}
$$

(ii) Multiply and divide given number by $\sqrt{ } 6+\sqrt{ } 5$

$$
\begin{aligned}
& \frac{1}{\sqrt{6}-\sqrt{5}} \\
& =\frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})} \\
& =\frac{\sqrt{6}+\sqrt{5}}{6-5} \\
& =\sqrt{6}+\sqrt{5}
\end{aligned}
$$

(iii) Multiply and divide given number by $\sqrt{ } 41+5$
(v) Multiply and divide given number by $2 \sqrt{ } 5+\sqrt{ } 3$

$$
\frac{1}{2 \sqrt{5}-\sqrt{3}}
$$

$$
=\frac{2 \sqrt{5}+\sqrt{3}}{(2 \sqrt{5}-\sqrt{3})(2 \sqrt{5}+\sqrt{3})}
$$

$$
=\frac{2 \sqrt{5}+\sqrt{3}}{20-3}
$$

$$
=\frac{2 \sqrt{5}+\sqrt{3}}{17}
$$

$$
\begin{aligned}
& \frac{16}{\sqrt{41}-5} \\
& =\frac{16 \times(\sqrt{41}+5)}{(\sqrt{41}-5)(\sqrt{41}+5)} \\
& =\frac{16 \sqrt{41}+80}{41-25} \\
& =\frac{16 \sqrt{41}+80}{16} \\
& =\frac{16(\sqrt{41}+5)}{16} \\
& =\sqrt{41}+5 \\
& \text { (iv) Multiply and divide given number by } 5 \sqrt{ } 3+3 \sqrt{ } 5 \\
& \frac{30}{5 \sqrt{3}-3 \sqrt{5}} \\
& =\frac{30 \times(5 \sqrt{3}+3 \sqrt{5})}{(5 \sqrt{3}-3 \sqrt{5})(5 \sqrt{3}+3 \sqrt{5})} \\
& =\frac{30 \times(5 \sqrt{3}+3 \sqrt{5})}{75-45} \\
& =\frac{30 \times(5 \sqrt{3}+3 \sqrt{5})}{30} \\
& =5 \sqrt{3}+3 \sqrt{5}
\end{aligned}
$$

(vi) Multiply and divide given number by $2 \sqrt{ } 2+\sqrt{ } 3$

$$
\begin{aligned}
& \frac{\sqrt{3}+1}{2 \sqrt{2}-\sqrt{3}} \\
= & \frac{(\sqrt{3}+1)(2 \sqrt{2}+\sqrt{3})}{(2 \sqrt{2}+\sqrt{3})(2 \sqrt{2}-\sqrt{3})} \\
= & \frac{(2 \sqrt{6}+3+2 \sqrt{2}+\sqrt{3})}{8-3} \\
= & \frac{(2 \sqrt{6}+3+2 \sqrt{2}+\sqrt{3})}{5}
\end{aligned}
$$

(vii) Multiply and divide given number by $6-4 \sqrt{ } 2$

$$
\begin{aligned}
& \frac{6-4 \sqrt{2}}{6+4 \sqrt{2}} \\
= & \frac{(6-4 \sqrt{2})(6-4 \sqrt{2})}{(6+4 \sqrt{2})(6-4 \sqrt{2})} \\
= & \frac{(6-4 \sqrt{2})^{2}}{36-32} \\
= & \frac{36-48 \sqrt{2}+32}{4} \\
= & \frac{68-48 \sqrt{2}}{4} \\
= & \frac{4(17-12 \sqrt{2})}{4} \\
= & 17-12 \sqrt{2}
\end{aligned}
$$

(viii) Multiply and divide given number by $2 \sqrt{ } 5+3$

$$
\begin{aligned}
& \frac{3 \sqrt{2}+1}{2 \sqrt{5}-3} \\
& =\frac{(3 \sqrt{2}+1) \times(2 \sqrt{5}+3)}{(2 \sqrt{5}-3)(2 \sqrt{5}+3)} \\
& =\frac{6 \sqrt{10}+9 \sqrt{2}+2 \sqrt{5}+3}{(20-9)} \\
& =\frac{6 \sqrt{10}+9 \sqrt{2}+2 \sqrt{5}+3}{11}
\end{aligned}
$$

(ix) Multiply and divide given number by $V\left(a^{\wedge} 2+b^{\wedge} 2\right)-a$

$$
\begin{aligned}
& \frac{b^{2}}{\sqrt{\left(a^{2}+b^{2}\right)}+a} \\
= & \frac{b^{2}\left(\sqrt{\left(a^{2}+b^{2}\right)}-a\right)}{\left(\sqrt{\left(a^{2}+b^{2}\right)}+a\right)\left(\sqrt{\left(a^{2}+b^{2}\right)}-a\right)} \\
= & \frac{b^{2}\left(\sqrt{\left(a^{2}+b^{2}\right)}-a\right)}{\left.\left(a^{2}+b^{2}\right)-a^{2}\right)} \\
= & \frac{b^{2}\left(\sqrt{\left(a^{2}+b^{2}\right)}-a\right)}{b^{2}}
\end{aligned}
$$

Question 4: Rationales the denominator and simplify:
(i) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$
(ii) $\frac{5+2 \sqrt{3}}{7+4 \sqrt{3}}$
(iii) $\frac{1+\sqrt{2}}{3-2 \sqrt{2}}$
(iv) $\frac{2 \sqrt{6}-\sqrt{5}}{3 \sqrt{5}-2 \sqrt{6}}$
(v) $\frac{4 \sqrt{3}+5 \sqrt{2}}{\sqrt{48}+\sqrt{18}}$
(vi) $\frac{2 \sqrt{3}-\sqrt{5}}{2 \sqrt{2}+3 \sqrt{3}}$

Solution:
[Use identities: $(a+b)(a-b)=a^{2}-b^{2} ;(a+b)^{2}=\left(a^{2}+2 a b+b^{2}\right.$ and $(a-b)^{2}=\left(a^{2}-2 a b+b^{2}\right]$
(i) Multiply both numerator and denominator by $\sqrt{ } 3-\sqrt{ } 2$ to rationalise the denominator.

$$
\begin{aligned}
& \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \\
= & \frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} \\
= & \frac{(\sqrt{3}-\sqrt{2})^{2}}{3-2} \\
= & \frac{3-2 \sqrt{3} \sqrt{2}+2}{1} \\
= & 5-2 \sqrt{6}
\end{aligned}
$$

(ii) Multiply both numerator and denominator by $7-4 \sqrt{ } 3$ to rationalise the denominator.

$$
\begin{aligned}
& \frac{5+2 \sqrt{3}}{7+4 \sqrt{3}} \\
= & \frac{(5+2 \sqrt{3})(7-4 \sqrt{3})}{(7+4 \sqrt{3})(7-4 \sqrt{3})} \\
= & \frac{(5+2 \sqrt{3})(7-4 \sqrt{3})}{49-48} \\
= & 35-20 \sqrt{3}+14 \sqrt{3}-24 \\
= & 11-6 \sqrt{3}
\end{aligned}
$$

(iii) Multiply both numerator and denominator by $3+2 \sqrt{ } 2$ to rationalise the denominator.

$$
\begin{aligned}
& \frac{1+\sqrt{2}}{3-2 \sqrt{2}} \\
& =\frac{(1+\sqrt{2})(3+2 \sqrt{2})}{(3-2 \sqrt{2})(3+2 \sqrt{2})} \\
& =\frac{(1+\sqrt{2})(3+2 \sqrt{2})}{9-8} \\
& =3+2 \sqrt{2}+3 \sqrt{2}+4 \\
& =7+5 \sqrt{2}
\end{aligned}
$$

(iv) Multiply both numerator and denominator by $3 \sqrt{ } 5+2 \sqrt{ } 6$ to rationalise the denominator.

$$
\begin{aligned}
& \frac{2 \sqrt{6}-\sqrt{5}}{3 \sqrt{5}-2 \sqrt{6}} \\
= & \frac{(2 \sqrt{6}-\sqrt{5})(3 \sqrt{5}+2 \sqrt{6})}{(3 \sqrt{5}-2 \sqrt{6})(3 \sqrt{5}+2 \sqrt{6})} \\
= & \frac{(2 \sqrt{6}-\sqrt{5})(3 \sqrt{5}+2 \sqrt{6})}{45-24} \\
= & \frac{(2 \sqrt{6}-\sqrt{5})(3 \sqrt{5}+2 \sqrt{6})}{21} \\
= & \frac{6 \sqrt{30}+24-15-2 \sqrt{30}}{21} \\
= & \frac{4 \sqrt{30}+9}{21}
\end{aligned}
$$

(v) Multiply both numerator and denominator by $\sqrt{ } 48-\sqrt{ } 18$ to rationalise the denominator.

$$
\begin{aligned}
& \frac{4 \sqrt{3}+5 \sqrt{2}}{\sqrt{48}+\sqrt{18}} \\
= & \frac{(4 \sqrt{3}+5 \sqrt{2})(\sqrt{48}-\sqrt{18})}{(\sqrt{48}+\sqrt{18})(\sqrt{48}-\sqrt{18})} \\
= & \frac{(4 \sqrt{3}+5 \sqrt{2})(\sqrt{48}-\sqrt{18})}{48-18} \\
= & \frac{48-12 \sqrt{6}+20 \sqrt{6}-30}{30} \\
= & \frac{18+8 \sqrt{6}}{30} \\
= & \frac{9+4 \sqrt{6}}{15}
\end{aligned}
$$

(vi) Multiply both numerator and denominator by $2 \sqrt{ } 2-3 \sqrt{ } 3$ to rationalise the denominator.

$$
\begin{aligned}
& \frac{2 \sqrt{3}-\sqrt{5}}{2 \sqrt{2}+3 \sqrt{3}} \\
= & \frac{(2 \sqrt{3}-\sqrt{5})(2 \sqrt{2}-3 \sqrt{3})}{(2 \sqrt{2}+3 \sqrt{3})(2 \sqrt{2}-3 \sqrt{3})} \\
= & \frac{(2 \sqrt{3}-\sqrt{5})(2 \sqrt{2}-3 \sqrt{3})}{8-27} \\
= & \frac{(4 \sqrt{6}-2 \sqrt{10})-18+3 \sqrt{15})}{-19} \\
= & \frac{(18-4 \sqrt{6}+2 \sqrt{10}-3 \sqrt{15})}{19}
\end{aligned}
$$

## Exercise VSAQs

Question 1: Write the value of $(2+\sqrt{ } 3)(2-\sqrt{ } 3)$.
Solution:
$(2+\sqrt{ } 3)(2-\sqrt{ } 3)$
$=(2)^{2}-(V 3)^{2}$
[Using identity : $(a+b)(a-b)=a^{2}-b^{2}$ ]
$=4-3$
$=1$

Question 2: Write the reciprocal of $5+\sqrt{ } 2$.

## Solution:

Reciprocal of $5+\sqrt{2}=\frac{1}{5+\sqrt{2}}$

## Rationalisation of fraction

Multiply and divide given fraction by 5 - V2

$$
\begin{aligned}
& =\frac{5-\sqrt{2}}{(5+\sqrt{2})(5-\sqrt{2})} \\
& =\frac{5-\sqrt{2}}{(5)^{2}-(\sqrt{2})^{2}} \\
& =\frac{5-\sqrt{2}}{25-2} \\
& =\frac{5-\sqrt{2}}{23}
\end{aligned}
$$

Question 3: Write the rationalisation factor of 7 - 3V5.
Solution:

Rationalisation factor of $7-3 \sqrt{ } 5$ is $7+3 \sqrt{ } 5$

Question 4: If

$$
\frac{\sqrt{3}-1}{\sqrt{3}+1}=x+y \sqrt{3}
$$

Find the values of $x$ and $y$.

## Solution:

[Using identities: $(a+b)(a-b)=a^{2}-b^{2}$ and $\left.(a-b)^{2}=a^{2}+b^{2}-2 a b\right]$

Rationalising Denominator

$$
\begin{gathered}
\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}=\frac{(\sqrt{3}-1)^{2}}{(\sqrt{3})^{2}-(1)^{2}} \\
\quad=\frac{3+1-2 \sqrt{3}}{3-1}=\frac{4-2 \sqrt{3}}{2}=2-\sqrt{3}
\end{gathered}
$$

Now,

$$
2-\sqrt{3}=x+y \sqrt{3}
$$

On comparing, $x=2, y=-1$

Question 5: If $\mathrm{x}=\mathrm{V} \mathbf{2 - 1}$, then write the value of $1 / \mathrm{x}$.
Solution:
$\mathrm{x}=\mathrm{v} 2-1$
or $1 / x=1 /(\sqrt{ } 2-1)$

Rationalising denominator, we have
$=1 /(\sqrt{ } 2-1) \times(v 2+1) /(v 2+1)$
$=(\sqrt{ } 2+1) /(2-1)$
$=\mathrm{V} 2+1$
Question 6: Simplify

$$
\sqrt{3+2 \sqrt{2}}
$$

## Solution:

$$
\begin{aligned}
& \sqrt{3+2 \sqrt{2}} \\
= & \sqrt{2+1+2 \sqrt{2}} \\
= & \sqrt{(\sqrt{2})^{2}+(1)^{2}+2 \times \sqrt{2} \times 1} \\
= & \sqrt{(\sqrt{2}+1)^{2}}=\sqrt{2}+1
\end{aligned}
$$

[ Because: $\left.(a+b)^{2}=a^{2}+b^{2}+2 a b\right]$

## Question 7: Simplify

$$
\sqrt{3-2 \sqrt{2}}
$$

## Solution:

$$
\begin{aligned}
& \sqrt{3-2 \sqrt{2}} \\
= & \sqrt{2+1-2 \sqrt{2}} \\
= & \sqrt{(\sqrt{2})^{2}+(1)^{2}-2 \times \sqrt{2} \times 1} \\
= & \sqrt{(\sqrt{2}-1)^{2}}=\sqrt{2}-1 \\
& {\left[\text { Because: }(a-b)^{2}=a^{2}+b^{2}-2 a b\right] }
\end{aligned}
$$

Question 8: If $a=\sqrt{ } 2+1$, then find the value of $a-1 / a$.
Solution:
Given: $a=\sqrt{ } 2+1$
$1 / a=1 /(v 2+1)$
$=1 /(\sqrt{ } 2+1) \times(V 2-1) /(\sqrt{ } 2-1)$
$=(\sqrt{ } 2-1) /\left((\sqrt{ } 2)^{\wedge} 2-(1)^{\wedge} 2\right)$
$=(\sqrt{ } 2-1) / 1$
$=\sqrt{ } 2-1$

Now,
$a-1 / a=(\sqrt{ } 2+1)-(v 2-1)$
$=2$
Question 9: If $x=2+\sqrt{3}$, find the value of $x+1 / x$.
Solution:
Given: $x=2+\sqrt{ } 3$
$1 / x=1 /(2+\sqrt{ } 3)$
$=1 /(2+\sqrt{ } 3) \times(2-\sqrt{ } 3) /(2-\sqrt{ } 3)$
$=(2-\sqrt{ } 3) /\left((2)^{\wedge} 2-(\sqrt{ } 3)^{\wedge} 2\right)$
$=(2-\sqrt{ } 3) /(4-3)$
$=(2-\sqrt{ } 3)$

Now,
$x+1 / x=(2+\sqrt{ } 3)+(2-\sqrt{ } 3)$
$=4$

Question 10: Write the rationalisation factor of V5-2.
Solution:
Rationalisation factor of $\mathrm{V} 5-2$ is $\mathrm{V} 5+2$
Question 11: If $x=3+2 \sqrt{2}$, then find the value of $\sqrt{x}-1 / \sqrt{x}$.

## Solution:

$$
\begin{aligned}
& x=3+2 v 2 \\
& \frac{1}{x}=\frac{1}{3+2 \sqrt{2}}=\frac{(3-2 \sqrt{2})}{(3+2 \sqrt{2})(3-2 \sqrt{2})} \\
& =\frac{3-2 \sqrt{2}}{(3)^{2}-(2 \sqrt{2})^{2}}=\frac{3-2 \sqrt{2}}{9-8}=\frac{3-2 \sqrt{2}}{1} \\
& x+\frac{1}{x}=3+2 \sqrt{2}+3-2 \sqrt{2}=6 \\
& \text { Now, }\left(v x-\frac{1}{v x}\right)^{2}=x+\frac{1}{x}-2 \\
& \quad=6-2=4=(2)^{2} \\
& \left(v x-\frac{1}{v x}\right)=2
\end{aligned}
$$

