EXERCISE 8.1 PAGE NO: 8.14

1. Solve the following system of equations by matrix method:

(i) 5x + 7y + 2 = 0
4x + 6y + 3 = 0
(ii) 5x + 2y = 3
3x + 2y = 5
(iii) 3x + 4y – 5 = 0
x – y + 3 = 0
(iv) 3x + y = 19
3x – y = 23
(v) 3x + 7y = 4
x + 2y = -1
(vi) 3x + y = 7
5x + 3y = 12

Solution:

(i) Given 5x + 7y + 2 = 0 and 4x + 6y + 3 = 0

The above system of equations can be written as

$$\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$ Or AX = B

Where A = $\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}$ B = $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 30 – 28 = 2

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1}\, 6 = 6$

$C_{12} = (-1)^{1+2}\, 4 = -4$

$C_{21} = (-1)^{2+1}\, 7 = -7$

$C_{22} = (-1)^{2+2}\, 5 = 5$

Also, adj A = $\begin{bmatrix} 6 & -4 \\ -7 & 5 \end{bmatrix}^T$

$= \begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$

Now, $X = A^{-1}B$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -12 + 21 \\ 8 - 15 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \frac{9}{2} \\ \frac{-7}{2} \end{bmatrix}$

Hence, x = 9/2 and y = -7/2

(ii) Given 5x + 2y = 3
3x + 2y = 5

The above system of equations can be written as

$\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 10 – 6 = 4

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1}\, 2 = 2$

$C_{12} = (-1)^{1+2}\, 3 = -3$

$C_{21} = (-1)^{2+1}\, 2 = -2$

$C_{22} = (-1)^{2+2}\, 5 = 5$

Also, adj A = $\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}^T$

$= \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

$A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

Now, $X = A^{-1}B$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 6-10 \\ -9+25 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Hence, x = -1 and y = 4

(iii) Given $3x + 4y - 5 = 0$

$x - y + 3 = 0$

The above system of equations can be written as

$\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}$ B = $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = – 3 – 4 = – 7

So, the above system has a unique solution, given by

X = $A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1} - 1 = -1$

$C_{12} = (-1)^{1+2}\ 1 = -1$

$C_{21} = (-1)^{2+1}\ 4 = -4$

$C_{22} = (-1)^{2+2}\ 3 = 3$

Also, adj A = $\begin{bmatrix} -1 & -1 \\ -4 & 3 \end{bmatrix}^T$

$= \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj A}$

$A^{-1} = \frac{1}{7}\begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$

Now, X = $A^{-1}B$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ -3 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -5 + 12 \\ -5 - 9 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -14 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Hence, X = 1 Y = – 2

(iv) Given 3x + y = 19

3x – y = 23

The above system of equations can be written as

$\begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 19 \\ 23 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix}$ B = $\begin{bmatrix} 19 \\ 23 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

$|A| = -3 - 3 = -6$

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1} - 1 = -1$

$C_{12} = (-1)^{1+2}\, 3 = -3$

$C_{21} = (-1)^{2+1}\, 1 = -1$

$C_{22} = (-1)^{2+2}\, 3 = 3$

Also, adj A = $\begin{bmatrix} -1 & -3 \\ -1 & 3 \end{bmatrix}^T$

= $\begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

$A^{-1} = \frac{1}{-6}\begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$

Now, $X = A^{-1}B$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-6}\begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 19 \\ 23 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-6}\begin{bmatrix} -19 - 23 \\ -57 + 69 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-6}\begin{bmatrix} -42 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$$

Hence, x = 7 and y = – 2

(v) Given 3x + 7y = 4

x + 2y = -1

The above system of equations can be written as

$$\begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$ Or AX = B

Where A = $\begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 6 – 7 = – 1

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1}\, 2 = 2$

$C_{12} = (-1)^{1+2}\, 1 = -1$

$C_{21} = (-1)^{2+1}\, 7 = -7$

$C_{22} = (-1)^{2+2}\, 3 = 3$

Also, adj A = $\begin{bmatrix} 2 & -1 \\ -7 & 3 \end{bmatrix}^T$

= $\begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

Now, $X = A^{-1}B$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 8 + 7 \\ -4 - 3 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 15 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -15 \\ 7 \end{bmatrix}$$

Hence, $X = -15$ $Y = 7$

(vi) Given $3x + y = 7$
$5x + 3y = 12$
The above system of equations can be written as

$$\begin{bmatrix} 3 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$ Or $AX = B$

Where $A = \begin{bmatrix} 3 & 1 \\ 5 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$ and $X = \begin{bmatrix} X \\ Y \end{bmatrix}$

$|A| = 9 - 5 = 4$

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1}\, 3 = 3$

$C_{12} = (-1)^{1+2}\, 5 = -5$

$C_{21} = (-1)^{2+1}\, 1 = -1$

$C_{22} = (-1)^{2+2}\, 3 = 3$

Also, adj A = $\begin{bmatrix} 3 & -5 \\ -1 & 3 \end{bmatrix}^T$

= $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}$

Now, $X = A^{-1}B$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 12 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 21 - 12 \\ -35 + 36 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 9 \\ 1 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \frac{9}{4} \\ \frac{1}{4} \end{bmatrix}$

Hence, $X = \frac{9}{4}$ $Y = \frac{1}{4}$

2. Solve the following system of equations by matrix method:

(i) x + y –z = 3
2x + 3y + z = 10
3x – y – 7z = 1
(ii) x + y + z = 3
2x – y + z = -1
2x + y – 3z = -9
(iii) 6x – 12y + 25z = 4
4x + 15y – 20z = 3
2x + 18y + 15z = 10

(iv) $3x + 4y + 7z = 14$
$2x - y + 3z = 4$
$x + 2y - 3z = 0$
(v) $(2/x) - (3/y) + (3/z) = 10$
$(1/x) + (1/y) + (1/z) = 10$
$(3/x) - (1/y) + (2/z) = 13$
(vi) $5x + 3y + z = 16$
$2x + y + 3z = 19$
$x + 2y + 4z = 25$
(vii) $3x + 4y + 2z = 8$
$2y - 3z = 3$
$x - 2y + 6z = -2$
(viii) $2x + y + z = 2$
$x + 3y - z = 5$
$3x + y - 2z = 6$
(ix) $2x + 6y = 2$
$3x - z = -8$
$2x - y + z = -3$
(x) $2y - z = 1$
$x - y + z = 2$
$2x - y = 0$
(xi) $8x + 4y + 3z = 18$
$2x + y + z = 5$
$x + 2y + z = 5$
(xii) $x + y + z = 6$
$x + 2z = 7$
$3x + y + z = 12$
(xiii) $(2/x) + (3/y) + (10/z) = 4$,
$(4/x) - (6/y) + (5/z) = 1$,
$(6/x) + (9/y) - (20/z) = 2$, $x, y, z \neq 0$
(xiv) $x - y + 2z = 7$
$3x + 4y - 5z = -5$
$2x - y + 3z = 12$

Solution:

(i) Given $x + y - z = 3$
$2x + 3y + z = 10$

3x – y – 7z = 1

The given system can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$ Or A X = B

A = $\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$

Now, |A| = $1\begin{vmatrix} 3 & 1 \\ -1 & -7 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 3 & -7 \end{vmatrix} - 1\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix}$

= (– 20) – 1(– 17) – 1(11)

= – 20 + 17 + 11 = 8

So, the above system has a unique solution, given by

X = $A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1} - 21 + 1 = -20$

$C_{21} = (-1)^{2+1} - 7 - 1 = 8$

$C_{31} = (-1)^{3+1}\ 1 + 3 = 4$

$C_{12} = (-1)^{1+2} - 14 - 3 = 17$

$C_{22} = (-1)^{2+1} - 7 + 3 = -4$

$C_{32} = (-1)^{3+1}\ 1 + 2 = -3$

$C_{13} = (-1)^{1+2} - 2 - 9 = -11$

$C_{23} = (-1)^{2+1} - 1 - 3 = 4$

$C_{33} = (-1)^{3+1}\ 3 - 2 = 1$

Adj A = $\begin{bmatrix} -20 & 17 & -11 \\ 8 & -4 & 4 \\ 4 & -3 & 1 \end{bmatrix}^T$

= $\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$

Now, X = $A^{-1}B$ = $\frac{1}{8}\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$

X = $\frac{1}{8}\begin{bmatrix} -60 + 80 + 4 \\ 51 - 40 - 3 \\ -33 + 40 + 1 \end{bmatrix}$

X = $\frac{1}{8}\begin{bmatrix} 24 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

Hence, X = 3, Y = 1 and Z = 1

(ii) Given x + y + z = 3
2x – y + z = -1
2x + y – 3z = -9
The given system can be written in matrix form as:

$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$ Or A X = B

A = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$

Now, |A| = $1\begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 2 & -3 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix}$

= (3 – 1) – 1(– 6 – 2) + 1(2 + 2)
= 2 + 8 + 4
= 14
So, the above system has a unique solution, given by
X = $A^{-1}B$
Cofactors of A are
$C_{11} = (-1)^{1+1}\ 3 - 1 = 2$
$C_{21} = (-1)^{2+1} - 3 - 1 = 4$
$C_{31} = (-1)^{3+1}\ 1 + 1 = 2$
$C_{12} = (-1)^{1+2} - 6 - 2 = 8$
$C_{22} = (-1)^{2+1} - 3 - 2 = -5$
$C_{32} = (-1)^{3+1}\ 1 - 2 = 1$
$C_{13} = (-1)^{1+2}\ 2 + 2 = 4$
$C_{23} = (-1)^{2+1}\ 1 - 2 = 1$

$C_{33} = (-1)^{3+1} - 1 - 2 = -3$

Adj A = $\begin{bmatrix} 2 & 8 & 4 \\ 4 & -5 & 1 \\ 2 & 1 & -3 \end{bmatrix}^T$

= $\begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}$

Now, X = $A^{-1}B$ = $\frac{1}{14}\begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$

X = $\frac{1}{14}\begin{bmatrix} -16 \\ 20 \\ 38 \end{bmatrix}$

X = $\begin{bmatrix} \frac{-8}{7} \\ \frac{10}{7} \\ \frac{19}{7} \end{bmatrix}$

Hence, X = $\frac{-8}{7}$, Y = $\frac{10}{7}$ and Z = $\frac{19}{7}$

(iii) Given 6x – 12y + 25z = 4

4x + 15y – 20z = 3

2x + 18y + 15z = 10

The given system can be written in matrix form as:

$\begin{bmatrix} 6 & -12 & 25 \\ 4 & 15 & -20 \\ 2 & 18 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$ Or A X = B

A = $\begin{bmatrix} 6 & -12 & 25 \\ 4 & 15 & -20 \\ 2 & 18 & 15 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$

Now, |A| = $6\begin{vmatrix} 15 & -20 \\ 18 & 15 \end{vmatrix} - 12\begin{vmatrix} 4 & -20 \\ 2 & 15 \end{vmatrix} + 25\begin{vmatrix} 4 & 15 \\ 2 & 18 \end{vmatrix}$

= 6(225 + 360) + 12(60 + 40) + 25(72 – 30)
= 3510 + 1200 + 1050
= 5760
So, the above system has a unique solution, given by
$X = A^{-1}B$
Cofactors of A are
$C_{11} = (-1)^{1+1}(225 + 360) = 585$
$C_{21} = (-1)^{2+1}(-180 - 450) = 630$
$C_{31} = (-1)^{3+1}(240 - 375) = -135$
$C_{12} = (-1)^{1+2}(60 + 40) = -100$
$C_{22} = (-1)^{2+1}(90 - 50) = 40$
$C_{32} = (-1)^{3+1}(-120 - 100) = 220$
$C_{13} = (-1)^{1+2}(72 - 30) = 42$
$C_{23} = (-1)^{2+1}(108 + 24) = -132$
$C_{33} = (-1)^{3+1}(90 + 48) = 138$

$$\text{Adj } A = \begin{bmatrix} 585 & -100 & 42 \\ 630 & 40 & -132 \\ -135 & 220 & 138 \end{bmatrix}^T$$

$$= \begin{bmatrix} 585 & 630 & -135 \\ -100 & 40 & 220 \\ 42 & -132 & 138 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{5760}\begin{bmatrix} 585 & 630 & -135 \\ -100 & 40 & 220 \\ 42 & -132 & 138 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$$

$$X = \frac{1}{5760}\begin{bmatrix} 2880 \\ 1920 \\ 1152 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

Hence, $X = \frac{1}{2}, Y = \frac{1}{3}$ and $Z = \frac{1}{5}$

(iv) Given 3x + 4y + 7z = 14
2x – y + 3z = 4
x + 2y – 3z = 0
The given system can be written in matrix form as:

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$ Or A X = B

A = $\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$

Now, |A| = $3\begin{vmatrix} -1 & 3 \\ 2 & -3 \end{vmatrix} - 4\begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix} + 7\begin{vmatrix} 2 & 3 \\ 2 & -3 \end{vmatrix}$

= 3(3 – 6) – 4(– 6 – 3) + 7(4 + 1)
= – 9 + 36 + 35
= 62
So, the above system has a unique solution, given by
$X = A^{-1}B$
Cofactors of A are
$C_{11} = (-1)^{1+1}\ 3 - 6 = -3$
$C_{21} = (-1)^{2+1} - 12 - 14 = 26$
$C_{31} = (-1)^{3+1} 12 + 7 = 19$
$C_{12} = (-1)^{1+2} - 6 - 3 = 9$
$C_{22} = (-1)^{2+1} - 3 - 7 = -10$
$C_{32} = (-1)^{3+1}\ 9 - 14 = 5$
$C_{13} = (-1)^{1+2}\ 4 + 1 = 5$
$C_{23} = (-1)^{2+1}\ 6 - 4 = -2$
$C_{33} = (-1)^{3+1} - 3 - 8 = -11$

Adj A = $\begin{bmatrix} -3 & 9 & 5 \\ 26 & -5 & -2 \\ 19 & 5 & -11 \end{bmatrix}^T$

= $\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

Now, $X = A^{-1}B = \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$

$X = \frac{1}{62}\begin{bmatrix} -42 + 104 + 0 \\ 126 - 64 + 0 \\ 70 - 8 + 0 \end{bmatrix}$

$X = \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix}$

$X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Hence, X = 1, Y = 1 and Z = 1

(v) Given (2/x) – (3/y) + (3/z) = 10
(1/x) + (1/y) + (1/z) = 10
(3/x) – (1/y) + (2/z) = 13
The given system can be written in matrix form as:

$\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$ Or A X = B

$A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$

Now, $|A| = 5\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$
= 5(4 – 6) – 3(8 – 3) + 1(4 – 2)
= – 10 – 15 + 3
= – 22
So, the above system has a unique solution, given by
$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1}(4-6) = -2$

$C_{21} = (-1)^{2+1}(12-2) = -10$

$C_{31} = (-1)^{3+1}(9-1) = 8$

$C_{12} = (-1)^{1+2}(8-3) = -5$

$C_{22} = (-1)^{2+1}\ 20 - 1 = 19$

$C_{32} = (-1)^{3+1}\ 15 - 2 = -13$

$C_{13} = (-1)^{1+2}(4-2) = 2$

$C_{23} = (-1)^{2+1}\ 10 - 3 = -7$

$C_{33} = (-1)^{3+1}\ 5 - 6 = -1$

$$\text{Adj } A = \begin{bmatrix} -2 & -5 & -3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & 13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{-22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}\begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$$

$$X = \frac{1}{-22}\begin{bmatrix} -32 - 190 + 200 \\ -80 + 361 - 325 \\ 48 - 133 - 25 \end{bmatrix}$$

$$X = \frac{1}{-22}\begin{bmatrix} -22 \\ -44 \\ -110 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

Hence, X = 1, Y = 2 and Z = 5

(vi) Given 5x + 3y + z = 16

2x + y + 3z = 19
x + 2y + 4z = 25

The given system can be written in matrix form as:

$$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$$ Or A X = B

A = $\begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$

Now, $|A| = 3\begin{vmatrix} 2 & -3 \\ -2 & 6 \end{vmatrix} - 4\begin{vmatrix} 0 & -3 \\ 1 & 6 \end{vmatrix} + 2\begin{vmatrix} 0 & 2 \\ 1 & -2 \end{vmatrix}$

= 3(12 – 6) – 4(0 + 3) + 2(0 – 2)
= 18 – 12 – 4
= 2

So, the above system has a unique solution, given by
$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1}(12 - 6) = 6$
$C_{21} = (-1)^{2+1}(24 + 4) = -28$
$C_{31} = (-1)^{3+1}(-12 - 4) = -16$
$C_{12} = (-1)^{1+2}(0 + 3) = -3$
$C_{22} = (-1)^{2+1}\ 18 - 2 = 16$
$C_{32} = (-1)^{3+1} - 9 - 0 = 9$
$C_{13} = (-1)^{1+2}(0 - 2) = -2$
$C_{23} = (-1)^{2+1}(-6 - 4) = 10$
$C_{33} = (-1)^{3+1}\ 6 - 0 = 6$

Adj A = $\begin{bmatrix} 6 & -3 & 2 \\ -28 & 16 & 10 \\ -16 & -9 & 6 \end{bmatrix}^T$

= $\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & -9 \\ 2 & 10 & 6 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

Now, X = A^{-1}B = $\frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & -9 \\ 2 & 10 & 6 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$

X = $\frac{1}{2}\begin{bmatrix} 48 - 84 + 32 \\ -24 + 48 - 18 \\ -16 + 30 - 12 \end{bmatrix}$

X = $\frac{1}{2}\begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix}$

X = $\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

Hence, X = – 2, Y = 3 and Z = 1

(vii) Given 3x + 4y + 2z = 8
2y – 3z = 3
x – 2y + 6z = -2
The given system can be written in matrix form as:

$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$ Or A X = B

A = $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$

Now, |A| = $2\begin{vmatrix} 3 & -1 \\ 1 & -2 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}$

= 2(– 6 + 1) – 1(– 2 + 3) + 1(1 – 9)
= – 10 – 1 – 8
= – 19
So, the above system has a unique solution, given by
X = A^{-1}B
Cofactors of A are
$C_{11} = (-1)^{1+1} - 6 + 1 = -5$

$C_{21} = (-1)^{2+1}(24 + 4) = -28$
$C_{31} = (-1)^{3+1} - 1 - 3 = -4$
$C_{12} = (-1)^{1+2} - 2 + 3 = -1$
$C_{22} = (-1)^{2+1} - 4 - 3 = -7$
$C_{32} = (-1)^{3+1} - 2 - 1 = 3$
$C_{13} = (-1)^{1+2} 1 - 9 = -8$
$C_{23} = (-1)^{2+1} 2 - 3 = -1$
$C_{33} = (-1)^{3+1} 6 - 1 = 5$

$\text{Adj } A = \begin{bmatrix} -5 & -1 & -8 \\ 3 & -7 & 1 \\ -4 & 3 & 5 \end{bmatrix}^T$

$= \begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, $X = A^{-1}B = \frac{1}{-19} \begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$

$X = \frac{1}{-19} \begin{bmatrix} -10 + 15 - 24 \\ -2 - 35 + 18 \\ -16 + 5 + 30 \end{bmatrix}$

$X = \frac{1}{-19} \begin{bmatrix} -19 \\ -19 \\ 19 \end{bmatrix}$

$X = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

Hence, X = 1, Y = 1 and Z = – 1

(viii) Given 2x + y + z = 2
x + 3y - z = 5
3x + y – 2z = 6

The given system can be written in matrix form as:

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$$ Or A X = B

A = $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$

Now, $|A| = 2\begin{vmatrix} 3 & -1 \\ 1 & -2 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}$

$= 2(-6 + 1) - 1(-2 + 3) + 1(1 - 9)$

$= -10 - 1 - 8$

$= -19$

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1} - 6 + 1 = -5$

$C_{21} = (-1)^{2+1}(24 + 4) = -28$

$C_{31} = (-1)^{3+1} - 1 - 3 = -4$

$C_{12} = (-1)^{1+2} - 2 + 3 = -1$

$C_{22} = (-1)^{2+1} - 4 - 3 = -7$

$C_{32} = (-1)^{3+1} - 2 - 1 = 3$

$C_{13} = (-1)^{1+2} 1 - 9 = -8$

$C_{23} = (-1)^{2+1} 2 - 3 = -1$

$C_{33} = (-1)^{3+1} 6 - 1 = 5$

Adj A = $\begin{bmatrix} -5 & -1 & -8 \\ 3 & -7 & 1 \\ -4 & 3 & 5 \end{bmatrix}^T$

= $\begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, X = A $^{-1}$B = $\frac{1}{-19}\begin{bmatrix}-5 & 3 & -4\\ -1 & -7 & 3\\ -8 & 1 & 5\end{bmatrix}\begin{bmatrix}2\\5\\6\end{bmatrix}$

X = $\frac{1}{-19}\begin{bmatrix}-10 + 15 - 24\\ -2 - 35 + 18\\ -16 + 5 + 30\end{bmatrix}$

X = $\frac{1}{-19}\begin{bmatrix}-19\\ -19\\ 19\end{bmatrix}$

X = $\begin{bmatrix}1\\ 1\\ -1\end{bmatrix}$

Hence, X = 1, Y = 1 and Z = – 1

(ix) Given 2x + 6y = 2
3x – z = -8
2x – y + z = -3
The given system can be written in matrix form as:

$\begin{bmatrix}2 & 6 & 0\\ 3 & 0 & -1\\ 2 & -1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}2\\ -8\\ -3\end{bmatrix}$ Or A X = B

A = $\begin{bmatrix}2 & 6 & 0\\ 3 & 0 & -1\\ 2 & -1 & 1\end{bmatrix}$, X = $\begin{bmatrix}x\\y\\z\end{bmatrix}$ and B = $\begin{bmatrix}2\\ -8\\ -3\end{bmatrix}$

Now, |A| = 2$\begin{vmatrix}0 & -1\\ -1 & 1\end{vmatrix} - 6\begin{vmatrix}3 & -1\\ 2 & 1\end{vmatrix} + 0$
= 2(0 – 1) – 6(3 + 2)
= – 2 – 30
= – 32
So, the above system has a unique solution, given by
X = A $^{-1}$B
Cofactors of A are
$C_{11} = (-1)^{1+1}\ 0 - 1 = -1$

$C_{21} = (-1)^{2+1} 6 + 0 = -6$
$C_{31} = (-1)^{3+1} - 6 = -6$
$C_{12} = (-1)^{1+2}\ 3 + 2 = 5$
$C_{22} = (-1)^{2+1}\ 2 - 0 = 2$
$C_{32} = (-1)^{3+1} - 2 - 0 = 2$
$C_{13} = (-1)^{1+2} - 3 - 0 = -3$
$C_{23} = (-1)^{2+1} - 2 - 12 = 14$
$C_{33} = (-1)^{3+1}\ 0 - 18 = -18$

$$\text{Adj } A = \begin{bmatrix} -1 & -5 & -3 \\ -6 & 2 & 14 \\ -6 & 2 & -18 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{-32}\begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}\begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$$

$$X = \frac{1}{32}\begin{bmatrix} -2 + 48 + 18 \\ -10 - 16 - 6 \\ -6 - 112 + 54 \end{bmatrix}$$

$$X = \frac{1}{32}\begin{bmatrix} 64 \\ -32 \\ -64 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

Hence, X = – 2, Y = 1 and Z = 2

(x) Given $2y - z = 1$
$x - y + z = 2$
$2x - y = 0$

The given system can be written in matrix form as:

$$\begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

AX = B

Now, $|A| = 0 - 2\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix}$

= 0 + 4 – 1

= 3

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1}\ 1 - 0 = 1$

$C_{21} = (-1)^{2+1} 1 - 2 = 1$

$C_{31} = (-1)^{3+1} 0 + 1 = 1$

$C_{12} = (-1)^{1+2} - 2 - 0 = 2$

$C_{22} = (-1)^{2+1} - 1 - 0 = -1$

$C_{32} = (-1)^{3+1}\ 0 - 2 = 2$

$C_{13} = (-1)^{1+2}\ 4 - 0 = 4$

$C_{23} = (-1)^{2+1}\ 2 - 0 = -2$

$C_{33} = (-1)^{3+1} - 1 + 2 = 1$

$\text{Adj } A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & -1 & -2 \\ 1 & 2 & 1 \end{bmatrix}^T$

$= \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, $X = A^{-1}B = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

$X = \frac{1}{3}\begin{bmatrix} 2+0+1 \\ 4-0+2 \\ 8-0+1 \end{bmatrix}$

$X = \frac{1}{3}\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

$X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Hence, X = 1, Y = 2 and Z = 3

(xi) Given 8x + 4y + 3z = 18
2x + y + z = 5
x + 2y + z = 5

The given system can be written in matrix form as:

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

AX = B

Now, $|A| = 8\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - 4\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$

= 8(– 1) – 4(1) + 3(3)
= – 8 – 4 + 9
= – 3
So, the above system has a unique solution, given by
$X = A^{-1}B$
Cofactors of A are
$C_{11} = (-1)^{1+1}\ 1 - 2 = -1$
$C_{21} = (-1)^{2+1}\ 4 - 6 = 2$
$C_{31} = (-1)^{3+1}\ 4 - 3 = 1$
$C_{12} = (-1)^{1+2}\ 2 - 1 = -1$
$C_{22} = (-1)^{2+1}\ 8 - 3 = 5$
$C_{32} = (-1)^{3+1}\ 8 - 6 = -2$

$C_{13} = (-1)^{1+2}\ 4 - 1 = 3$
$C_{23} = (-1)^{2+1}\ 16 - 4 = -12$
$C_{33} = (-1)^{3+1}\ 8 - 8 = 0$

$$\text{Adj } A = \begin{bmatrix} -1 & -1 & 3 \\ 2 & 5 & -12 \\ 1 & -2 & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{-3}\begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}\begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

$$X = -\frac{1}{3}\begin{bmatrix} -18 + 10 + 5 \\ -18 + 25 - 10 \\ 54 - 60 + 0 \end{bmatrix}$$

$$X = \frac{1}{-3}\begin{bmatrix} -3 \\ -3 \\ -6 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Hence, X = 1, Y = 1 and Z = 2

(xii) Given x + y + z = 6
x + 2z = 7
3x + y + z = 12
The given system can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 17 \\ 12 \end{bmatrix}$$

A X = B

Now, $|A| = 1\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

= 1(– 2) – 1(1 – 6) + 1(1)

= – 2 + 5 + 1

= 4

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are

$C_{11} = (-1)^{1+1}\ 0 - 2 = -2$

$C_{21} = (-1)^{2+1}\ 1 - 1 = 0$

$C_{31} = (-1)^{3+1}\ 2 - 0 = 2$

$C_{12} = (-1)^{1+2}\ 1 - 6 = 5$

$C_{22} = (-1)^{2+1}\ 1 - 3 = -2$

$C_{32} = (-1)^{3+1}\ 2 - 1 = -1$

$C_{13} = (-1)^{1+2}\ 1 - 0 = 1$

$C_{23} = (-1)^{2+1}\ 1 - 3 = 2$

$C_{33} = (-1)^{3+1}\ 0 - 1 = -1$

$\text{Adj } A = \begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}^T$

$= \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, $X = A^{-1}B = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$X = \frac{1}{4}\begin{bmatrix} -12 + 0 + 24 \\ 30 - 14 - 12 \\ 6 + 14 - 12 \end{bmatrix}$

$X = \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix}$

X = $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

Hence, X = 3, Y = 1 and Z = 2

(xiii) Given (2/x) + (3/y) + (10/z) = 4,
(4/x) – (6/y) + (5/z) = 1,
(6/x) + (9/y) – (20/z) = 2, x, y, z ≠ 0

The given system can be written in matrix form as:

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

AX = B
Now,
|A| = 2(75) – 3(– 110) + 10(72)
= 150 + 330 + 720
= 1200
So, the above system has a unique solution, given by
$X = A^{-1}B$
Cofactors of A are
$C_{11} = (-1)^{1+1}\ 120 - 45 = 75$
$C_{21} = (-1)^{2+1} - 60 - 90 = 150$
$C_{31} = (-1)^{3+1}\ 15 + 60 = 75$
$C_{12} = (-1)^{1+2} - 80 - 30 = 110$
$C_{22} = (-1)^{2+1} - 40 - 60 = -100$
$C_{32} = (-1)^{3+1}\ 10 - 40 = 30$
$C_{13} = (-1)^{1+2}\ 36 + 36 = 72$
$C_{23} = (-1)^{2+1}\ 18 - 18 = 0$
$C_{33} = (-1)^{3+1} - 12 - 12 = -24$

Adj A = $\begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T$

= $\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, $X = A^{-1}B = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$X = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$

$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$

Hence, X = 2, Y = 3 and Z = 5

(xiv) Given x – y + 2z = 7
3x + 4y – 5z = -5
2x – y + 3z = 12
The given system can be written in matrix form as:

$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

A X = B
Now,
|A| = 1(12 – 5) + 1(9 + 10) + 2(– 3 – 8)
= 7 + 19 – 22
= 4
So, the above system has a unique solution, given by
$X = A^{-1}B$
Cofactors of A are
$C_{11} = (-1)^{1+1}\ 12 - 5 = 7$
$C_{21} = (-1)^{2+1} - 3 + 2 = 1$
$C_{31} = (-1)^{3+1}\ 5 - 8 = -3$
$C_{12} = (-1)^{1+2}\ 9 + 10 = -19$

$C_{22} = (-1)^{2+1}\ 3 - 4 = -1$
$C_{32} = (-1)^{3+1} - 5 - 6 = 11$
$C_{13} = (-1)^{1+2} - 3 - 8 = -11$
$C_{23} = (-1)^{2+1} - 1 + 2 = -1$
$C_{33} = (-1)^{3+1}\ 4 + 3 = 7$

Adj A = $\begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & -7 \end{bmatrix}^T$

= $\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, $X = A^{-1}B = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

$X = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

Hence, X = 2, Y = 1 and Z = 3

3. Show that each one of the following systems of linear equations is consistent and also find their solutions:

(i) 6x + 4y = 2
9x + 6y = 3
(ii) 2x + 3y = 5
6x + 9y = 15
(iii) 5x + 3y + 7z = 4
3x + 26y + 2z = 9
7x + 2y + 10z = 5
(v) x + y + z = 6

x + 2y + 3z = 14
x + 4y + 7z = 30
(vi) 2x + 2y – 2z = 1
4x + 4y – z = 2
6x + 6y + 2z = 3

Solution:
(i) Given 6x + 4y = 2
9x + 6y = 3
The above system of equations can be written as

$$\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$ Or AX = B

Where A = $\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}$ B = $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 36 – 36 = 0
So, A is singular, Now X will be consistence if (Adj A) x B = 0
$C_{11} = (-1)^{1+1}\, 6 = 6$
$C_{12} = (-1)^{1+2}\, 9 = -9$
$C_{21} = (-1)^{2+1}\, 4 = -4$
$C_{22} = (-1)^{2+2}\, 6 = 6$

Also, adj A = $\begin{bmatrix} 6 & -9 \\ -4 & 6 \end{bmatrix}^T$

$= \begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}$

(Adj A).B = $\begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 12 - 12 \\ -18 + 18 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Thus, AX = B will be infinite solution,

Let y = k

Hence, 6x = 2 – 4k or 9x = 3 – 6k

X = $\frac{1-2k}{3}$

Hence, X = $\frac{1-2k}{3}$, Y = k

(ii) Given 2x + 3y = 5

6x + 9y = 15

The above system of equations can be written as

$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$ B = $\begin{bmatrix} 5 \\ 15 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 18 – 18 = 0

So, A is singular,

Now X will be consistence if (Adj A) x B = 0

$C_{11} = (-1)^{1+1}\, 9 = 9$

$C_{12} = (-1)^{1+2}\, 6 = -6$

$C_{21} = (-1)^{2+1}\, 3 = -3$

$C_{22} = (-1)^{2+2}\, 2 = 2$

Also, adj A = $\begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T$

$= \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$

(Adj A).B = $\left[\begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}\right]\begin{bmatrix} 5 \\ 15 \end{bmatrix}$

$= \begin{bmatrix} 45 - 45 \\ -30 + 30 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Thus, AX = B will be infinite solution,

Let y = k

Hence,

$2x = 5 - 3k$ or $X = \frac{5-3k}{2}$

$x = 15 - 9k$ or $X = \frac{5-3k}{2}$

Hence, $X = \frac{5-3k}{2}$, Y = k

(iii) Given 5x + 3y + 7z = 4
3x + 26y + 2z = 9
7x + 2y + 10z = 5
This can be written as:

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

$|A| = 5(260 - 4) - 3(30 - 14) + 7(6 - 182)$
$= 5(256) - 3(16) + 7(176)$
$|A| = 0$
So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:
(Adj A) x B≠0 or (Adj A) x B = 0
Cofactors of A are
$C_{11} = (-1)^{1+1}\ 260 - 4 = 256$
$C_{21} = (-1)^{2+1}\ 30 - 14 = -16$
$C_{31} = (-1)^{3+1}\ 6 - 182 = -176$
$C_{12} = (-1)^{1+2}\ 30 - 14 = -16$
$C_{22} = (-1)^{2+1}\ 50 - 49 = 1$
$C_{32} = (-1)^{3+1}\ 10 - 21 = 11$
$C_{13} = (-1)^{1+2}\ 6 - 182 = -176$
$C_{23} = (-1)^{2+1}\ 10 - 21 = 11$
$C_{33} = (-1)^{3+1}\ 130 - 9 = 121$

Adj A = $\begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}^T$

= $\begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}$

Adj A x B = $\begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now, AX = B has infinite many solution

Let z = k

Then, 5x + 3y = 4 – 7k

3x + 26y = 9 – 2k

This can be written as

$\begin{bmatrix} 5 & 3 \\ 3 & 26 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4-7k \\ 9-2k \end{bmatrix}$

|A| = 121

Adj A = $\begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}$

Now, X = $A^{-1}B = \frac{1}{|A|}\text{Adj } A \times B$

= $\frac{1}{121}\begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 4-7k \\ 9-2k \end{bmatrix}$

= $\frac{1}{121}\begin{bmatrix} 77-176k \\ 11k+33 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{7-16k}{11} \\ \frac{k+3}{11} \end{bmatrix}$

There values of x, y and z satisfy the third equation

(v) Given x + y + z = 6

x + 2y + 3z = 14

x + 4y + 7z = 30

This can be written as:

$$\begin{bmatrix}1 & 1 & 1\\1 & 2 & 3\\1 & 4 & 7\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}6\\14\\30\end{bmatrix}$$

|A| = 1(2) – 1(4) + 1(2)

= 2 – 4 + 2

|A| = 0

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:

(Adj A) x B≠0 or (Adj A) x B = 0

Cofactors of A are

$C_{11} = (-1)^{1+1}\ 14 - 12 = 2$

$C_{21} = (-1)^{2+1}\ 7 - 4 = -3$

$C_{31} = (-1)^{3+1}\ 3 - 2 = 1$

$C_{12} = (-1)^{1+2}\ 7 - 3 = -4$

$C_{22} = (-1)^{2+1}\ 7 - 1 = 6$

$C_{32} = (-1)^{3+1}\ 3 - 1 = 2$

$C_{13} = (-1)^{1+2}\ 4 - 2 = 2$

$C_{23} = (-1)^{2+1}\ 4 - 1 = -3$

$C_{33} = (-1)^{3+1}\ 2 - 1 = 1$

Adj A = $\begin{bmatrix}2 & -4 & 2\\-3 & 6 & -3\\1 & -2 & 1\end{bmatrix}^T$

= $\begin{bmatrix}2 & -3 & 1\\-4 & 1 & -2\\2 & -3 & 1\end{bmatrix}$

Adj A x B = $\begin{bmatrix}2 & -3 & 1\\-4 & 1 & -2\\2 & -3 & 1\end{bmatrix}\begin{bmatrix}6\\14\\30\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

Now, AX = B has infinite many solution

Let z = k

Then, x + y = 6 – k

x + 2y = 14 – 3k

This can be written as:

$$\begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}6-k\\14-3k\end{bmatrix}$$

|A| = 1

Adj A = $\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}$

Now, X = $A^{-1}B = \frac{1}{|A|}\text{Adj A} \times B$

$= \frac{1}{1}\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}\begin{bmatrix}6-k\\14-3k\end{bmatrix}$

$= \frac{1}{1}\begin{bmatrix}12-2k-14+3k\\-6+k+14-3k\end{bmatrix}$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-2+k\\8-2k\end{bmatrix}$$

There values of x, y and z satisfy the third equation

Hence, x = k – 2, y = 8 – 2k, z = k

(vi) Given x + y + z = 6

x + 2y + 3z = 14

x + 4y + 7z = 30

This can be written as

$$\begin{bmatrix}2 & 2 & -2\\4 & 4 & -1\\6 & 6 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

|A| = 2(14) – 2(14) – 2(0)

|A| = 0

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:

(Adj A) x B≠0 or (Adj A) x B = 0

Cofactors of A are:

$C_{11} = (-1)^{1+1}8 + 6 = 14$

$C_{21} = (-1)^{2+1}\ 4 + 12 = -16$
$C_{31} = (-1)^{3+1} - 2 + 8 = 6$
$C_{12} = (-1)^{1+2}\ 8 + 6 = -14$
$C_{22} = (-1)^{2+1}\ 4 + 12 = 16$
$C_{32} = (-1)^{3+1} - 2 + 8 = -6$
$C_{13} = (-1)^{1+2}\ 24 - 24 = 0$
$C_{23} = (-1)^{2+1}\ 12 - 12 = 0$
$C_{33} = (-1)^{3+1}\ 8 - 8 = 0$

$$\text{Adj } A = \begin{bmatrix} 14 & -14 & 0 \\ -16 & 16 & 0 \\ 6 & -6 & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Adj } A \times B = \begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, AX = B has infinite many solution

Let z = k

Then, 2x + 2y = 1 + 2k

4x + 4y = 2 + k

This can be written as:

$$\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 + 2k \\ 2 + k \end{bmatrix}$$

Hence, $|A| = 0 \angle = 0$

Hence, the given equation doesn't satisfy.

4. Show that each one of the following systems of linear equations is consistent:
(i) 2x + 5y = 7
6x + 15y = 13
(ii) 2x + 3y = 5
6x + 9y = 10

(iii) 4x – 2y = 3
6x – 3y = 5
(iv) 4x – 5y – 2z = 2
5x – 4y + 2z = -2
2x + 2y + 8z = -1
(v) 3x – y – 2z = 2
2y – z = -1
3x – 5y = 3
(vi) x + y – 2z = 5
x – 2y + z = -2
-2x + y + z = 4

Solution:

(i) Given 2x + 5y = 7

6x + 15y = 13

The above system of equations can be written as

$\begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix}$ B = $\begin{bmatrix} 7 \\ 13 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 30 – 30 = 0

So, A is singular,

Now X will be consistence if (Adj A) x B = 0

$C_{11} = (-1)^{1+1}\, 15 = 15$

$C_{12} = (-1)^{1+2}\, 6 = -6$

$C_{21} = (-1)^{2+1}\, 5 = -5$

$C_{22} = (-1)^{2+2}\, 2 = 2$

Also, adj A = $\begin{bmatrix} 15 & -6 \\ -5 & 2 \end{bmatrix}^T$

= $\begin{bmatrix} 15 & -5 \\ -5 & 2 \end{bmatrix}$

(Adj A).B = $\begin{bmatrix} 15 & -5 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 13 \end{bmatrix}$

= $\begin{bmatrix} 105-65 \\ -35+26 \end{bmatrix} = \begin{bmatrix} 40 \\ -16 \end{bmatrix}$

$\neq 0$

Hence, the given system is inconsistent.

(ii) Given 2x + 3y = 5
6x + 9y = 10
The above system of equations can be written as

$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$ Or AX = B

Where A = $\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$ B = $\begin{bmatrix} 5 \\ 10 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 18 – 18 = 0
So, A is singular,
Now X will be consistence if (Adj A) x B = 0
$C_{11} = (-1)^{1+1}\ 9 = 9$
$C_{12} = (-1)^{1+2}\ 6 = -6$
$C_{21} = (-1)^{2+1}\ 3 = -3$
$C_{22} = (-1)^{2+2}\ 2 = 2$

Also, adj A = $\begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T$

= $\begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$

(Adj A).B = $\begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \end{bmatrix}$

= $\begin{bmatrix} 45-30 \\ -30+20 \end{bmatrix} = \begin{bmatrix} 15 \\ -10 \end{bmatrix} \neq 0$

Hence, the given system is inconsistent.

(iii) Given 4x – 2y = 3
6x – 3y = 5
The above system of equations can be written as

$$\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$ Or AX = B

Where A = $\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$ B = $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = – 12 + 12 = 0
So, A is singular,
Now X will be consistence if (Adj A) x B = 0
$C_{11} = (-1)^{1+1} - 3 = -3$
$C_{12} = (-1)^{1+2}\, 6 = -6$
$C_{21} = (-1)^{2+1} - 2 = 2$
$C_{22} = (-1)^{2+2}\, 4 = 4$

Also, adj A = $\begin{bmatrix} -3 & -6 \\ 2 & 4 \end{bmatrix}^T$

$= \begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}$

(Adj A).B = $\begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$= \begin{bmatrix} -9 + 10 \\ -18 + 20 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Hence, the given system is inconsistent.

(iv) Given 4x – 5y – 2z = 2
5x – 4y + 2z = -2
2x + 2y + 8z = -1

$$\begin{bmatrix} 4 & -5 & -2 \\ 5 & -4 & 2 \\ 2 & 2 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$$

|A| = 4(– 36) + 5(36) – 2(18)
|A| = 0
Cofactors of A are:

$C_{11} = (-1)^{1+1} - 32 - 4 = -36$
$C_{21} = (-1)^{2+1} - 40 + 4 = -36$
$C_{31} = (-1)^{3+1} - 10 - 8 = -18$
$C_{12} = (-1)^{1+2}\ 40 - 4 = -36$
$C_{22} = (-1)^{2+1}\ 32 + 4 = 36$
$C_{32} = (-1)^{3+1}\ 8 + 10 = -18$
$C_{13} = (-1)^{1+2}\ 10 + 8 = 18$
$C_{23} = (-1)^{2+1}\ 8 + 10 = -18$
$C_{33} = (-1)^{3+1} - 16 + 25 = 9$

$$\text{Adj } A = \begin{bmatrix} -36 & -36 & 18 \\ 36 & 36 & -18 \\ -18 & -18 & 9 \end{bmatrix}^T$$

$$= \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}$$

$$\text{Adj } A \times B = \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -72 - 72 + 18 \\ -72 - 72 + 18 \\ 36 + 36 - 9 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, the above system is inconsistent.

(v) Given $3x - y - 2z = 2$
$2y - z = -1$
$3x - 5y = 3$

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$|A| = 3(-5) + 1(3) - 2(-6)$
$|A| = 0$
Cofactors of A are
$C_{11} = (-1)^{1+1}\ 0 - 5 = -5$
$C_{21} = (-1)^{2+1}\ 0 - 10 = 10$

$C_{31} = (-1)^{3+1}\ 1 + 4 = 5$
$C_{12} = (-1)^{1+2}\ 0 + 3 = -3$
$C_{22} = (-1)^{2+1}\ 0 + 6 = 6$
$C_{32} = (-1)^{3+1} -3 - 0 = 3$
$C_{13} = (-1)^{1+2}\ 0 - 6 = -6$
$C_{23} = (-1)^{2+1} -15 + 3 = 12$
$C_{33} = (-1)^{3+1}\ 6 - 0 = 6$

$$\text{Adj A} = \begin{bmatrix} -5 & 3 & -6 \\ 10 & 6 & 12 \\ 5 & 3 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$\text{Adj A x B} = \begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -10 - 10 + 15 \\ 6 - 6 + 9 \\ -12 - 12 + 18 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, the above system is inconsistent.

(vi) Given x + y – 2z = 5
x – 2y + z = -2
-2x + y + z = 4

$$\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

$|A| = 1(-3) - 1(3) - 2(-3) = -3 - 3 + 6$
$|A| = 0$
Cofactors of A are:
$C_{11} = (-1)^{1+1} -2 - 1 = -3$
$C_{21} = (-1)^{2+1}\ 1 + 2 = -3$
$C_{31} = (-1)^{3+1}\ 1 - 4 = -3$
$C_{12} = (-1)^{1+2}\ 1 + 2 = -3$
$C_{22} = (-1)^{2+1}\ 1 - 4 = -3$

$C_{32} = (-1)^{3+1}\ 1 + 2 = -3$
$C_{13} = (-1)^{1+2}\ 1 - 4 = -3$
$C_{23} = (-1)^{2+1}\ 1 + 2 = -3$
$C_{33} = (-1)^{3+1} - 2 - 1 = -3$

$$\text{Adj } A = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}$$

$$\text{Adj } A \times B = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -15 + 6 - 12 \\ -15 + 6 - 12 \\ -15 + 6 - 12 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, the above system is inconsistent.

5. $If A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ *and* $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & 1 & -5 \end{bmatrix}$ *are two square matrices.*
Find AB and hence solve the system of linear equations :
x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7

Solution:

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 + 4 + 0 & 2 - 2 + 0 & -4 + 4 + 0 \\ 4 - 12 + 8 & 4 + 6 - 4 & -8 - 12 + 20 \\ 0 - 4 + 4 & 0 + 2 - 2 & 0 - 4 + 10 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Now, we can see that it is AB = 6I. Where I is the unit Matrix

Or, $A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

A X = B

Or, $X = A^{-1}B$

$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix}$$

$= \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$

$X = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

Hence, x = 2, y = – 1 and z = 4

6. $If\ A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix},\ find\ A^{-1}\ and\ hence\ solve\ the\ system\ of\ linear\ equations:$

2x – 3y + 5z = 11, 3x + 2y – 4z = -5, x + y – 2z = -3.

Solution:

$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$|A| = 2(0) + 3(-2) + 5(1)$

$= -1$

Now, the cofactors of A

$C_{11} = (-1)^{1+1} - 4 + 4 = 0$

$C_{21} = (-1)^{2+1}\ 6 - 5 = -1$

$C_{31} = (-1)^{3+1}\ 12 - 10 = 2$

$C_{12} = (-1)^{1+2} - 6 + 4 = 2$

$C_{22} = (-1)^{2+1} - 4 - 5 = -9$

$C_{32} = (-1)^{3+1} - 8 - 15 = 23$

$C_{13} = (-1)^{1+2}\ 3 - 2 = 1$

$C_{23} = (-1)^{2+1}\ 2 + 3 = -5$

$C_{33} = (-1)^{3+1}\ 4 + 9 = 13$

$$\text{Adj } A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Now the given equation can be written as:

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

A X = B

Or, $X = A^{-1}B$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0-5+6 \\ -22+45+69 \\ -11-25+39 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, x = 1, y = 2 and z = 3

7. $Find\ A^{-2},\ if\ A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}.\ Hence\ solve\ the\ following\ system\ of\ linear\ equations:$

x + 2y + 5z = 10, x – y – z = -2, 2x + 3y – z = -11.

Solution:

Given

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$$

|A| = 1(1 + 3) + 2(– 1 + 2) + 5(3 + 2)

= 4 + 2 + 25

= 27

Now, the cofactors of A

$C_{11} = (-1)^{1+1}\ 1 + 3 = 4$

$C_{21} = (-1)^{2+1} - 2 - 15 = 17$

$C_{31} = (-1)^{3+1} - 2 + 5 = 3$

$C_{12} = (-1)^{1+2} - 1 + 2 = -1$

$C_{22} = (-1)^{2+1} - 1 - 10 = -11$

$C_{32} = (-1)^{3+1} - 1 - 5 = 6$

$C_{13} = (-1)^{1+2}\ 3 + 2 = 5$

$C_{23} = (-1)^{2+1}\ 3 - 4 = 1$

$C_{33} = (-1)^{3+1} - 1 - 2 = -3$

$$Adj\ A = \begin{bmatrix} 4 & -1 & 5 \\ 17 & -11 & 1 \\ 3 & 6 & -3 \end{bmatrix}^T = \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

$A^{-1} = \frac{1}{27}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

A X = B

Or, $X = A^{-1}B$

$= \frac{1}{27}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}\begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 40 - 34 - 33 \\ -10 + 22 - 66 \\ 50 - 2 + 33 \end{bmatrix}$$

$X = \frac{1}{27}\begin{bmatrix} -27 \\ -54 \\ 81 \end{bmatrix}$

$X = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$

Hence, x = – 1, y = – 2 and z = 3

8. (i) $If\ A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, $find\ A^{-1}. Using\ A^{-1},\ solve\ the system\ of\ linear\ equations:$

$x - 2y = 10, 2x + y + 3z = 8, -2y + z = 7$

Solution:

Given

A = $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$

|A| = 1(1 + 6) + 2(2 – 0) + 0

= 7 + 4

= 11

Now, the cofactors of A

$C_{11} = (-1)^{1+1}\ 1 + 6 = 7$

$C_{21} = (-1)^{2+1} - 2 - 0 = 2$

$C_{31} = (-1)^{3+1} - 6 - 0 = -6$

$C_{12} = (-1)^{1+2}\ 2 - 0 = -2$

$C_{22} = (-1)^{2+1}\ 1 - 0 = 1$

$C_{32} = (-1)^{3+1}\ 3 - 0 = -3$

$C_{13} = (-1)^{1+2} - 4 - 0 = -4$

$C_{23} = (-1)^{2+1} - 2 - 0 = 2$

$C_{33} = (-1)^{3+1}\ 1 + 4 = 5$

Adj A = $\begin{bmatrix} 7 & 2 & -4 \\ -2 & 1 & -3 \\ -6 & 2 & 5 \end{bmatrix}^T = \begin{bmatrix} 7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{11}\begin{bmatrix} 7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & 3 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

A X = B

Or, $X = A^{-1}B$

$= \frac{1}{11}\begin{bmatrix} 7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix}$

$X = \frac{1}{11}\begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix}$

$X = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$

Hence, x = 4, y = – 3 and z = 1

$(ii)\ A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix},\ find\ A^{-1}\ and\ hence\ solve\ the system\ of\ linear\ equations:$

$3x - 4y + 2z = -1, 2x + 3y + 5z = 7, x + z = 2$

Solution:

Given

$A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$

|A| = 3(3 – 0) + 4(2 – 5) + 2(0 – 3)

= 9 – 12 – 6

= – 9

Now, the cofactors of A

$C_{11} = (-1)^{1+1}\ 3 - 0 = 3$

$C_{21} = (-1)^{2+1} - 4 - 0 = 4$

$C_{31} = (-1)^{3+1} - 20 - 6 = -26$

$C_{12} = (-1)^{1+2}\ 2 - 5 = 3$

$C_{22} = (-1)^{2+1}\ 3 - 2 = 1$

$C_{32} = (-1)^{3+1}\ 15 - 4 = -11$

$C_{13} = (-1)^{1+2}\ 0 - 3 = -3$

$C_{23} = (-1)^{2+1}\, 0 + 4 = -4$

$C_{33} = (-1)^{3+1}\, 9 + 8 = 17$

$$\text{Adj A} = \begin{bmatrix} 3 & 3 & -3 \\ 4 & 1 & -4 \\ -26 & -4 & 27 \end{bmatrix}^T = \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj A}$$

$$A^{-1} = \frac{1}{-9}\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$$

Now the given equation can be written as:

$$\begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

A X = B

Or, $X = A^{-1}B$

$$= \frac{1}{-9}\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & 11 \\ -3 & -4 & 17 \end{bmatrix}\begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-9}\begin{bmatrix} -3 + 28 - 52 \\ 21 + 7 + 22 \\ 3 - 28 + 34 \end{bmatrix}$$

$$X = \frac{1}{-9}\begin{bmatrix} -27 \\ -18 \\ 9 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

Hence, x = 3, y = 2 and z = – 1

$(iii)\ A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix},\ and\ B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}\ find\ AB.\ Hence\ solve\ the system\ of\ linear\ equations:$

$x - 2y = 10, 2x + y + 3z = 8\ and\ -2y + z = 7$

Solution:

$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} 7+4-0 & 2-2+0 & -6+6+0 \\ 14-2-12 & 4+1+6 & -12-3+15 \\ 0-4+4 & 0-2+2 & 0+6+5 \end{bmatrix}$

$AB = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

Now, we can see that it is AB = 11I. Where I is the unit Matrix

Or, $A^{-1} = \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

A X = B

Or, $X = A^{-1}B$

$= \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 70+16-42 \\ -20+8-21 \\ -40+16+35 \end{bmatrix}$$

$= \frac{1}{11}\begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix}$

$X = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$

Hence, x = 4, y = – 3 and z = 1

$(iv)\ If\ A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix},\ find\ A^{-1}. Using\ A^{-1}\ solve\ the\ system\ of\ linear equations:$

$x - 2y = 10, 2x - y - z = 8, -2y + z = 7$

Solution:

Given

$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

|A| = 1(– 1 – 1) – 2(– 2 – 0) + 0

= – 2 + 4

= 2

Now, the cofactors of A

$C_{11} = (-1)^{1+1} - 1 - 1 = -2$

$C_{21} = (-1)^{2+1}\ 2 - 0 = 2$

$C_{31} = (-1)^{3+1} - 2 - 0 = -2$

$C_{12} = (-1)^{1+2}\ 2 - 0 = -2$

$C_{22} = (-1)^{2+1}\ 1 - 0 = 1$

$C_{32} = (-1)^{3+1} - 1 - 0 = 1$

$C_{13} = (-1)^{1+2} - 2 - 0 = -2$

$C_{23} = (-1)^{2+1} - 1 - 0 = 1$

$C_{33} = (-1)^{3+1} - 1 + 4 = 3$

$Adj\ A = \begin{bmatrix} -2 & -2 & -2 \\ 2 & 1 & 1 \\ -2 & 1 & 3 \end{bmatrix}^T = \begin{bmatrix} -2 & 2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

A X = B

Or, $X = A^{-1}B$

$= \frac{1}{2}\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 10 - 16 + 0 \\ 20 - 8 - 7 \\ 0 - 16 + 7 \end{bmatrix}$$

$X = \frac{1}{2}\begin{bmatrix} -6 \\ 5 \\ -9 \end{bmatrix}$

Hence, x = – 3, y = 2.5 and z = – 4.5

$(v)\ Given\ A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix},\ B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$
$find\ BA\ and\ Use\ this\ to\ solve\ the\ system\ of\ linear\ equations\ y + 2z = 7, x - y = 3, 2x + 3y + 4z = 17$

Solution:

Given

$B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}$

BA = $\begin{bmatrix} 2+4-0 & 2-2+0 & -4+4+0 \\ -4-12+16 & 4+6-4 & -8-12+20 \\ 0-4+8 & 0-2+2 & 0-4+10 \end{bmatrix}$

BA = $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

Now, we can see that it is BA = 6I. Where I is the unit Matrix

Or, $B^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 0 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 17 \end{bmatrix}$$

A X = B

Or, X = $B^{-1}A$

$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \\ 17 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 14+6-68 \\ -28+6-68 \\ 28-3+85 \end{bmatrix}$$

$= \frac{1}{6}\begin{bmatrix} -48 \\ -90 \\ 110 \end{bmatrix}$

X = $\begin{bmatrix} -8 \\ -15 \\ \frac{110}{6} \end{bmatrix}$

Hence, x = – 8, y = – 15 and z = $\frac{110}{6}$

9. The sum of three numbers is 2. If twice the second number is added to the sum of first and third, the sum is 1. By adding second and third number to five times the first number, we get 6. Find the three numbers by using matrices.

Solution:

Let the numbers are x, y, z

$x + y + z = 2$ …… (i)

Also, $2y + (x + z) + 1$

$x + 2y + z = 1$ …… (ii)

Again,

$x + z + 5(x) = 6$

$5x + y + z = 6$ …… (iii)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

A X = B

$|A| = 1(1) - 1(-4) + 1(-9)$

$= 1 + 4 - 9$

$= -4$

Hence, the unique solution given by $x = A^{-1}B$

$C_{11} = (-1)^{1+1}(2-1) = 1$

$C_{12} = (-1)^{1+2}(1-5) = 4$

$C_{13} = (-1)^{1+3}(1-10) = -9$

$C_{21} = (-1)^{2+1}(1-1) = 0$

$C_{22} = (-1)^{2+2}(1-5) = -4$

$C_{23} = (-1)^{2+3}(1-5) = 4$

$C_{31} = (-1)^{3+1}(1-2) = -1$

$C_{32} = (-1)^{3+2}(1-1) = 0$

$C_{33} = (-1)^{3+3}(2-1) = 1$

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj } A)\,B$$

$$\text{Adj } A = \begin{bmatrix} 1 & 4 & -9 \\ 0 & -4 & 4 \\ -1 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}$$

$$X = \frac{1}{-4}\begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

$$X = \frac{1}{-4}\begin{bmatrix} 2-6 \\ 8-4 \\ -18+4+6 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} -4 \\ 4 \\ -8 \end{bmatrix}$$

Hence, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

10. An amount of ₹10,000 is put into three investments at the rate of 10, 12 and 15% per annum. The combined incomes are ₹1310 and the combined income of first and second investment is ₹ 190 short of the income from the third. Find the investment in each using matrix method.

Solution:

Let the numbers are x, y, and z

x + y + z = 10,000 …… (i)

Also,

0.1x + 0.12y + 0.15z = 1310 …… (ii)

Again,

0.1x + 0.12y – 0.15z = – 190 …… (iii)

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.12 & 0.15 \\ 0.1 & 0.12 & -0.15 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

A X = B

|A| = 1(– 0.036) – 1(– 0.03) + 1(0)

= – 0.006

Hence, the unique solution given by $x = A^{-1}B$

$C_{11} = -0.036$

$C_{12} = 0.27$

$C_{13} = 0$

$C_{21} = 0.27$
$C_{22} = -0.25$
$C_{23} = -0.02$
$C_{31} = 0.03$
$C_{32} = -0.05$
$C_{33} = 0.02$

$X = A^{-1} B = \frac{1}{|A|} (\text{adj } A)\ B$

$$\text{Adj } A = \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.27 & -0.25 & -0.05 \\ 0.03 & -0.02 & 0.02 \end{bmatrix}^T = \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix}$$

$$X = \frac{1}{-0.006} \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix} \begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

$$X = \frac{1}{-0.006} \begin{bmatrix} -12 \\ -18 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2000 \\ 3000 \\ 5000 \end{bmatrix}$$

Hence, x = Rs 2000, y = Rs 3000 and z = Rs 5000