

EXERCISE 8.1

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1. Solve the following system of equations by matrix method:

(i) $5x + 7y + 2 = 0$

$4x + 6y + 3 = 0$

(ii) $5x + 2y = 3$

$3x + 2y = 5$

(iii) $3x + 4y - 5 = 0$

$x - y + 3 = 0$

(iv) $3x + y = 19$

$3x - y = 23$

(v) $3x + 7y = 4$

$x + 2y = -1$

(vi) $3x + y = 7$

$5x + 3y = 12$

Solution:

(i) Given $5x + 7y + 2 = 0$ and $4x + 6y + 3 = 0$

The above system of equations can be written as

$$\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix} \text{ Or } AX = B$$

Where $A = \begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}$ $B = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$ and $X = \begin{bmatrix} X \\ Y \end{bmatrix}$

$$|A| = 30 - 28 = 2$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the cofactor of a_{ij} in A , then

$$C_{11} = (-1)^{1+1} 6 = 6$$

$$C_{12} = (-1)^{1+2} 4 = -4$$

$$C_{21} = (-1)^{2+1} 7 = -7$$

$$C_{22} = (-1)^{2+2} 5 = 5$$

Where $A = \begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$ and $X = \begin{bmatrix} X \\ Y \end{bmatrix}$

$$|A| = -3 - 4 = -7$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the cofactor of a_{ij} in A , then

$$C_{11} = (-1)^{1+1} 1 = 1$$

$$C_{12} = (-1)^{1+2} 1 = -1$$

$$C_{21} = (-1)^{2+1} 4 = -4$$

$$C_{22} = (-1)^{2+2} 3 = 3$$

Also, $\text{adj } A = \begin{bmatrix} -1 & -1 \\ -4 & 3 \end{bmatrix}^T$

$$= \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{-7} \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-7} \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-7} \begin{bmatrix} -5 + 12 \\ -5 - 9 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-7} \begin{bmatrix} 7 \\ -14 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Hence, $X = 1$ $Y = -2$

(iv) Given $3x + y = 19$

$3x - y = 23$

The above system of equations can be written as

$$\begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 19 \\ 23 \end{bmatrix} \text{ Or } AX = B$$

Where $A = \begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 19 \\ 23 \end{bmatrix}$ and $X = \begin{bmatrix} X \\ Y \end{bmatrix}$

$$|A| = -3 - 3 = -6$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the cofactor of a_{ij} in A , then

$$C_{11} = (-1)^{1+1} - 1 = -1$$

$$C_{12} = (-1)^{1+2} 3 = -3$$

$$C_{21} = (-1)^{2+1} 1 = -1$$

$$C_{22} = (-1)^{2+2} 3 = 3$$

Also, $\text{adj } A = \begin{bmatrix} -1 & -3 \\ -1 & 3 \end{bmatrix}^T$

$$= \begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{-6} \begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-6} \begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 19 \\ 23 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-6} \begin{bmatrix} -19 - 23 \\ -57 + 69 \end{bmatrix}$$

$$\begin{aligned} &= 6(225 + 360) + 12(60 + 40) + 25(72 - 30) \\ &= 3510 + 1200 + 1050 \\ &= 5760 \end{aligned}$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} (225 + 360) = 585$$

$$C_{21} = (-1)^{2+1} (-180 - 450) = 630$$

$$C_{31} = (-1)^{3+1} (240 - 375) = -135$$

$$C_{12} = (-1)^{1+2} (60 + 40) = -100$$

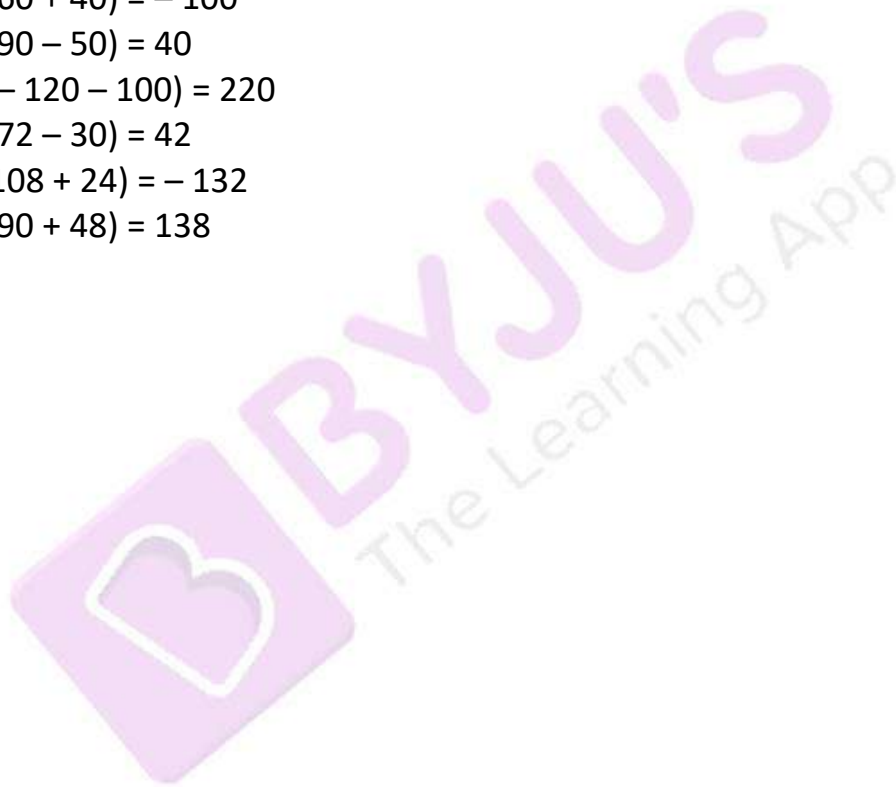
$$C_{22} = (-1)^{2+1} (90 - 50) = 40$$

$$C_{32} = (-1)^{3+1} (-120 - 100) = 220$$

$$C_{13} = (-1)^{1+2} (72 - 30) = 42$$

$$C_{23} = (-1)^{2+1} (108 + 24) = -132$$

$$C_{33} = (-1)^{3+1} (90 + 48) = 138$$



$$\begin{aligned} \text{(iv) Given } 3x + 4y + 7z &= 14 \\ 2x - y + 3z &= 4 \\ x + 2y - 3z &= 0 \end{aligned}$$

The given system can be written in matrix form as:

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix} \text{ Or } AX = B$$

$$A = \begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \text{Now, } |A| &= 3 \begin{vmatrix} -1 & 3 \\ 2 & -3 \end{vmatrix} - 4 \begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix} + 7 \begin{vmatrix} 2 & 3 \\ 2 & -3 \end{vmatrix} \\ &= 3(3 - 6) - 4(-6 - 3) + 7(4 + 1) \\ &= -9 + 36 + 35 \\ &= 62 \end{aligned}$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} 3 - 6 = -3$$

$$C_{21} = (-1)^{2+1} - 12 - 14 = 26$$

$$C_{31} = (-1)^{3+1} 12 + 7 = 19$$

$$C_{12} = (-1)^{1+2} - 6 - 3 = 9$$

$$C_{22} = (-1)^{2+2} - 3 - 7 = -10$$

$$C_{32} = (-1)^{3+2} 9 - 14 = 5$$

$$C_{13} = (-1)^{1+3} 4 + 1 = 5$$

$$C_{23} = (-1)^{2+3} 6 - 4 = -2$$

$$C_{33} = (-1)^{3+3} - 3 - 8 = -11$$

$$\text{Adj } A = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -5 & -2 \\ 19 & 5 & -11 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{62} \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix} \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

$$X = \frac{1}{62} \begin{bmatrix} -42 + 104 + 0 \\ 126 - 64 + 0 \\ 70 - 8 + 0 \end{bmatrix}$$

$$X = \frac{1}{62} \begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Hence, $X = 1$, $Y = 1$ and $Z = 1$

(v) Given $(2/x) - (3/y) + (3/z) = 10$

$(1/x) + (1/y) + (1/z) = 10$

$(3/x) - (1/y) + (2/z) = 13$

The given system can be written in matrix form as:

$$\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix} \text{ Or } A X = B$$

$$A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$$

$$\begin{aligned} \text{Now, } |A| &= 5 \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} - 3 \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \\ &= 5(4 - 6) - 3(8 - 3) + 1(4 - 2) \\ &= -10 - 15 + 3 \\ &= -22 \end{aligned}$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} (4 - 6) = -2$$

$$C_{21} = (-1)^{2+1} (12 - 2) = -10$$

$$C_{31} = (-1)^{3+1} (9 - 1) = 8$$

$$C_{12} = (-1)^{1+2} (8 - 3) = -5$$

$$C_{22} = (-1)^{2+2} 20 - 1 = 19$$

$$C_{32} = (-1)^{3+2} 15 - 2 = -13$$

$$C_{13} = (-1)^{1+3} (4 - 2) = 2$$

$$C_{23} = (-1)^{2+3} 10 - 3 = -7$$

$$C_{33} = (-1)^{3+3} 5 - 6 = -1$$

$$\text{Adj A} = \begin{bmatrix} -2 & -5 & -3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj A}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{-22} \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix} \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$$

$$X = \frac{1}{-22} \begin{bmatrix} -32 - 190 + 200 \\ -80 + 361 - 325 \\ 48 - 133 - 25 \end{bmatrix}$$

$$X = \frac{1}{-22} \begin{bmatrix} -22 \\ -44 \\ -110 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

Hence, $X = 1$, $Y = 2$ and $Z = 5$

(vi) Given $5x + 3y + z = 16$

$$2x + y + 3z = 19$$

$$x + 2y + 4z = 25$$

The given system can be written in matrix form as:

$$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix} \text{ Or } AX = B$$

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$$

$$\begin{aligned} \text{Now, } |A| &= 3 \begin{vmatrix} 2 & -3 \\ -2 & 6 \end{vmatrix} - 4 \begin{vmatrix} 0 & -3 \\ 1 & 6 \end{vmatrix} + 2 \begin{vmatrix} 0 & 2 \\ 1 & -2 \end{vmatrix} \\ &= 3(12 - 6) - 4(0 + 3) + 2(0 - 2) \\ &= 18 - 12 - 4 \\ &= 2 \end{aligned}$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Cofactors of A are

$$C_{11} = (-1)^{1+1}(12 - 6) = 6$$

$$C_{21} = (-1)^{2+1}(24 + 4) = -28$$

$$C_{31} = (-1)^{3+1}(-12 - 4) = -16$$

$$C_{12} = (-1)^{1+2}(0 + 3) = -3$$

$$C_{22} = (-1)^{2+2}18 - 2 = 16$$

$$C_{32} = (-1)^{3+2} - 9 - 0 = 9$$

$$C_{13} = (-1)^{1+3}(0 - 2) = -2$$

$$C_{23} = (-1)^{2+3}(-6 - 4) = 10$$

$$C_{33} = (-1)^{3+3}6 - 0 = 6$$

$$\text{Adj } A = \begin{bmatrix} 6 & -3 & 2 \\ -28 & 16 & 10 \\ -16 & -9 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & -9 \\ 2 & 10 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{2} \begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & -9 \\ 2 & 10 & 6 \end{bmatrix} \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$$

$$X = \frac{1}{2} \begin{bmatrix} 48 - 84 + 32 \\ -24 + 48 - 18 \\ -16 + 30 - 12 \end{bmatrix}$$

$$X = \frac{1}{2} \begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

Hence, $X = -2$, $Y = 3$ and $Z = 1$

(vii) Given $3x + 4y + 2z = 8$

$2y - 3z = 3$

$x - 2y + 6z = -2$

The given system can be written in matrix form as:

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \text{ Or } AX = B$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$$

$$\text{Now, } |A| = 2 \begin{vmatrix} 3 & -1 \\ 1 & -2 \end{vmatrix} - 1 \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} + 1 \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}$$

$$= 2(-6 + 1) - 1(-2 + 3) + 1(1 - 9)$$

$$= -10 - 1 - 8$$

$$= -19$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} - 6 + 1 = -5$$

$$C_{21} = (-1)^{2+1}(24 + 4) = -28$$

$$C_{31} = (-1)^{3+1} - 1 - 3 = -4$$

$$C_{12} = (-1)^{1+2} - 2 + 3 = -1$$

$$C_{22} = (-1)^{2+1} - 4 - 3 = -7$$

$$C_{32} = (-1)^{3+1} - 2 - 1 = 3$$

$$C_{13} = (-1)^{1+2} 1 - 9 = -8$$

$$C_{23} = (-1)^{2+1} 2 - 3 = -1$$

$$C_{33} = (-1)^{3+1} 6 - 1 = 5$$

$$\text{Adj } A = \begin{bmatrix} -5 & -1 & -8 \\ 3 & -7 & 1 \\ -4 & 3 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{-19} \begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$$

$$X = \frac{1}{-19} \begin{bmatrix} -10 + 15 - 24 \\ -2 - 35 + 18 \\ -16 + 5 + 30 \end{bmatrix}$$

$$X = \frac{1}{-19} \begin{bmatrix} -19 \\ -19 \\ 19 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Hence, $X = 1$, $Y = 1$ and $Z = -1$

(viii) Given $2x + y + z = 2$

$$x + 3y - z = 5$$

$$3x + y - 2z = 6$$

The given system can be written in matrix form as:

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \text{ Or } AX = B$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$$

$$\text{Now, } |A| = 2 \begin{vmatrix} 3 & -1 \\ 1 & -2 \end{vmatrix} - 1 \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} + 1 \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}$$

$$= 2(-6 + 1) - 1(-2 + 3) + 1(1 - 9)$$

$$= -10 - 1 - 8$$

$$= -19$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} - 6 + 1 = -5$$

$$C_{21} = (-1)^{2+1}(24 + 4) = -28$$

$$C_{31} = (-1)^{3+1} - 1 - 3 = -4$$

$$C_{12} = (-1)^{1+2} - 2 + 3 = -1$$

$$C_{22} = (-1)^{2+2} - 4 - 3 = -7$$

$$C_{32} = (-1)^{3+2} - 2 - 1 = 3$$

$$C_{13} = (-1)^{1+3} 1 - 9 = -8$$

$$C_{23} = (-1)^{2+3} 2 - 3 = -1$$

$$C_{33} = (-1)^{3+3} 6 - 1 = 5$$

$$\text{Adj } A = \begin{bmatrix} -5 & -1 & -8 \\ 3 & -7 & 1 \\ -4 & 3 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{-19} \begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$$

$$X = \frac{1}{-19} \begin{bmatrix} -10 + 15 - 24 \\ -2 - 35 + 18 \\ -16 + 5 + 30 \end{bmatrix}$$

$$X = \frac{1}{-19} \begin{bmatrix} -19 \\ -19 \\ 19 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Hence, $X = 1$, $Y = 1$ and $Z = -1$

(ix) Given $2x + 6y = 2$

$$3x - z = -8$$

$$2x - y + z = -3$$

The given system can be written in matrix form as:

$$\begin{bmatrix} 2 & 6 & 0 \\ 3 & 0 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix} \text{ Or } AX = B$$

$$A = \begin{bmatrix} 2 & 6 & 0 \\ 3 & 0 & -1 \\ 2 & -1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$$

$$\text{Now, } |A| = 2 \begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix} - 6 \begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} + 0$$

$$= 2(0 - 1) - 6(3 + 2)$$

$$= -2 - 30$$

$$= -32$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} 0 - 1 = -1$$

$$C_{21} = (-1)^{2+1}6 + 0 = -6$$

$$C_{31} = (-1)^{3+1} - 6 = -6$$

$$C_{12} = (-1)^{1+2}3 + 2 = 5$$

$$C_{22} = (-1)^{2+1}2 - 0 = 2$$

$$C_{32} = (-1)^{3+1} - 2 - 0 = 2$$

$$C_{13} = (-1)^{1+2} - 3 - 0 = -3$$

$$C_{23} = (-1)^{2+1} - 2 - 12 = 14$$

$$C_{33} = (-1)^{3+1}0 - 18 = -18$$

$$\text{Adj } A = \begin{bmatrix} -1 & -5 & -3 \\ -6 & 2 & 14 \\ -6 & 2 & -18 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{-32} \begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix} \begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$$

$$X = \frac{1}{-32} \begin{bmatrix} -2 + 48 + 18 \\ -10 - 16 - 6 \\ -6 - 112 + 54 \end{bmatrix}$$

$$X = \frac{1}{-32} \begin{bmatrix} 64 \\ -32 \\ -64 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $X = -2$, $Y = 1$ and $Z = 2$

(x) Given $2y - z = 1$

$$x - y + z = 2$$

$$2x - y = 0$$

The given system can be written in matrix form as:

$$\begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$AX = B$$

$$\begin{aligned} \text{Now, } |A| &= 0 \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} - 1 \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} \\ &= 0 + 4 - 1 \\ &= 3 \end{aligned}$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} 1 - 0 = 1$$

$$C_{21} = (-1)^{2+1} 1 - 2 = 1$$

$$C_{31} = (-1)^{3+1} 0 + 1 = 1$$

$$C_{12} = (-1)^{1+2} - 2 - 0 = 2$$

$$C_{22} = (-1)^{2+2} - 1 - 0 = -1$$

$$C_{32} = (-1)^{3+2} 0 - 2 = 2$$

$$C_{13} = (-1)^{1+3} 4 - 0 = 4$$

$$C_{23} = (-1)^{2+3} 2 - 0 = -2$$

$$C_{33} = (-1)^{3+3} - 1 + 2 = 1$$

$$\text{Adj } A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & -1 & -2 \\ 1 & 2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$X = \frac{1}{3} \begin{bmatrix} 2 + 0 + 1 \\ 4 - 0 + 2 \\ 8 - 0 + 1 \end{bmatrix}$$

$$X = \frac{1}{3} \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $X = 1$, $Y = 2$ and $Z = 3$

(xi) Given $8x + 4y + 3z = 18$

$$2x + y + z = 5$$

$$x + 2y + z = 5$$

The given system can be written in matrix form as:

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

$$AX = B$$

$$\begin{aligned} \text{Now, } |A| &= 8 \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - 4 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 3 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \\ &= 8(-1) - 4(1) + 3(3) \\ &= -8 - 4 + 9 \\ &= -3 \end{aligned}$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} 1 - 2 = -1$$

$$C_{21} = (-1)^{2+1} 4 - 6 = 2$$

$$C_{31} = (-1)^{3+1} 4 - 3 = 1$$

$$C_{12} = (-1)^{1+2} 2 - 1 = -1$$

$$C_{22} = (-1)^{2+2} 8 - 3 = 5$$

$$C_{32} = (-1)^{3+2} 8 - 6 = -2$$

$$C_{13} = (-1)^{1+2} 4 - 1 = 3$$

$$C_{23} = (-1)^{2+1} 16 - 4 = -12$$

$$C_{33} = (-1)^{3+1} 8 - 8 = 0$$

$$\text{Adj } A = \begin{bmatrix} -1 & -1 & 3 \\ 2 & 5 & -12 \\ 1 & -2 & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{-3} \begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix} \begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

$$X = \frac{1}{-3} \begin{bmatrix} -18 + 10 + 5 \\ -18 + 25 - 10 \\ 54 - 60 + 0 \end{bmatrix}$$

$$X = \frac{1}{-3} \begin{bmatrix} -3 \\ -3 \\ -6 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $X = 1$, $Y = 1$ and $Z = 2$

(xii) Given $x + y + z = 6$

$$x + 2z = 7$$

$$3x + y + z = 12$$

The given system can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$A X = B$$

$$\begin{aligned} \text{Now, } |A| &= 1 \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} \\ &= 1(-2) - 1(1 - 6) + 1(1) \\ &= -2 + 5 + 1 \\ &= 4 \end{aligned}$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} 0 - 2 = -2$$

$$C_{21} = (-1)^{2+1} 1 - 1 = 0$$

$$C_{31} = (-1)^{3+1} 2 - 0 = 2$$

$$C_{12} = (-1)^{1+2} 1 - 6 = 5$$

$$C_{22} = (-1)^{2+2} 1 - 3 = -2$$

$$C_{32} = (-1)^{3+2} 2 - 1 = -1$$

$$C_{13} = (-1)^{1+3} 1 - 0 = 1$$

$$C_{23} = (-1)^{2+3} 1 - 3 = 2$$

$$C_{33} = (-1)^{3+3} 0 - 1 = -1$$

$$\text{Adj } A = \begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{4} \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$X = \frac{1}{4} \begin{bmatrix} -12 + 0 + 24 \\ 30 - 14 - 12 \\ 6 + 14 - 12 \end{bmatrix}$$

$$X = \frac{1}{4} \begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $X = 3$, $Y = 1$ and $Z = 2$

(xiii) Given $(2/x) + (3/y) + (10/z) = 4$,
 $(4/x) - (6/y) + (5/z) = 1$,
 $(6/x) + (9/y) - (20/z) = 2$, $x, y, z \neq 0$

The given system can be written in matrix form as:

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$AX = B$$

Now,

$$\begin{aligned} |A| &= 2(75) - 3(-110) + 10(72) \\ &= 150 + 330 + 720 \\ &= 1200 \end{aligned}$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} 120 - 45 = 75$$

$$C_{21} = (-1)^{2+1} - 60 - 90 = 150$$

$$C_{31} = (-1)^{3+1} 15 + 60 = 75$$

$$C_{12} = (-1)^{1+2} - 80 - 30 = 110$$

$$C_{22} = (-1)^{2+1} - 40 - 60 = -100$$

$$C_{32} = (-1)^{3+1} 10 - 40 = 30$$

$$C_{13} = (-1)^{1+2} 36 + 36 = 72$$

$$C_{23} = (-1)^{2+1} 18 - 18 = 0$$

$$C_{33} = (-1)^{3+1} - 12 - 12 = -24$$

$$\text{Adj } A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T$$

$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$X = \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \\ 1 \\ 5 \end{bmatrix}$$

Hence, $X = 2$, $Y = 3$ and $Z = 5$

(xiv) Given $x - y + 2z = 7$

$$3x + 4y - 5z = -5$$

$$2x - y + 3z = 12$$

The given system can be written in matrix form as:

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$A X = B$$

Now,

$$|A| = 1(12 - 5) + 1(9 + 10) + 2(-3 - 8)$$

$$= 7 + 19 - 22$$

$$= 4$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} 12 - 5 = 7$$

$$C_{21} = (-1)^{2+1} -3 + 2 = 1$$

$$C_{31} = (-1)^{3+1} 5 - 8 = -3$$

$$C_{12} = (-1)^{1+2} 9 + 10 = -19$$

$$C_{22} = (-1)^{2+1} 3 - 4 = -1$$

$$C_{32} = (-1)^{3+1} - 5 - 6 = 11$$

$$C_{13} = (-1)^{1+2} - 3 - 8 = -11$$

$$C_{23} = (-1)^{2+1} - 1 + 2 = -1$$

$$C_{33} = (-1)^{3+1} 4 + 3 = 7$$

$$\text{Adj } A = \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & -7 \end{bmatrix}^T$$

$$= \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$X = \frac{1}{4} \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hence, $X = 2$, $Y = 1$ and $Z = 3$

3. Show that each one of the following systems of linear equations is consistent and also find their solutions:

(i) $6x + 4y = 2$

$9x + 6y = 3$

(ii) $2x + 3y = 5$

$6x + 9y = 15$

(iii) $5x + 3y + 7z = 4$

$3x + 26y + 2z = 9$

$7x + 2y + 10z = 5$

(v) $x + y + z = 6$

$$x + 2y + 3z = 14$$

$$x + 4y + 7z = 30$$

$$(vi) 2x + 2y - 2z = 1$$

$$4x + 4y - z = 2$$

$$6x + 6y + 2z = 3$$

Solution:

(i) Given $6x + 4y = 2$

$$9x + 6y = 3$$

The above system of equations can be written as

$$\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \text{ Or } AX = B$$

Where $A = \begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $X = \begin{bmatrix} X \\ Y \end{bmatrix}$

$$|A| = 36 - 36 = 0$$

So, A is singular, Now X will be consistence if $(\text{Adj } A) \times B = 0$

$$C_{11} = (-1)^{1+1} 6 = 6$$

$$C_{12} = (-1)^{1+2} 9 = -9$$

$$C_{21} = (-1)^{2+1} 4 = -4$$

$$C_{22} = (-1)^{2+2} 6 = 6$$

Also, $\text{adj } A = \begin{bmatrix} 6 & -9 \\ -4 & 6 \end{bmatrix}^T$

$$= \begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}$$

$$(\text{Adj } A) \cdot B = \begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 12 - 12 \\ -18 + 18 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Thus, $AX = B$ will be infinite solution,

Let $y = k$

Hence, $6x = 2 - 4k$ or $9x = 3 - 6k$

$$X = \frac{1-2k}{3}$$

Hence, $X = \frac{1-2k}{3}$, $Y = k$

(ii) Given $2x + 3y = 5$

$6x + 9y = 15$

The above system of equations can be written as

$$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix} \text{ Or } AX = B$$

$$\text{Where } A = \begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix} \text{ } B = \begin{bmatrix} 5 \\ 15 \end{bmatrix} \text{ and } X = \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$|A| = 18 - 18 = 0$$

So, A is singular,

Now X will be consistence if $(\text{Adj } A) \times B = 0$

$$C_{11} = (-1)^{1+1} 9 = 9$$

$$C_{12} = (-1)^{1+2} 6 = -6$$

$$C_{21} = (-1)^{2+1} 3 = -3$$

$$C_{22} = (-1)^{2+2} 2 = 2$$

$$\text{Also, adj } A = \begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$$

$$(\text{Adj } A) \cdot B = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 15 \end{bmatrix}$$

$$= \begin{bmatrix} 45 - 45 \\ -30 + 30 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Thus, $AX = B$ will be infinite solution,

Let $y = k$

Hence,

$$2x = 5 - 3k \text{ or } X = \frac{5-3k}{2}$$

$$x = 15 - 9k \text{ or } X = \frac{5-3k}{2}$$

$$\text{Hence, } X = \frac{5-3k}{2}, Y = k$$

(iii) Given $5x + 3y + 7z = 4$

$$3x + 26y + 2z = 9$$

$$7x + 2y + 10z = 5$$

This can be written as:

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

$$|A| = 5(260 - 4) - 3(30 - 14) + 7(6 - 182)$$

$$= 5(256) - 3(16) + 7(176)$$

$$|A| = 0$$

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:

$$(\text{Adj } A) \times B \neq 0 \text{ or } (\text{Adj } A) \times B = 0$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} 260 - 4 = 256$$

$$C_{21} = (-1)^{2+1} 30 - 14 = -16$$

$$C_{31} = (-1)^{3+1} 6 - 182 = -176$$

$$C_{12} = (-1)^{1+2} 30 - 14 = -16$$

$$C_{22} = (-1)^{2+2} 50 - 49 = 1$$

$$C_{32} = (-1)^{3+2} 10 - 21 = 11$$

$$C_{13} = (-1)^{1+3} 6 - 182 = -176$$

$$C_{23} = (-1)^{2+3} 10 - 21 = 11$$

$$C_{33} = (-1)^{3+3} 130 - 9 = 121$$

$$\text{Adj } A = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}^T$$

$$= \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}$$

$$\text{Adj } A \times B = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, $AX = B$ has infinite many solution

Let $z = k$

Then, $5x + 3y = 4 - 7k$

$3x + 26y = 9 - 2k$

This can be written as

$$\begin{bmatrix} 5 & 3 \\ 3 & 26 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 - 7k \\ 9 - 2k \end{bmatrix}$$

$$|A| = 121$$

$$\text{Adj } A = \begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{|A|} \text{Adj } A \times B$$

$$= \frac{1}{121} \begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 4 - 7k \\ 9 - 2k \end{bmatrix}$$

$$= \frac{1}{121} \begin{bmatrix} 77 - 176k \\ 11k + 33 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{7 - 16k}{11} \\ \frac{k + 3}{11} \\ 11 \end{bmatrix}$$

There values of x , y and z satisfy the third equation

(v) Given $x + y + z = 6$

$$x + 2y + 3z = 14$$

$$x + 4y + 7z = 30$$

This can be written as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

$$|A| = 1(2) - 1(4) + 1(2)$$

$$= 2 - 4 + 2$$

$$|A| = 0$$

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:

$$(\text{Adj } A) \times B \neq 0 \text{ or } (\text{Adj } A) \times B = 0$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} 14 - 12 = 2$$

$$C_{21} = (-1)^{2+1} 7 - 4 = -3$$

$$C_{31} = (-1)^{3+1} 3 - 2 = 1$$

$$C_{12} = (-1)^{1+2} 7 - 3 = -4$$

$$C_{22} = (-1)^{2+2} 7 - 1 = 6$$

$$C_{32} = (-1)^{3+2} 3 - 1 = 2$$

$$C_{13} = (-1)^{1+3} 4 - 2 = 2$$

$$C_{23} = (-1)^{2+3} 4 - 1 = -3$$

$$C_{33} = (-1)^{3+3} 2 - 1 = 1$$

$$\text{Adj } A = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 6 & -3 \\ 1 & -2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & -3 & 1 \\ -4 & 1 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$\text{Adj } A \times B = \begin{bmatrix} 2 & -3 & 1 \\ -4 & 1 & -2 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, $AX = B$ has infinite many solution

Let $z = k$

Then, $x + y = 6 - k$

$$x + 2y = 14 - 3k$$

This can be written as:

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 - k \\ 14 - 3k \end{bmatrix}$$

$$|A| = 1$$

$$\text{Adj } A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{|A|} \text{Adj } A \times B$$

$$= \frac{1}{1} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 6 - k \\ 14 - 3k \end{bmatrix}$$

$$= \frac{1}{1} \begin{bmatrix} 12 - 2k - 14 + 3k \\ -6 + k + 14 - 3k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 + k \\ 8 - 2k \end{bmatrix}$$

There values of x, y and z satisfy the third equation

$$\text{Hence, } x = k - 2, y = 8 - 2k, z = k$$

(vi) Given $x + y + z = 6$

$$x + 2y + 3z = 14$$

$$x + 4y + 7z = 30$$

This can be written as

$$\begin{bmatrix} 2 & 2 & -2 \\ 4 & 4 & -1 \\ 6 & 6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$|A| = 2(14) - 2(14) - 2(0)$$

$$|A| = 0$$

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:

$$(\text{Adj } A) \times B \neq 0 \text{ or } (\text{Adj } A) \times B = 0$$

Cofactors of A are:

$$C_{11} = (-1)^{1+1}8 + 6 = 14$$

$$C_{21} = (-1)^{2+1} 4 + 12 = -16$$

$$C_{31} = (-1)^{3+1} - 2 + 8 = 6$$

$$C_{12} = (-1)^{1+2} 8 + 6 = -14$$

$$C_{22} = (-1)^{2+1} 4 + 12 = 16$$

$$C_{32} = (-1)^{3+1} - 2 + 8 = -6$$

$$C_{13} = (-1)^{1+2} 24 - 24 = 0$$

$$C_{23} = (-1)^{2+1} 12 - 12 = 0$$

$$C_{33} = (-1)^{3+1} 8 - 8 = 0$$

$$\text{Adj A} = \begin{bmatrix} 14 & -14 & 0 \\ -16 & 16 & 0 \\ 6 & -6 & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Adj A} \times B = \begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, $AX = B$ has infinite many solution

Let $z = k$

$$\text{Then, } 2x + 2y = 1 + 2k$$

$$4x + 4y = 2 + k$$

This can be written as:

$$\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 + 2k \\ 2 + k \end{bmatrix}$$

$$\text{Hence, } |A| = 0 \neq 0$$

Hence, the given equation doesn't satisfy.

4. Show that each one of the following systems of linear equations is consistent:

(i) $2x + 5y = 7$

$6x + 15y = 13$

(ii) $2x + 3y = 5$

$6x + 9y = 10$

(iii) $4x - 2y = 3$

$6x - 3y = 5$

(iv) $4x - 5y - 2z = 2$

$5x - 4y + 2z = -2$

$2x + 2y + 8z = -1$

(v) $3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

(vi) $x + y - 2z = 5$

$x - 2y + z = -2$

$-2x + y + z = 4$

Solution:

(i) Given $2x + 5y = 7$

$6x + 15y = 13$

The above system of equations can be written as

$$\begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \end{bmatrix} \text{ Or } AX = B$$

Where $A = \begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix}$ $B = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$ and $X = \begin{bmatrix} X \\ Y \end{bmatrix}$

$|A| = 30 - 30 = 0$

So, A is singular,

Now X will be consistence if $(\text{Adj } A) \times B = 0$

$C_{11} = (-1)^{1+1} 15 = 15$

$C_{12} = (-1)^{1+2} 6 = -6$

$C_{21} = (-1)^{2+1} 5 = -5$

$C_{22} = (-1)^{2+2} 2 = 2$

Also, $\text{adj } A = \begin{bmatrix} 15 & -6 \\ -5 & 2 \end{bmatrix}^T$

$= \begin{bmatrix} 15 & -5 \\ -5 & 2 \end{bmatrix}$

$$(\text{Adj } A).B = \begin{bmatrix} 15 & -5 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ 13 \end{bmatrix}$$

$$= \begin{bmatrix} 105 - 65 \\ -35 + 26 \end{bmatrix} = \begin{bmatrix} 40 \\ -16 \end{bmatrix}$$

$\neq 0$

Hence, the given system is inconsistent.

(ii) Given $2x + 3y = 5$

$6x + 9y = 10$

The above system of equations can be written as

$$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix} \text{ Or } AX = B$$

$$\text{Where } A = \begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 10 \end{bmatrix} \text{ and } X = \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$|A| = 18 - 18 = 0$$

So, A is singular,

Now X will be consistence if $(\text{Adj } A) \times B = 0$

$$C_{11} = (-1)^{1+1} 9 = 9$$

$$C_{12} = (-1)^{1+2} 6 = -6$$

$$C_{21} = (-1)^{2+1} 3 = -3$$

$$C_{22} = (-1)^{2+2} 2 = 2$$

$$\text{Also, adj } A = \begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$$

$$(\text{Adj } A).B = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 45 - 30 \\ -30 + 20 \end{bmatrix} = \begin{bmatrix} 15 \\ -10 \end{bmatrix} \neq 0$$

Hence, the given system is inconsistent.

(iii) Given $4x - 2y = 3$

$6x - 3y = 5$

The above system of equations can be written as

$$\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \text{ Or } AX = B$$

Where $A = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and $X = \begin{bmatrix} X \\ Y \end{bmatrix}$

$$|A| = -12 + 12 = 0$$

So, A is singular,

Now X will be consistence if $(\text{Adj } A) \times B = 0$

$$C_{11} = (-1)^{1+1} - 3 = -3$$

$$C_{12} = (-1)^{1+2} 6 = -6$$

$$C_{21} = (-1)^{2+1} - 2 = 2$$

$$C_{22} = (-1)^{2+2} 4 = 4$$

Also, $\text{adj } A = \begin{bmatrix} -3 & -6 \\ 2 & 4 \end{bmatrix}^T$

$$= \begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}$$

$$(\text{Adj } A) \cdot B = \begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} -9 + 10 \\ -18 + 20 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Hence, the given system is inconsistent.

(iv) Given $4x - 5y - 2z = 2$

$5x - 4y + 2z = -2$

$2x + 2y + 8z = -1$

$$\begin{bmatrix} 4 & -5 & -2 \\ 5 & -4 & 2 \\ 2 & 2 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$$

$$|A| = 4(-36) + 5(36) - 2(18)$$

$$|A| = 0$$

Cofactors of A are:

$$C_{11} = (-1)^{1+1} - 32 - 4 = -36$$

$$C_{21} = (-1)^{2+1} - 40 + 4 = -36$$

$$C_{31} = (-1)^{3+1} - 10 - 8 = -18$$

$$C_{12} = (-1)^{1+2} 40 - 4 = -36$$

$$C_{22} = (-1)^{2+2} 32 + 4 = 36$$

$$C_{32} = (-1)^{3+2} 8 + 10 = -18$$

$$C_{13} = (-1)^{1+3} 10 + 8 = 18$$

$$C_{23} = (-1)^{2+3} 8 + 10 = -18$$

$$C_{33} = (-1)^{3+3} - 16 + 25 = 9$$

$$\text{Adj A} = \begin{bmatrix} -36 & -36 & 18 \\ 36 & 36 & -18 \\ -18 & -18 & 9 \end{bmatrix}^T$$

$$= \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}$$

$$\text{Adj A} \times B = \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -72 - 72 + 18 \\ -72 - 72 + 18 \\ 36 + 36 - 9 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, the above system is inconsistent.

(v) Given $3x - y - 2z = 2$

$$2y - z = -1$$

$$3x - 5y = 3$$

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$|A| = 3(-5) + 1(3) - 2(-6)$$

$$|A| = 0$$

Cofactors of A are

$$C_{11} = (-1)^{1+1} 0 - 5 = -5$$

$$C_{21} = (-1)^{2+1} 0 - 10 = 10$$

$$C_{31} = (-1)^{3+1} 1 + 4 = 5$$

$$C_{12} = (-1)^{1+2} 0 + 3 = -3$$

$$C_{22} = (-1)^{2+1} 0 + 6 = 6$$

$$C_{32} = (-1)^{3+1} -3 - 0 = 3$$

$$C_{13} = (-1)^{1+2} 0 - 6 = -6$$

$$C_{23} = (-1)^{2+1} -15 + 3 = 12$$

$$C_{33} = (-1)^{3+1} 6 - 0 = 6$$

$$\text{Adj A} = \begin{bmatrix} -5 & 3 & -6 \\ 10 & 6 & 12 \\ 5 & 3 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$\text{Adj A} \times \text{B} = \begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -10 - 10 + 15 \\ 6 - 6 + 9 \\ -12 - 12 + 18 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, the above system is inconsistent.

(vi) Given $x + y - 2z = 5$

$$x - 2y + z = -2$$

$$-2x + y + z = 4$$

$$\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

$$|A| = 1(-3) - 1(3) - 2(-3) = -3 - 3 + 6$$

$$|A| = 0$$

Cofactors of A are:

$$C_{11} = (-1)^{1+1} -2 - 1 = -3$$

$$C_{21} = (-1)^{2+1} 1 + 2 = -3$$

$$C_{31} = (-1)^{3+1} 1 - 4 = -3$$

$$C_{12} = (-1)^{1+2} 1 + 2 = -3$$

$$C_{22} = (-1)^{2+1} 1 - 4 = -3$$

$$C_{32} = (-1)^{3+1} 1 + 2 = -3$$

$$C_{13} = (-1)^{1+2} 1 - 4 = -3$$

$$C_{23} = (-1)^{2+1} 1 + 2 = -3$$

$$C_{33} = (-1)^{3+1} - 2 - 1 = -3$$

$$\text{Adj A} = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}$$

$$\text{Adj A} \times B = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -15 + 6 - 12 \\ -15 + 6 - 12 \\ -15 + 6 - 12 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, the above system is inconsistent.

5. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & 1 & -5 \end{bmatrix}$ are two square matrices.

Find AB and hence solve the system of linear equations :

$$x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$$

Solution:

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 + 4 + 0 & 2 - 2 + 0 & -4 + 4 + 0 \\ 4 - 12 + 8 & 4 + 6 - 4 & -8 - 12 + 20 \\ 0 - 4 + 4 & 0 + 2 - 2 & 0 - 4 + 10 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Now, we can see that it is $AB = 6I$. Where I is the unit Matrix

$$\text{Or, } A^{-1} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$AX = B$$

$$\text{Or, } X = A^{-1}B$$

$$= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

Hence, $x = 2$, $y = -1$ and $z = 4$

6. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations :

$$2x - 3y + 5z = 11, 3x + 2y - 4z = -5, x + y - 2z = -3.$$

Solution:

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A| = 2(0) + 3(-2) + 5(1) \\ = -1$$

Now, the cofactors of A

$$C_{11} = (-1)^{1+1} - 4 + 4 = 0$$

$$C_{21} = (-1)^{2+1} 6 - 5 = -1$$

$$C_{31} = (-1)^{3+1} 12 - 10 = 2$$

$$C_{12} = (-1)^{1+2} - 6 + 4 = 2$$

$$C_{22} = (-1)^{2+2} - 4 - 5 = -9$$

$$C_{32} = (-1)^{3+2} - 8 - 15 = 23$$

$$C_{13} = (-1)^{1+3} 3 - 2 = 1$$

$$C_{23} = (-1)^{2+3} 2 + 3 = -5$$

$$C_{33} = (-1)^{3+3} 4 + 9 = 13$$

$$\text{Adj } A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Now the given equation can be written as:

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$A X = B$$

$$\text{Or, } X = A^{-1}B$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 + 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1$, $y = 2$ and $z = 3$

7. Find A^{-2} , if $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$. Hence solve the following system of linear equations :

$$x + 2y + 5z = 10, x - y - z = -2, 2x + 3y - z = -11.$$

Solution:

Given

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$$

$$\begin{aligned} |A| &= 1(1 + 3) + 2(-1 + 2) + 5(3 + 2) \\ &= 4 + 2 + 25 \\ &= 27 \end{aligned}$$

Now, the cofactors of A

$$C_{11} = (-1)^{1+1} 1 + 3 = 4$$

$$C_{21} = (-1)^{2+1} - 2 - 15 = 17$$

$$C_{31} = (-1)^{3+1} - 2 + 5 = 3$$

$$C_{12} = (-1)^{1+2} - 1 + 2 = -1$$

$$C_{22} = (-1)^{2+2} - 1 - 10 = -11$$

$$C_{32} = (-1)^{3+2} - 1 - 5 = 6$$

$$C_{13} = (-1)^{1+3} 3 + 2 = 5$$

$$C_{23} = (-1)^{2+3} 3 - 4 = 1$$

$$C_{33} = (-1)^{3+3} - 1 - 2 = -3$$

$$\text{Adj } A = \begin{bmatrix} 4 & -1 & 5 \\ 17 & -11 & 1 \\ 3 & 6 & -3 \end{bmatrix}^T = \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{27} \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

$$A X = B$$

$$\text{Or, } X = A^{-1}B$$

$$= \frac{1}{27} \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix} \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{27} \begin{bmatrix} 40 - 34 - 33 \\ -10 + 22 - 66 \\ 50 - 2 + 33 \end{bmatrix}$$

$$X = \frac{1}{27} \begin{bmatrix} -27 \\ -54 \\ 81 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$$

Hence, $x = -1$, $y = -2$ and $z = 3$

8. (i) If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} . Using A^{-1} , solve the system of linear equations :

$$x - 2y = 10, 2x + y + 3z = 8, -2y + z = 7$$

Solution:

Given

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$$

$$\begin{aligned} |A| &= 1(1 + 6) + 2(2 - 0) + 0 \\ &= 7 + 4 \\ &= 11 \end{aligned}$$

Now, the cofactors of A

$$C_{11} = (-1)^{1+1} 1 + 6 = 7$$

$$C_{21} = (-1)^{2+1} - 2 - 0 = 2$$

$$C_{31} = (-1)^{3+1} - 6 - 0 = -6$$

$$C_{12} = (-1)^{1+2} 2 - 0 = -2$$

$$C_{22} = (-1)^{2+2} 1 - 0 = 1$$

$$C_{32} = (-1)^{3+2} 3 - 0 = -3$$

$$C_{13} = (-1)^{1+3} - 4 - 0 = -4$$

$$C_{23} = (-1)^{2+3} - 2 - 0 = 2$$

$$C_{33} = (-1)^{3+3} 1 + 4 = 5$$

$$\text{Adj } A = \begin{bmatrix} 7 & 2 & -4 \\ -2 & 1 & -3 \\ -6 & 2 & 5 \end{bmatrix}^T = \begin{bmatrix} 7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{11} \begin{bmatrix} 7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5 \end{bmatrix}$$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & 3 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$A X = B$$

$$\text{Or, } X = A^{-1}B$$

$$= \frac{1}{11} \begin{bmatrix} 7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix}$$

$$X = \frac{1}{11} \begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix}$$

$$X = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

Hence, $x = 4$, $y = -3$ and $z = 1$

(ii) $A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations :

$$3x - 4y + 2z = -1, 2x + 3y + 5z = 7, x + z = 2$$

Solution:

Given

$$A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} |A| &= 3(3 - 0) + 4(2 - 5) + 2(0 - 3) \\ &= 9 - 12 - 6 \\ &= -9 \end{aligned}$$

Now, the cofactors of A

$$C_{11} = (-1)^{1+1} 3 - 0 = 3$$

$$C_{21} = (-1)^{2+1} - 4 - 0 = 4$$

$$C_{31} = (-1)^{3+1} - 20 - 6 = -26$$

$$C_{12} = (-1)^{1+2} 2 - 5 = 3$$

$$C_{22} = (-1)^{2+2} 3 - 2 = 1$$

$$C_{32} = (-1)^{3+2} 15 - 4 = -11$$

$$C_{13} = (-1)^{1+3} 0 - 3 = -3$$

$$C_{23} = (-1)^{2+1} 0 + 4 = -4$$

$$C_{33} = (-1)^{3+1} 9 + 8 = 17$$

$$\text{Adj } A = \begin{bmatrix} 3 & 3 & -3 \\ 4 & 1 & -4 \\ -26 & -4 & 27 \end{bmatrix}^T = \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{-9} \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$$

Now the given equation can be written as:

$$\begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

$$A X = B$$

$$\text{Or, } X = A^{-1}B$$

$$= \frac{1}{-9} \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix} \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-9} \begin{bmatrix} -3 + 28 - 52 \\ 21 + 7 + 22 \\ 3 - 28 + 34 \end{bmatrix}$$

$$X = \frac{1}{-9} \begin{bmatrix} -27 \\ -18 \\ 9 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

Hence, $x = 3$, $y = 2$ and $z = -1$

(iii) $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, and $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$ find AB . Hence solve the system of linear equations :

$x - 2y = 10, 2x + y + 3z = 8$ and $-2y + z = 7$

Solution:

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 7 + 4 - 0 & 2 - 2 + 0 & -6 + 6 + 0 \\ 14 - 2 - 12 & 4 + 1 + 6 & -12 - 3 + 15 \\ 0 - 4 + 4 & 0 - 2 + 2 & 0 + 6 + 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Now, we can see that it is $AB = 11I$. Where I is the unit Matrix

$$\text{Or, } A^{-1} = \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$AX = B$$

$$\text{Or, } X = A^{-1}B$$

$$= \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix}$$

$$X = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

Hence, $x = 4$, $y = -3$ and $z = 1$

(iv) If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} . Using A^{-1} solve the system of linear equations :

$$x - 2y = 10, 2x - y - z = 8, -2y + z = 7$$

Solution:

Given

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\begin{aligned} |A| &= 1(-1-1) - 2(-2-0) + 0 \\ &= -2 + 4 \\ &= 2 \end{aligned}$$

Now, the cofactors of A

$$C_{11} = (-1)^{1+1} - 1 - 1 = -2$$

$$C_{21} = (-1)^{2+1} 2 - 0 = 2$$

$$C_{31} = (-1)^{3+1} - 2 - 0 = -2$$

$$C_{12} = (-1)^{1+2} 2 - 0 = -2$$

$$C_{22} = (-1)^{2+2} 1 - 0 = 1$$

$$C_{32} = (-1)^{3+2} - 1 - 0 = 1$$

$$C_{13} = (-1)^{1+3} - 2 - 0 = -2$$

$$C_{23} = (-1)^{2+3} - 1 - 0 = 1$$

$$C_{33} = (-1)^{3+3} - 1 + 4 = 3$$

$$\text{Adj } A = \begin{bmatrix} -2 & -2 & -2 \\ 2 & 1 & 1 \\ -2 & 1 & 3 \end{bmatrix}^T = \begin{bmatrix} -2 & 2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{2} \begin{bmatrix} -2 & 2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$A X = B$$

$$\text{Or, } X = A^{-1}B$$

$$= \frac{1}{2} \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 10 - 16 + 0 \\ 20 - 8 - 7 \\ 0 - 16 + 7 \end{bmatrix}$$

$$X = \frac{1}{2} \begin{bmatrix} -6 \\ 5 \\ -9 \end{bmatrix}$$

Hence, $x = -3$, $y = 2.5$ and $z = -4.5$

(v) Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

find BA and Use this to solve the system of linear equations $y + 2z = 7$, $x - y = 3$, $2x + 3y + 4z = 17$

Solution:

Given

$$B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 + 4 - 0 & 2 - 2 + 0 & -4 + 4 + 0 \\ -4 - 12 + 16 & 4 + 6 - 4 & -8 - 12 + 20 \\ 0 - 4 + 8 & 0 - 2 + 2 & 0 - 4 + 10 \end{bmatrix}$$

$$BA = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Now, we can see that it is $BA = 6I$. Where I is the unit Matrix

$$\text{Or, } B^{-1} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}$$

Now the given equation can be written as:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 0 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 17 \end{bmatrix}$$

$$AX = B$$

$$\text{Or, } X = B^{-1}A$$

$$= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 17 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 14 + 6 - 68 \\ -28 + 6 - 68 \\ 28 - 3 + 85 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} -48 \\ -90 \\ 110 \end{bmatrix}$$

$$X = \begin{bmatrix} -8 \\ -15 \\ \frac{110}{6} \end{bmatrix}$$

Hence, $x = -8$, $y = -15$ and $z = \frac{110}{6}$

9. The sum of three numbers is 2. If twice the second number is added to the sum of first and third, the sum is 1. By adding second and third number to five times the first number, we get 6. Find the three numbers by using matrices.

Solution:

Let the numbers are x, y, z

$$x + y + z = 2 \dots\dots (i)$$

Also, $2y + (x + z) + 1$

$$x + 2y + z = 1 \dots\dots (ii)$$

Again,

$$x + z + 5(x) = 6$$

$$5x + y + z = 6 \dots\dots (iii)$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

$$A X = B$$

$$|A| = 1(1) - 1(-4) + 1(-9)$$

$$= 1 + 4 - 9$$

$$= -4$$

Hence, the unique solution given by $x = A^{-1}B$

$$C_{11} = (-1)^{1+1} (2 - 1) = 1$$

$$C_{12} = (-1)^{1+2} (1 - 5) = 4$$

$$C_{13} = (-1)^{1+3} (1 - 10) = -9$$

$$C_{21} = (-1)^{2+1} (1 - 1) = 0$$

$$C_{22} = (-1)^{2+2} (1 - 5) = -4$$

$$C_{23} = (-1)^{2+3} (1 - 5) = 4$$

$$C_{31} = (-1)^{3+1} (1 - 2) = -1$$

$$C_{32} = (-1)^{3+2} (1 - 1) = 0$$

$$C_{33} = (-1)^{3+3} (2 - 1) = 1$$

$$X = A^{-1} B = \frac{1}{|A|} (\text{adj } A) B$$

$$\text{Adj } A = \begin{bmatrix} 1 & 4 & -9 \\ 0 & -4 & 4 \\ -1 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}$$

$$X = \frac{1}{-4} \begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

$$X = \frac{1}{-4} \begin{bmatrix} 2 - 6 \\ 8 - 4 \\ -18 + 4 + 6 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} -4 \\ 4 \\ -8 \end{bmatrix}$$

$$\text{Hence, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

10. An amount of ₹10,000 is put into three investments at the rate of 10, 12 and 15% per annum. The combined incomes are ₹1310 and the combined income of first and second investment is ₹ 190 short of the income from the third. Find the investment in each using matrix method.

Solution:

Let the numbers are x, y, and z

$$x + y + z = 10,000 \dots\dots (i)$$

Also,

$$0.1x + 0.12y + 0.15z = 1310 \dots\dots (ii)$$

Again,

$$0.1x + 0.12y - 0.15z = -190 \dots\dots (iii)$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.12 & 0.15 \\ 0.1 & 0.12 & -0.15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

$$A X = B$$

$$|A| = 1(-0.036) - 1(-0.03) + 1(0)$$

$$= -0.006$$

Hence, the unique solution given by $x = A^{-1}B$

$$C_{11} = -0.036$$

$$C_{12} = 0.27$$

$$C_{13} = 0$$

$$C_{21} = 0.27$$

$$C_{22} = -0.25$$

$$C_{23} = -0.02$$

$$C_{31} = 0.03$$

$$C_{32} = -0.05$$

$$C_{33} = 0.02$$

$$X = A^{-1} B = \frac{1}{|A|} (\text{adj } A) B$$

$$\text{Adj } A = \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.27 & -0.25 & -0.05 \\ 0.03 & -0.02 & 0.02 \end{bmatrix}^T = \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix}$$

$$X = \frac{1}{-0.006} \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix} \begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

$$X = \frac{1}{-0.006} \begin{bmatrix} -12 \\ -18 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2000 \\ 3000 \\ 5000 \end{bmatrix}$$

Hence, $x = \text{Rs } 2000$, $y = \text{Rs } 3000$ and $z = \text{Rs } 5000$