1. Solve the following system of equations by matrix method:
(i) $5 \mathrm{x}+7 \mathrm{y}+2=0$
$4 \mathrm{x}+6 \mathrm{y}+3=0$
(ii) $5 x+2 y=3$
$3 x+2 y=5$
(iii) $3 x+4 y-5=0$
$x-y+3=0$
(iv) $3 x+y=19$
$3 x-y=23$
(v) $3 x+7 y=4$
$x+2 y=-1$
(vi) $3 x+y=7$
$5 x+3 y=12$

## Solution:

(i) Given $5 x+7 y+2=0$ and $4 x+6 y+3=0$

The above system of equations can be written as
$\left[\begin{array}{ll}5 & 7 \\ 4 & 6\end{array}\right]\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{l}-2 \\ -3\end{array}\right]$ Or $\mathrm{AX}=\mathrm{B}$
Where $\mathrm{A}=\left[\begin{array}{ll}5 & 7 \\ 4 & 6\end{array}\right] \mathrm{B}=\left[\begin{array}{l}-2 \\ -3\end{array}\right]$ and $\mathrm{X}=\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]$
$|A|=30-28=2$
So, the above system has a unique solution, given by
$\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}$
Let $\mathrm{C}_{\mathrm{ij}}$ be the cofactor of $\mathrm{a}_{\mathrm{ij}}$ in A , then
$\mathrm{C}_{11}=(-1)^{1+1} 6=6$
$\mathrm{C}_{12}=(-1)^{1+2} 4=-4$
$\mathrm{C}_{21}=(-1)^{2+1} 7=-7$
$\mathrm{C}_{22}=(-1)^{2+2} 5=5$

Also, $\operatorname{adj} A=\left[\begin{array}{cc}6 & -4 \\ -7 & 5\end{array}\right]^{T}$
$=\left[\begin{array}{cc}6 & -7 \\ -4 & 5\end{array}\right]$
$A^{-1}=\frac{1}{|A|} \operatorname{adj} \mathrm{A}$
$A^{-1}=\frac{1}{2}\left[\begin{array}{cc}6 & -7 \\ -4 & 5\end{array}\right]$
Now, $X=A^{-1} B$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\frac{1}{2}\left[\begin{array}{cc}6 & -7 \\ -4 & 5\end{array}\right]\left[\begin{array}{l}-2 \\ -3\end{array}\right]$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\frac{1}{2}\left[\begin{array}{c}-12+21 \\ 8-15\end{array}\right]$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{c}\frac{9}{2} \\ \frac{-7}{2}\end{array}\right]$
Hence, $x=9 / 2$ and $y=-7 / 2$
(ii) Given $5 x+2 y=3$
$3 x+2 y=5$
The above system of equations can be written as
$\left[\begin{array}{ll}5 & 2 \\ 3 & 2\end{array}\right]\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{l}3 \\ 5\end{array}\right]$ Or $\mathrm{AX}=\mathrm{B}$
Where $A=\left[\begin{array}{ll}5 & 2 \\ 3 & 2\end{array}\right] B=\left[\begin{array}{l}3 \\ 5\end{array}\right]$ and $X=\left[\begin{array}{l}X \\ Y\end{array}\right]$
$|A|=10-6=4$
So, the above system has a unique solution, given by
$X=A^{-1} B$
Let $C_{i j}$ be the cofactor of $a_{i j}$ in $A$, then

$$
\begin{aligned}
& C_{11}=(-1)^{1+1} 2=2 \\
& C_{12}=(-1)^{1+7} 3=-3 \\
& C_{21}=(-1)^{2+1} 2=-2 \\
& C_{22}=(-1)^{2+2} 5=5
\end{aligned}
$$

Also, $\operatorname{adj} A=\left[\begin{array}{cc}2 & -3 \\ -2 & 5\end{array}\right]^{T}$

$$
=\left[\begin{array}{cc}
2 & -2 \\
-3 & 5
\end{array}\right]
$$

$$
\mathrm{A}^{-1}=\frac{1}{|\mathrm{~A}|} \operatorname{adj} \mathrm{A}
$$

$$
\mathrm{A}^{-1}=\frac{1}{4}\left[\begin{array}{cc}
2 & -2 \\
-3 & 5
\end{array}\right]
$$

Now, $X=A^{-1} B$

$$
\left[\begin{array}{l}
\mathrm{X} \\
\mathrm{Y}
\end{array}\right]=\frac{1}{4}\left[\begin{array}{cc}
2 & -2 \\
-3 & 5
\end{array}\right]\left[\begin{array}{l}
3 \\
5
\end{array}\right]
$$

$$
\left[\begin{array}{l}
\mathrm{X} \\
\mathrm{Y}
\end{array}\right]=\frac{1}{4}\left[\begin{array}{c}
6-10 \\
-9+25
\end{array}\right]
$$

$$
\left[\begin{array}{c}
\mathrm{X} \\
\mathrm{Y}
\end{array}\right]=\frac{1}{4}\left[\begin{array}{c}
-4 \\
16
\end{array}\right]
$$

$$
\left[\begin{array}{l}
\mathrm{X} \\
\mathrm{Y}
\end{array}\right]=\left[\begin{array}{c}
-1 \\
4
\end{array}\right]
$$

Hence, $x=-1$ and $y=4$
(iii) Given $3 x+4 y-5=0$
$x-y+3=0$
The above system of equations can be written as
$\left[\begin{array}{cc}3 & 4 \\ 1 & -1\end{array}\right]\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{c}5 \\ -3\end{array}\right]$ Or $\mathrm{AX}=\mathrm{B}$

Where $\mathrm{A}=\left[\begin{array}{cc}3 & 4 \\ 1 & -1\end{array}\right] \mathrm{B}=\left[\begin{array}{c}5 \\ -3\end{array}\right]$ and $\mathrm{X}=\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]$
$|A|=-3-4=-7$
So, the above system has a unique solution, given by
$X=A^{-1} B$
Let $\mathrm{C}_{\mathrm{ij}}$ be the cofactor of $\mathrm{a}_{\mathrm{ij}}$ in A , then
$\mathrm{C}_{11}=(-1)^{1+1}-1=-1$
$\mathrm{C}_{12}=(-1)^{1+2} 1=-1$
$C_{21}=(-1)^{2+1} 4=-4$
$\mathrm{C}_{22}=(-1)^{2+2} 3=3$
Also, $\operatorname{adj} A=\left[\begin{array}{cc}-1 & -1 \\ -4 & 3\end{array}\right]^{T}$
$=\left[\begin{array}{cc}-1 & -4 \\ -1 & 3\end{array}\right]$
$A^{-1}=\frac{1}{|A|} \operatorname{adj} \mathrm{A}$
$\mathrm{A}^{-1}=\frac{1}{7}\left[\begin{array}{cc}-1 & -4 \\ -1 & 3\end{array}\right]$
Now, $X=A^{-1} B$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\frac{1}{7}\left[\begin{array}{cc}-1 & -4 \\ -1 & 3\end{array}\right]\left[\begin{array}{c}5 \\ -3\end{array}\right]$
$\left[\begin{array}{l}X \\ \mathrm{Y}\end{array}\right]=\frac{1}{7}\left[\begin{array}{c}-5+12 \\ -5-9\end{array}\right]$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\frac{1}{7}\left[\begin{array}{c}7 \\ -14\end{array}\right]$
$\left[\begin{array}{c}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{c}1 \\ -2\end{array}\right]$
Hence, $X=1 Y=-2$
(iv) Given $3 x+y=19$
$3 x-y=23$
The above system of equations can be written as

$$
\left[\begin{array}{cc}
3 & 1 \\
3 & -1
\end{array}\right]\left[\begin{array}{l}
\mathrm{X} \\
\mathrm{Y}
\end{array}\right]=\left[\begin{array}{l}
19 \\
23
\end{array}\right] \text { Or } \mathrm{AX}=\mathrm{B}
$$

Where $\mathrm{A}=\left[\begin{array}{cc}3 & 1 \\ 3 & -1\end{array}\right] \mathrm{B}=\left[\begin{array}{l}19 \\ 23\end{array}\right]$ and $\mathrm{X}=\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]$

$$
|A|=-3-3=-6
$$

So, the above system has a unique solution, given by
$X=A^{-1} B$
Let $\mathrm{C}_{\mathrm{ij}}$ be the cofactor of $\mathrm{a}_{\mathrm{ij}}$ in A , then
$\mathrm{C}_{11}=(-1)^{1+1}-1=-1$
$\mathrm{C}_{12}=(-1)^{1+2} 3=-3$
$\mathrm{C}_{21}=(-1)^{2+1} 1=-1$
$\mathrm{C}_{22}=(-1)^{2+2} 3=3$
Also, $\operatorname{adj} \mathrm{A}=\left[\begin{array}{cc}-1 & -3 \\ -1 & 3\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{cc}-1 & -1 \\ -3 & 3\end{array}\right]$
$A^{-1}=\frac{1}{|A|} \operatorname{adj} A$
$A^{-1}=\frac{1}{-6}\left[\begin{array}{cc}-1 & -1 \\ -3 & 3\end{array}\right]$
Now, $X=A^{-1} B$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\frac{1}{-6}\left[\begin{array}{cc}-1 & -1 \\ -3 & 3\end{array}\right]\left[\begin{array}{l}19 \\ 23\end{array}\right]$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\frac{1}{-6}\left[\begin{array}{c}-19-23 \\ -57+69\end{array}\right]$
$\left[\begin{array}{c}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\frac{1}{-6}\left[\begin{array}{c}-42 \\ 12\end{array}\right]$
$\left[\begin{array}{c}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{c}7 \\ -2\end{array}\right]$
Hence, $x=7$ and $y=-2$
(v) Given $3 x+7 y=4$
$x+2 y=-1$
The above system of equations can be written as
$\left[\begin{array}{ll}3 & 7 \\ 1 & 2\end{array}\right]\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{c}4 \\ -1\end{array}\right]$ Or $\mathrm{AX}=\mathrm{B}$
Where $\mathrm{A}=\left[\begin{array}{ll}3 & 7 \\ 1 & 2\end{array}\right] \mathrm{B}=\left[\begin{array}{c}4 \\ -1\end{array}\right]$ and $\mathrm{X}=\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]$

$$
|A|=6-7=-1
$$

So, the above system has a unique solution, given by
$X=A^{-1} B$
Let $\mathrm{C}_{\mathrm{ij}}$ be the cofactor of $\mathrm{a}_{\mathrm{ij}}$ in A , then
$\mathrm{C}_{11}=(-1)^{1+1} 2=2$
$\mathrm{C}_{12}=(-1)^{1+2} 1=-1$
$\mathrm{C}_{21}=(-1)^{2+1} 7=-7$
$\mathrm{C}_{22}=(-1)^{2+2} 3=3$
Also, $\operatorname{adj} \mathrm{A}=\left[\begin{array}{cc}2 & -1 \\ -7 & 3\end{array}\right]^{T}$
$=\left[\begin{array}{cc}2 & -7 \\ -1 & 3\end{array}\right]$
$A^{-1}=\frac{1}{|A|} \operatorname{adj} \mathrm{A}$

Now, $X=A^{-1} B$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\frac{1}{-1}\left[\begin{array}{cc}2 & -7 \\ -1 & 3\end{array}\right]\left[\begin{array}{c}4 \\ -1\end{array}\right]$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\frac{1}{-1}\left[\begin{array}{c}8+7 \\ -4-3\end{array}\right]$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\frac{1}{-1}\left[\begin{array}{l}15 \\ -7\end{array}\right]$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{c}-15 \\ 7\end{array}\right]$
Hence, $X=-15 Y=7$
(vi) Given $3 x+y=7$
$5 x+3 y=12$
The above system of equations can be written as
$\left[\begin{array}{ll}3 & 1 \\ 5 & 3\end{array}\right]\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{c}7 \\ 12\end{array}\right]$ Or $\mathrm{AX}=\mathrm{B}$
Where $A=\left[\begin{array}{ll}3 & 1 \\ 5 & 3\end{array}\right] B=\left[\begin{array}{c}7 \\ 12\end{array}\right]$ and $X=\left[\begin{array}{c}X \\ Y\end{array}\right]$
$|A|=9-5=4$
So, the above system has a unique solution, given by
$X=A^{-1} B$
Let $\mathrm{C}_{\mathrm{ij}}$ be the cofactor of $\mathrm{a}_{\mathrm{ij}}$ in A , then
$C_{11}=(-1)^{1+1} 3=3$
$C_{12}=(-1)^{1+2} 5=-5$
$C_{21}=(-1)^{2+1} 1=-1$
$C_{22}=(-1)^{2+2} 3=3$

Also, $\operatorname{adj} A=\left[\begin{array}{cc}3 & -5 \\ -1 & 3\end{array}\right]^{T}$
$=\left[\begin{array}{cc}2 & -1 \\ -5 & 3\end{array}\right]$
$\mathrm{A}^{-1}=\frac{1}{|\mathrm{~A}|} \operatorname{adj} \mathrm{A}$
$A^{-1}=\frac{1}{4}\left[\begin{array}{cc}3 & -1 \\ -5 & 3\end{array}\right]$
Now, $X=A^{-1} B$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\frac{1}{4}\left[\begin{array}{cc}3 & -1 \\ -5 & 3\end{array}\right]\left[\begin{array}{c}7 \\ 12\end{array}\right]$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\frac{1}{4}\left[\begin{array}{c}21-12 \\ -35+36\end{array}\right]$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\frac{1}{4}\left[\begin{array}{l}9 \\ 1\end{array}\right]$
$\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{l}\frac{9}{4} \\ \frac{1}{4}\end{array}\right]$
Hence, $\mathrm{X}=\stackrel{9}{4} \mathrm{Y}=\frac{1}{4}$
2. Solve the following system of equations by matrix method:
(i) $x+y-z=3$
$2 x+3 y+z=10$
$3 x-y-7 z=1$
(ii) $x+y+z=3$
$2 x-y+z=-1$
$2 x+y-3 z=-9$
(iii) $6 x-12 y+25 z=4$
$4 x+15 y-20 z=3$
$2 x+18 y+15 z=10$
(iv) $3 x+4 y+7 z=14$
$2 x-y+3 z=4$
$x+2 y-3 z=0$
(v) $(2 / x)-(3 / y)+(3 / z)=10$
$(1 / x)+(1 / y)+(1 / z)=10$
$(3 / x)-(1 / y)+(2 / z)=13$
(vi) $5 x+3 y+z=16$
$2 x+y+3 z=19$
$x+2 y+4 z=25$
(vii) $3 x+4 y+2 z=8$
$2 y-3 z=3$
$x-2 y+6 z=-2$
(viii) $2 x+y+z=2$
$x+3 y-z=5$
$3 x+y-2 z=6$
(ix) $2 x+6 y=2$
$3 x-z=-8$
$2 x-y+z=-3$
(x) $2 y-z=1$
$x-y+z=2$
$2 x-y=0$
(xi) $8 x+4 y+3 z=18$
$2 x+y+z=5$
$x+2 y+z=5$
(xii) $x+y+z=6$
$x+2 z=7$
$3 x+y+z=12$
(xiii) $(2 / x)+(3 / y)+(10 / z)=4$,
$(4 / x)-(6 / y)+(5 / z)=1$,
$(6 / x)+(9 / y)-(20 / z)=2, x, y, z \neq 0$
(xiv) $x-y+2 z=7$
$3 x+4 y-5 z=-5$
$2 x-y+3 z=12$

## Solution:

(i) Given $x+y-z=3$
$2 x+3 y+z=10$
$3 x-y-7 z=1$
The given system can be written in matrix form as:

$$
\left[\begin{array}{ccc}
1 & 1 & -1 \\
2 & 3 & 1 \\
3 & -1 & -7
\end{array}\right]\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]=\left[\begin{array}{c}
3 \\
10 \\
1
\end{array}\right] \text { Or } \mathrm{AX}=\mathrm{B}
$$

$A=\left[\begin{array}{ccc}1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7\end{array}\right], X=\left[\begin{array}{l}x \\ y \\ z\end{array}\right]$ and $B=\left[\begin{array}{c}3 \\ 10 \\ 1\end{array}\right]$
Now, $\left.|A|=1 \begin{array}{cc}3 & 1 \\ -1 & -7\end{array}|-1| \begin{array}{cc}2 & 1 \\ 3 & -7\end{array}|-1| \begin{array}{cc}2 & 3 \\ 3 & -1\end{array} \right\rvert\,$
$=(-20)-1(-17)-1(11)$
$=-20+17+11=8$
So, the above system has a unique solution, given by
$X=A{ }^{-1} B$
Cofactors of $A$ are
$C_{11}=(-1)^{1+1}-21+1=-20$
$C_{21}=(-1)^{2+1}-7-1=8$
$C_{31}=(-1)^{3+1} 1+3=4$
$C_{12}=(-1)^{1+2}-14-3=17$
$C_{22}=(-1)^{2+1}-7+3=-4$
$C_{32}=(-1)^{3+1} 1+2=-3$
$C_{13}=(-1)^{1+2}-2-9=-11$
$C_{23}=(-1)^{2+1}-1-3=4$
$C_{33}=(-1)^{3+1} 3-2=1$
Adj $A=\left[\begin{array}{ccc}-20 & 17 & -11 \\ 8 & -4 & 4 \\ 4 & -3 & 1\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}-20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1\end{array}\right]$
Now, $\left.\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{\frac{1}{8}}{}{ }^{\frac{2}{-20}} \begin{array}{ccc}8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1\end{array}\right]\left[\begin{array}{c}3 \\ 10 \\ 1\end{array}\right]$
$\mathrm{X}=\frac{1}{8}\left[\begin{array}{c}-60+80+4 \\ 51-40-3 \\ -33+40+1\end{array}\right]$
$\mathrm{X}=\frac{1}{8}\left[\begin{array}{c}24 \\ 8 \\ 8\end{array}\right]=\left[\begin{array}{l}3 \\ 1 \\ 1\end{array}\right]$
Hence, $\mathrm{X}=3, \mathrm{Y}=1$ and $\mathrm{Z}=1$
(ii) Given $x+y+z=3$
$2 x-y+z=-1$
$2 x+y-3 z=-9$
The given system can be written in matrix form as:
$\left[\begin{array}{ccc}1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{c}3 \\ -1 \\ -9\end{array}\right]$ Or AX=B
$A=\left[\begin{array}{ccc}1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3\end{array}\right], X=\left[\begin{array}{l}x \\ y \\ z\end{array}\right]$ and $B=\left[\begin{array}{c}3 \\ -1 \\ -9\end{array}\right]$
Now, $\left.|\mathrm{A}|=1 \begin{array}{cc}-1 & 1 \\ 1 & -3\end{array}|-1| \begin{array}{cc}2 & 1 \\ 2 & -3\end{array}|+1| \begin{array}{ll}2 & 1 \\ 2 & 1\end{array} \right\rvert\,$
$=(3-1)-1(-6-2)+1(2+2)$
$=2+8+4$
$=14$
So, the above system has a unique solution, given by
$X=A^{-1} B$
Cofactors of A are
$\mathrm{C}_{11}=(-1)^{1+1} 3-1=2$
$\mathrm{C}_{21}=(-1)^{2+1}-3-1=4$
$\mathrm{C}_{31}=(-1)^{3+1} 1+1=2$
$\mathrm{C}_{12}=(-1)^{1+2}-6-2=8$
$\mathrm{C}_{22}=(-1)^{2+1}-3-2=-5$
$C_{32}=(-1)^{3+1} 1-2=1$
$\mathrm{C}_{13}=(-1)^{1+2} 2+2=4$
$C_{23}=(-1)^{2+1} 1-2=1$
$C_{33}=(-1)^{3+1}-1-2=-3$
$\operatorname{Adj} A=\left[\begin{array}{ccc}2 & 8 & 4 \\ 4 & -5 & 1 \\ 2 & 1 & -3\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3\end{array}\right]$
Now, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{14}\left[\begin{array}{ccc}2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3\end{array}\right]\left[\begin{array}{c}3 \\ -1 \\ -9\end{array}\right]$
$X=\frac{1}{14}\left[\begin{array}{c}-16 \\ 20 \\ 38\end{array}\right]$
$X=\left[\begin{array}{c}\frac{-8}{7} \\ \frac{10}{7} \\ \frac{19}{7}\end{array}\right]$
Hence, $X=\frac{-8}{7}, Y=\frac{10}{7}$ and $Z=\frac{19}{7}$
(iii) Given $6 x-12 y+25 z=4$
$4 x+15 y-20 z=3$
$2 x+18 y+15 z=10$
The given system can be written in matrix form as:

$$
\left[\begin{array}{ccc}
6 & -12 & 25 \\
4 & 15 & -20 \\
2 & 18 & 15
\end{array}\right]\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]=\left[\begin{array}{c}
4 \\
3 \\
10
\end{array}\right]_{\text {Or } \mathrm{AX}=\mathrm{B}}
$$

$\mathrm{A}=\left[\begin{array}{ccc}6 & -12 & 25 \\ 4 & 15 & -20 \\ 2 & 18 & 15\end{array}\right], \mathrm{X}=\left[\begin{array}{l}\mathrm{X} \\ \mathrm{y} \\ \mathrm{z}\end{array}\right]$ and $\mathrm{B}=\left[\begin{array}{c}4 \\ 3 \\ 10\end{array}\right]$
Now, $|A|=6\left|\begin{array}{cc}15 & -20 \\ 18 & 15\end{array}\right|-12\left|\begin{array}{cc}4 & -20 \\ 2 & 15\end{array}\right|+25\left|\begin{array}{ll}4 & 15 \\ 2 & 18\end{array}\right|$
$=6(225+360)+12(60+40)+25(72-30)$
$=3510+1200+1050$
= 5760
So, the above system has a unique solution, given by
$X=A{ }^{-1} B$
Cofactors of $A$ are
$\mathrm{C}_{11}=(-1)^{1+1}(225+360)=585$
$C_{21}=(-1)^{2+1}(-180-450)=630$
$C_{31}=(-1)^{3+1}(240-375)=-135$
$\mathrm{C}_{12}=(-1)^{1+2}(60+40)=-100$
$C_{22}=(-1)^{2+1}(90-50)=40$
$C_{32}=(-1)^{3+1}(-120-100)=220$
$C_{13}=(-1)^{1+2}(72-30)=42$
$C_{23}=(-1)^{2+1}(108+24)=-132$
$C_{33}=(-1)^{3+1}(90+48)=138$
$\operatorname{Adj} A=\left[\begin{array}{ccc}585 & -100 & 42 \\ 630 & 40 & -132 \\ -135 & 220 & 138\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}585 & 630 & -135 \\ -100 & 40 & 220 \\ 12 & -132 & 138\end{array}\right]$
Now, $X=A^{-1} B=\frac{1}{5760}\left[\begin{array}{ccc}585 & 630 & -135 \\ -100 & 40 & 220 \\ 42 & -132 & 138\end{array}\right]\left[\begin{array}{c}4 \\ 3 \\ 10\end{array}\right]$
$X=\frac{1}{5760}\left[\begin{array}{l}2880 \\ 1920 \\ 1152\end{array}\right]$
$X=\left[\begin{array}{c}\frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5}\end{array}\right]$
Hence, $X=\frac{1}{2}, Y=\frac{1}{3}$ and $Z=\frac{1}{5}$
(iv) Given $3 x+4 y+7 z=14$
$2 x-y+3 z=4$
$x+2 y-3 z=0$
The given system can be written in matrix form as:

$$
\left[\begin{array}{ccc}
3 & 4 & 7 \\
2 & -1 & 3 \\
1 & 2 & -3
\end{array}\right]\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]=\left[\begin{array}{c}
14 \\
4 \\
0
\end{array}\right] \text { Or } \mathrm{AX}=\mathrm{B}
$$

$$
A=\left[\begin{array}{ccc}
3 & 4 & 7 \\
2 & -1 & 3 \\
1 & 2 & -3
\end{array}\right], X=\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right] \text { and } B=\left[\begin{array}{c}
14 \\
4 \\
0
\end{array}\right]
$$

Now, $|\mathrm{A}|=3\left|\begin{array}{cc}-1 & 3 \\ 2 & -3\end{array}\right|-4\left|\begin{array}{cc}2 & 3 \\ 1 & -3\end{array}\right|+7\left|\begin{array}{cc}2 & 3 \\ 2 & -3\end{array}\right|$
$=3(3-6)-4(-6-3)+7(4+1)$
$=-9+36+35$
$=62$
So, the above system has a unique solution, given by
$X=A^{-1} B$

## Cofactors of $A$ are

$\mathrm{C}_{11}=(-1)^{1+1} 3-6=-3$
$\mathrm{C}_{21}=(-1)^{2+1}-12-14=26$
$\mathrm{C}_{31}=(-1)^{3+1} 12+7=19$
$\mathrm{C}_{12}=(-1)^{1+2}-6-3=9$
$\mathrm{C}_{22}=(-1)^{2+1}-3-7=-10$
$\mathrm{C}_{32}=(-1)^{3+1} 9-14=5$
$\mathrm{C}_{13}=(-1)^{1+2} 4+1=5$
$\mathrm{C}_{23}=(-1)^{2+1} 6-4=-2$
$\mathrm{C}_{33}=(-1)^{3+1}-3-8=-11$
$\operatorname{Adj} \mathrm{A}=\left[\begin{array}{ccc}-3 & 9 & 5 \\ 26 & -5 & -2 \\ 19 & 5 & -11\end{array}\right]^{\mathrm{T}}$

$$
=\left[\begin{array}{ccc}
-3 & 26 & 19 \\
9 & -16 & 5 \\
5 & -2 & -11
\end{array}\right]
$$

$A^{-1}=\frac{1}{|\mathrm{~A}|} \operatorname{adj} \mathrm{A}$
Now, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}={ }^{\frac{1}{62}}\left[\begin{array}{ccc}-3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11\end{array}\right]\left[\begin{array}{c}14 \\ 4 \\ 0\end{array}\right]$
$X={ }^{62}\left[\begin{array}{c}-42+104+0 \\ 126-64+0 \\ 70-8+0\end{array}\right]$
$X=\frac{1}{62}\left[\begin{array}{l}62 \\ 62 \\ 62\end{array}\right]$
$X=\left[\begin{array}{l}1 \\ 1 \\ 1\end{array}\right]$
Hence, $\mathrm{X}=1, \mathrm{Y}=1$ and $\mathrm{Z}=1$
(v) Given $(2 / x)-(3 / y)+(3 / z)=10$
$(1 / x)+(1 / y)+(1 / z)=10$
$(3 / x)-(1 / y)+(2 / z)=13$
The given system can be written in matrix form as:

$$
\left[\begin{array}{lll}
5 & 3 & 1 \\
2 & 1 & 3 \\
1 & 2 & 4
\end{array}\right]\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]=\left[\begin{array}{l}
16 \\
19 \\
25
\end{array}\right]_{\text {Or } \mathrm{AX}=\mathrm{B}}
$$

$$
A=\left[\begin{array}{lll}
5 & 3 & 1 \\
2 & 1 & 3 \\
1 & 2 & 4
\end{array}\right], X=\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right] \text { and } B=\left[\begin{array}{c}
16 \\
19 \\
25
\end{array}\right]
$$

Now, $|A|=5\left|\begin{array}{ll}1 & 3 \\ 2 & 4\end{array}\right|-3\left|\begin{array}{ll}2 & 3 \\ 1 & 4\end{array}\right|+1\left|\begin{array}{ll}2 & 1 \\ 1 & 2\end{array}\right|$
$=5(4-6)-3(8-3)+1(4-2)$
$=-10-15+3$
$=-22$
So, the above system has a unique solution, given by
$X=A{ }^{-1} B$

Cofactors of $A$ are
$\mathrm{C}_{11}=(-1)^{1+1}(4-6)=-2$
$\mathrm{C}_{21}=(-1)^{2+1}(12-2)=-10$
$\mathrm{C}_{31}=(-1)^{3+1}(9-1)=8$
$\mathrm{C}_{12}=(-1)^{1+2}(8-3)=-5$
$C_{22}=(-1)^{2+1} 20-1=19$
$\mathrm{C}_{32}=(-1)^{3+1} 15-2=-13$
$\mathrm{C}_{13}=(-1)^{1+2}(4-2)=2$
$C_{23}=(-1)^{2+1} 10-3=-7$
$\mathrm{C}_{33}=(-1)^{3+1} 5-6=-1$
$\operatorname{Adj} A=\left[\begin{array}{ccc}-2 & -5 & -3 \\ -10 & 19 & -7 \\ 8 & -13 & -1\end{array}\right]^{\mathrm{T}}$

$$
=\left[\begin{array}{ccc}
-2 & -10 & 8 \\
-5 & 19 & 13 \\
3 & -7 & -1
\end{array}\right]
$$

$A^{-1}=\frac{1}{|A|} \operatorname{adj} A$
Now, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{-22}\left[\begin{array}{ccc}-2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1\end{array}\right]\left[\begin{array}{l}16 \\ 19 \\ 25\end{array}\right]$
$\mathrm{X}=\frac{1}{-22}\left[\begin{array}{c}-32-190+200 \\ -80+361-325 \\ 48-133-25\end{array}\right]$
$X=\frac{1}{-22}\left[\begin{array}{c}-22 \\ -44 \\ -110\end{array}\right]$

$$
X=\left[\begin{array}{l}
1 \\
2 \\
5
\end{array}\right]
$$

Hence, $X=1, Y=2$ and $Z=5$
(vi) Given $5 x+3 y+z=16$
$2 x+y+3 z=19$
$x+2 y+4 z=25$
The given system can be written in matrix form as:
$\left[\begin{array}{ccc}3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{c}8 \\ 3 \\ -2\end{array}\right]$ Or $A X=B$
$A=\left[\begin{array}{ccc}3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6\end{array}\right], X=\left[\begin{array}{l}x \\ y \\ z\end{array}\right]$ and $B=\left[\begin{array}{c}8 \\ 3 \\ -2\end{array}\right]$
Now, $|A|=3\left|\begin{array}{cc}2 & -3 \\ -2 & 6\end{array}\right|-4\left|\begin{array}{cc}0 & -3 \\ 1 & 6\end{array}\right|+2\left|\begin{array}{cc}0 & 2 \\ 1 & -2\end{array}\right|$
$=3(12-6)-4(0+3)+2(0-2)$
$=18-12-4$
$=2$
So, the above system has a unique solution, given by
$X=A{ }^{-1} B$
Cofactors of $A$ are
$C_{11}=(-1)^{1+1}(12-6)=6$
$C_{21}=(-1)^{2+1}(24+4)=-28$
$C_{31}=(-1)^{3+1}(-12-4)=-16$
$C_{12}=(-1)^{1+2}(0+3)=-3$
$C_{22}=(-1)^{2+1} 18-2=16$
$C_{32}=(-1)^{3+1}-9-0=9$
$\mathrm{C}_{13}=(-1)^{1+2}(0-2)=-2$
$C_{23}=(-1)^{2+1}(-6-4)=10$
$C_{33}=(-1)^{3+1} 6-0=6$
Adj $A=\left[\begin{array}{ccc}6 & -3 & 2 \\ -28 & 16 & 10 \\ -16 & -9 & 6\end{array}\right]^{T}$
$=\left[\begin{array}{ccc}6 & -28 & -16 \\ -3 & 16 & -9 \\ 2 & 10 & 6\end{array}\right]$
$A^{-1}=\frac{1}{|A|} \operatorname{adj} \mathrm{A}$

Now, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{2}\left[\begin{array}{ccc}6 & -28 & -16 \\ -3 & 16 & -9 \\ 2 & 10 & 6\end{array}\right]\left[\begin{array}{c}8 \\ 3 \\ -2\end{array}\right]$
$X=\frac{1}{2}\left[\begin{array}{c}48-84+32 \\ -24+48-18 \\ -16+30-12\end{array}\right]$
$X=\frac{1}{2}\left[\begin{array}{c}-4 \\ 6 \\ 2\end{array}\right]$
$X=\left[\begin{array}{c}-2 \\ 3 \\ 1\end{array}\right]$
Hence, $X=-2, Y=3$ and $Z=1$
(vii) Given $3 x+4 y+2 z=8$
$2 y-3 z=3$
$x-2 y+6 z=-2$
The given system can be written in matrix form as:

$$
\left[\begin{array}{ccc}
2 & 1 & 1 \\
1 & 3 & -1 \\
3 & 1 & -2
\end{array}\right]\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]=\left[\begin{array}{l}
2 \\
5 \\
6
\end{array}\right]_{\text {Or } \mathrm{AX}=\mathrm{B}}
$$

$A=\left[\begin{array}{ccc}2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2\end{array}\right], X=\left[\begin{array}{l}\mathrm{x} \\ \mathrm{y} \\ \mathrm{z}\end{array}\right]$ and $\mathrm{B}=\left[\begin{array}{l}2 \\ 5 \\ 6\end{array}\right]$
Now, $|\mathrm{A}|=2\left|\begin{array}{ll}3 & -1 \\ 1 & -2\end{array}\right|-1\left|\begin{array}{ll}1 & -1 \\ 3 & -2\end{array}\right|+1\left|\begin{array}{ll}1 & 3 \\ 3 & 1\end{array}\right|$
$=2(-6+1)-1(-2+3)+1(1-9)$
$=-10-1-8$
$=-19$
So, the above system has a unique solution, given by
$X=A{ }^{-1} B$
Cofactors of $A$ are
$C_{11}=(-1)^{1+1}-6+1=-5$
$C_{21}=(-1)^{2+1}(24+4)=-28$
$C_{31}=(-1)^{3+1}-1-3=-4$
$C_{12}=(-1)^{1+2}-2+3=-1$
$C_{22}=(-1)^{2+1}-4-3=-7$
$C_{32}=(-1)^{3+1}-2-1=3$
$C_{13}=(-1)^{1+2} 1-9=-8$
$\mathrm{C}_{23}=(-1)^{2+1} 2-3=-1$
$C_{33}=(-1)^{3+1} 6-1=5$
Adj $A=\left[\begin{array}{ccc}-5 & -1 & -8 \\ 3 & -7 & 1 \\ -4 & 3 & 5\end{array}\right]^{T}$
$=\left[\begin{array}{ccc}-5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5\end{array}\right]$
$\mathrm{A}^{-1}=\frac{1}{|\mathrm{~A}|} \operatorname{adj} \mathrm{A}$
Now, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{-19}\left[\begin{array}{ccc}-5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5\end{array}\right]\left[\begin{array}{l}2 \\ 5 \\ 6\end{array}\right]$
$X=\frac{1}{-19}\left[\begin{array}{l}-10+15-24 \\ -2-35+18 \\ -16+5+30\end{array}\right]$
$X=\frac{1}{-19}\left[\begin{array}{c}-19 \\ -19 \\ 19\end{array}\right]$
$X=\left[\begin{array}{c}1 \\ 1 \\ -1\end{array}\right]$
Hence, $X=1, Y=1$ and $Z=-1$
(viii) Given $2 x+y+z=2$
$x+3 y-z=5$
$3 x+y-2 z=6$

The given system can be written in matrix form as:

$$
\begin{aligned}
& {\left[\begin{array}{ccc}
2 & 1 & 1 \\
1 & 3 & -1 \\
3 & 1 & -2
\end{array}\right]\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]=\left[\begin{array}{l}
2 \\
5 \\
6
\end{array}\right] \text { Or } \mathrm{XX}=\mathrm{B}} \\
& \mathrm{~A}=\left[\begin{array}{ccc}
2 & 1 & 1 \\
1 & 3 & -1 \\
3 & 1 & -2
\end{array}\right], \mathrm{X}=\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right] \text { and } \mathrm{B}=\left[\begin{array}{l}
2 \\
5 \\
6
\end{array}\right] \\
& \text { Now, }|\mathrm{A}|=2^{\left|\begin{array}{ll}
3 & -1 \\
1 & -2
\end{array}\right|-1\left|\begin{array}{ll}
1 & -1 \\
3 & -2
\end{array}\right|+1\left|\begin{array}{ll}
1 & 3 \\
3 & 1
\end{array}\right|} \\
& =2(-6+1)-1(-2+3)+1(1-9) \\
& =-10-1-8 \\
& =-19
\end{aligned}
$$

So, the above system has a unique solution, given by
$X=A{ }^{-1} B$
Cofactors of $A$ are
$\mathrm{C}_{11}=(-1)^{1+1}-6+1=-5$
$C_{21}=(-1)^{2+1}(24+4)=-28$
$\mathrm{C}_{31}=(-1)^{3+1}-1-3=-4$
$C_{12}=(-1)^{1+2}-2+3=-1$
$C_{22}=(-1)^{2+1}-4-3=-7$
$C_{32}=(-1)^{3+1}-2-1=3$
$C_{13}=(-1)^{1+2} 1-9=-8$
$C_{23}=(-1)^{2+1} 2-3=-1$
$C_{33}=(-1)^{3+1} 6-1=5$
Adj $A=\left[\begin{array}{ccc}-5 & -1 & -8 \\ 3 & -7 & 1 \\ -4 & 3 & 5\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}-5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5\end{array}\right]$
$A^{-1}=\frac{1}{|A|} \operatorname{adj} \mathrm{A}$

Now, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{-19}\left[\begin{array}{ccc}-5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5\end{array}\right]\left[\begin{array}{l}2 \\ 5 \\ 6\end{array}\right]$
$\mathrm{X}=\frac{1}{-19}\left[\begin{array}{l}-10+15-24 \\ -2-35+18 \\ -16+5+30\end{array}\right]$
$X=\frac{1}{-19}\left[\begin{array}{c}-19 \\ -19 \\ 19\end{array}\right]$
$\mathrm{X}=\left[\begin{array}{c}1 \\ 1 \\ -1\end{array}\right]$
Hence, $\mathrm{X}=1, \mathrm{Y}=1$ and $\mathrm{Z}=-1$
(ix) Given $2 x+6 y=2$
$3 x-z=-8$
$2 x-y+z=-3$
The given system can be written in matrix form as:
$\left[\begin{array}{ccc}2 & 6 & 0 \\ 3 & 0 & -1 \\ 2 & -1 & 1\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{c}2 \\ -8 \\ -3\end{array}\right]$ Or $A X=B$
$\mathrm{A}=\left[\begin{array}{ccc}2 & 6 & 0 \\ 3 & 0 & -1 \\ 2 & -1 & 1\end{array}\right], \mathrm{X}=\left[\begin{array}{l}\mathrm{x} \\ \mathrm{y} \\ \mathrm{z}\end{array}\right]$ and $\mathrm{B}=\left[\begin{array}{c}2 \\ -8 \\ -3\end{array}\right]$
Now, $\left.|A|=2^{\mid c c} \begin{array}{cc}0 & -1 \\ -1 & 1\end{array}|-6| \begin{array}{cc}3 & -1 \\ 2 & 1\end{array} \right\rvert\,+0$
$=2(0-1)-6(3+2)$
$=-2-30$
$=-32$
So, the above system has a unique solution, given by
$X=A^{-1} B$
Cofactors of $A$ are
$\mathrm{C}_{11}=(-1)^{1+1} 0-1=-1$
$C_{21}=(-1)^{2+1} 6+0=-6$
$C_{31}=(-1)^{3+1}-6=-6$
$C_{12}=(-1)^{1+2} 3+2=5$
$C_{22}=(-1)^{2+1} 2-0=2$
$C_{32}=(-1)^{3+1}-2-0=2$
$\mathrm{C}_{13}=(-1)^{1+2}-3-0=-3$
$C_{23}=(-1)^{2+1}-2-12=14$
$C_{33}=(-1)^{3+1} 0-18=-18$
$\operatorname{Adj} A=\left[\begin{array}{ccc}-1 & -5 & -3 \\ -6 & 2 & 14 \\ -6 & 2 & -18\end{array}\right]^{T}$
$=\left[\begin{array}{ccc}-1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18\end{array}\right]$
$A^{-1}=\frac{1}{|A|} \operatorname{adj} \mathrm{A}$
Now, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{-32}\left[\begin{array}{ccc}-1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18\end{array}\right]\left[\begin{array}{c}2 \\ -8 \\ -3\end{array}\right]$
$\mathrm{X}=\frac{1}{32}\left[\begin{array}{c}-2+48+18 \\ -10-16-6 \\ -6-112+54\end{array}\right]$
$X=\frac{1}{32}\left[\begin{array}{c}64 \\ -32 \\ -64\end{array}\right]$
$X=\left[\begin{array}{c}-2 \\ 1 \\ 2\end{array}\right]$
Hence, $X=-2, Y=1$ and $Z=2$
(x) Given $2 y-z=1$
$x-y+z=2$
$2 x-y=0$

The given system can be written in matrix form as:

$$
\left[\begin{array}{ccc}
0 & 2 & -1 \\
1 & -1 & 1 \\
2 & -1 & 0
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{l}
1 \\
2 \\
0
\end{array}\right]
$$

$A X=B$
Now, $|\mathrm{A}|=0^{-2}\left|\begin{array}{ll}1 & 1 \\ 2 & 0\end{array}\right|-1\left|\begin{array}{ll}1 & -1 \\ 2 & -1\end{array}\right|$
$=0+4-1$
$=3$
So, the above system has a unique solution, given by
$X=A^{-1} B$
Cofactors of $A$ are
$\mathrm{C}_{11}=(-1)^{1+1} 1-0=1$
$\mathrm{C}_{21}=(-1)^{2+1} 1-2=1$
$\mathrm{C}_{31}=(-1)^{3+1} 0+1=1$
$\mathrm{C}_{12}=(-1)^{1+2}-2-0=2$
$\mathrm{C}_{22}=(-1)^{2+1}-1-0=-1$
$\mathrm{C}_{32}=(-1)^{3+1} 0-2=2$
$\mathrm{C}_{13}=(-1)^{1+2} 4-0=4$
$\mathrm{C}_{23}=(-1)^{2+1} 2-0=-2$
$C_{33}=(-1)^{3+1}-1+2=1$
$\operatorname{Adj} \mathrm{A}=\left[\begin{array}{ccc}1 & 2 & 4 \\ 1 & -1 & -2 \\ 1 & 2 & 1\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}1 & 1 & 1 \\ 2 & -1 & 2 \\ 4 & -2 & 1\end{array}\right]$
$A^{-1}=\frac{1}{|A|} \operatorname{adj} \mathrm{A}$
Now, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\stackrel{\frac{1}{3}}{ }=\left[\begin{array}{ccc}1 & 1 & 1 \\ 2 & -1 & 2 \\ 4 & -2 & 1\end{array}\right]\left[\begin{array}{l}2 \\ 0 \\ 1\end{array}\right]$

$$
X={ }^{\frac{1}{3}}\left[\begin{array}{l}
2+0+1 \\
4-0+2 \\
8-0+1
\end{array}\right]
$$

$$
\mathrm{X}={ }^{\frac{1}{3}}\left[\begin{array}{l}
3 \\
6 \\
9
\end{array}\right]
$$

$X=\left[\begin{array}{l}1 \\ 2 \\ 3\end{array}\right]$
Hence, $X=1, Y=2$ and $Z=3$
(xi) Given $8 x+4 y+3 z=18$
$2 x+y+z=5$
$x+2 y+z=5$
The given system can be written in matrix form as:
$\left[\begin{array}{lll}8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{c}18 \\ 5 \\ 5\end{array}\right]$
$A X=B$
Now, $|A|=8\left|\begin{array}{ll}1 & 1 \\ 2 & 1\end{array}\right|-4\left|\begin{array}{ll}2 & 1 \\ 1 & 1\end{array}\right|+3\left|\begin{array}{ll}2 & 1 \\ 1 & 2\end{array}\right|$
$=8(-1)-4(1)+3(3)$
$=-8-4+9$
$=-3$
So, the above system has a unique solution, given by
$X=A^{-1} B$
Cofactors of $A$ are
$\mathrm{C}_{11}=(-1)^{1+1} 1-2=-1$
$\mathrm{C}_{21}=(-1)^{2+1} 4-6=2$
$\mathrm{C}_{31}=(-1)^{3+1} 4-3=1$
$\mathrm{C}_{12}=(-1)^{1+2} 2-1=-1$
$C_{22}=(-1)^{2+1} 8-3=5$
$\mathrm{C}_{32}=(-1)^{3+1} 8-6=-2$
$C_{13}=(-1)^{1+2} 4-1=3$
$C_{23}=(-1)^{2+1} 16-4=-12$
$C_{33}=(-1)^{3+1} 8-8=0$
$\operatorname{Adj} A=\left[\begin{array}{ccc}-1 & -1 & 3 \\ 2 & 5 & -12 \\ 1 & -2 & 0\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}-1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0\end{array}\right]$
$\mathrm{A}^{-1}=\frac{1}{|\mathrm{~A}|} \operatorname{adj} \mathrm{A}$

$X=\quad \begin{gathered}-\frac{1}{3}\end{gathered}\left[\begin{array}{c}-18+10+5 \\ -18+25-10 \\ 54-60+0\end{array}\right]$
$X={ }^{\frac{1}{-3}}\left[\begin{array}{l}-3 \\ -3 \\ -6\end{array}\right]$
$X=\left[\begin{array}{l}1 \\ 1 \\ 2\end{array}\right]$
Hence, $\mathrm{X}=1, \mathrm{Y}=1$ and $\mathrm{Z}=2$
(xii) Given $x+y+z=6$
$x+2 z=7$
$3 x+y+z=12$
The given system can be written in matrix form as:

$$
\left[\begin{array}{lll}
1 & 1 & 1 \\
1 & 0 & 2 \\
3 & 1 & 1
\end{array}\right]\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]=\left[\begin{array}{c}
6 \\
17 \\
12
\end{array}\right]
$$

$A X=B$

Now, $|A|=1\left|\begin{array}{ll}0 & 2 \\ 1 & 1\end{array}\right|-1\left|\begin{array}{ll}1 & 2 \\ 3 & 1\end{array}\right|+1\left|\begin{array}{ll}1 & 0 \\ 3 & 1\end{array}\right|$
$=1(-2)-1(1-6)+1(1)$
$=-2+5+1$
$=4$
So, the above system has a unique solution, given by
$X=A{ }^{-1} B$
Cofactors of $A$ are
$\mathrm{C}_{11}=(-1)^{1+1} 0-2=-2$
$\mathrm{C}_{21}=(-1)^{2+1} 1-1=0$
$\mathrm{C}_{31}=(-1)^{3+1} 2-0=2$
$\mathrm{C}_{12}=(-1)^{1+2} 1-6=5$
$\mathrm{C}_{22}=(-1)^{2+1} 1-3=-2$
$\mathrm{C}_{32}=(-1)^{3+1} 2-1=-1$
$\mathrm{C}_{13}=(-1)^{1+2} 1-0=1$
$\mathrm{C}_{23}=(-1)^{2+1} 1-3=2$
$\mathrm{C}_{33}=(-1)^{3+1} 0-1=-1$
$\operatorname{Adj} \mathrm{A}=\left[\begin{array}{ccc}-2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}-2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1\end{array}\right]$
$A^{-1}=\frac{1}{|A|} \operatorname{adj} \mathrm{A}$
Now, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{4}\left[\begin{array}{ccc}-2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1\end{array}\right]\left[\begin{array}{c}6 \\ 7 \\ 12\end{array}\right]$
$\mathrm{X}=\frac{1}{4}\left[\begin{array}{c}-12+0+24 \\ 30-14-12 \\ 6+14-12\end{array}\right]$
$X=\begin{gathered}\frac{1}{4}\end{gathered}\left[\begin{array}{c}12 \\ 4 \\ 8\end{array}\right]$

$$
X=\left[\begin{array}{l}
3 \\
1 \\
2
\end{array}\right]
$$

Hence, $\mathrm{X}=3, \mathrm{Y}=1$ and $\mathrm{Z}=2$
(xiii) Given $(2 / x)+(3 / y)+(10 / z)=4$,
$(4 / x)-(6 / y)+(5 / z)=1$,
$(6 / x)+(9 / y)-(20 / z)=2, x, y, z \neq 0$
The given system can be written in matrix form as:
$\left[\begin{array}{ccc}2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20\end{array}\right]\left[\begin{array}{l}\mathrm{u} \\ \mathrm{v} \\ \mathrm{W}\end{array}\right]=\left[\begin{array}{l}4 \\ 1 \\ 2\end{array}\right]$
$A X=B$
Now,
$|A|=2(75)-3(-110)+10(72)$
$=150+330+720$
= 1200
So, the above system has a unique solution, given by
$X=A{ }^{-1} B$
Cofactors of $A$ are
$C_{11}=(-1)^{1+1} 120-45=75$
$C_{21}=(-1)^{2+1}-60-90=150$
$C_{31}=(-1)^{3+1} 15+60=75$
$C_{12}=(-1)^{1+2}-80-30=110$
$\mathrm{C}_{22}=(-1)^{2+1}-40-60=-100$
$C_{32}=(-1)^{3+1} 10-40=30$
$C_{13}=(-1)^{1+2} 36+36=72$
$C_{23}=(-1)^{2+1} 18-18=0$
$C_{33}=(-1)^{3+1}-12-12=-24$
Adj $A=\left[\begin{array}{ccc}75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24\end{array}\right]$

Now, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\stackrel{1}{1200}\left[\begin{array}{ccc}75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24\end{array}\right]\left[\begin{array}{l}4 \\ 1 \\ 2\end{array}\right]$
$X=\frac{1}{1200}\left[\begin{array}{l}600 \\ 400 \\ 240\end{array}\right]$
$\left[\begin{array}{c}\mathrm{u} \\ \mathrm{v} \\ \mathrm{W}\end{array}\right]=\left[\begin{array}{c}\frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5}\end{array}\right]$
Hence, $\mathrm{X}=2, \mathrm{Y}=3$ and $\mathrm{Z}=5$
(xiv) Given $x-y+2 z=7$
$3 x+4 y-5 z=-5$
$2 x-y+3 z=12$
The given system can be written in matrix form as:
$\left[\begin{array}{ccc}1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{c}7 \\ -5 \\ 12\end{array}\right]$
$A X=B$
Now,
$|A|=1(12-5)+1(9+10)+2(-3-8)$
$=7+19-22$
$=4$
So, the above system has a unique solution, given by
$X=A^{-1} B$
Cofactors of $A$ are
$\mathrm{C}_{11}=(-1)^{1+1} 12-5=7$
$C_{21}=(-1)^{2+1}-3+2=1$
$C_{31}=(-1)^{3+1} 5-8=-3$
$C_{12}=(-1)^{1+2} 9+10=-19$
$C_{22}=(-1)^{2+1} 3-4=-1$
$C_{32}=(-1)^{3+1}-5-6=11$
$C_{13}=(-1)^{1+2}-3-8=-11$
$C_{23}=(-1)^{2+1}-1+2=-1$
$C_{33}=(-1)^{3+1} 4+3=7$
Adj $A=\left[\begin{array}{ccc}7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & -7\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7\end{array}\right]$
$A^{-1}=\frac{1}{|A|} \operatorname{adj} A$
Now, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{4}\left[\begin{array}{ccc}7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7\end{array}\right]\left[\begin{array}{c}7 \\ -5 \\ 12\end{array}\right]$
$X=\frac{1}{4}\left[\begin{array}{c}8 \\ 4 \\ 12\end{array}\right]$

$$
\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]=\left[\begin{array}{l}
2 \\
1 \\
3
\end{array}\right]
$$

Hence, $X=2, Y=1$ and $Z=3$
3. Show that each one of the following systems of linear equations is consistent and also find their solutions:
(i) $6 x+4 y=2$
$9 x+6 y=3$
(ii) $2 x+3 y=5$
$6 x+9 y=15$
(iii) $5 x+3 y+7 z=4$
$3 x+26 y+2 z=9$
$7 x+2 y+10 z=5$
(v) $x+y+z=6$
$x+2 y+3 z=14$
$x+4 y+7 z=30$
(vi) $2 x+2 y-2 z=1$
$4 x+4 y-z=2$
$6 x+6 y+2 z=3$

## Solution:

(i) Given $6 x+4 y=2$
$9 x+6 y=3$
The above system of equations can be written as
$\left[\begin{array}{ll}6 & 4 \\ 9 & 6\end{array}\right]\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{l}2 \\ 3\end{array}\right]$ Or AX $=\mathrm{B}$
Where $\mathrm{A}=\left[\begin{array}{ll}6 & 4 \\ 9 & 6\end{array}\right] \mathrm{B}=\left[\begin{array}{l}2 \\ 3\end{array}\right]$ and $\mathrm{X}=\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]$
$|A|=36-36=0$
So, $A$ is singular, Now $X$ will be consistence if $(\operatorname{Adj} A) \times B=0$
$\mathrm{C}_{11}=(-1)^{1+1} 6=6$
$\mathrm{C}_{12}=(-1)^{1+2} 9=-9$
$\mathrm{C}_{21}=(-1)^{2+1} 4=-4$
$\mathrm{C}_{22}=(-1)^{2+2} 6=6$
Also, $\operatorname{adj} A=\left[\begin{array}{cc}6 & -9 \\ -4 & 6\end{array}\right]^{T}$
$=\left[\begin{array}{cc}6 & -4 \\ -9 & 6\end{array}\right]$
(Adj A).B $=\left[\begin{array}{cc}6 & -4 \\ -9 & 6\end{array}\right]\left[\begin{array}{l}2 \\ 3\end{array}\right]$
$=\left[\begin{array}{c}12-12 \\ -18+18\end{array}\right]=\left[\begin{array}{l}0 \\ 0\end{array}\right]$
Thus, $\mathrm{AX}=\mathrm{B}$ will be infinite solution,

Let $\mathrm{y}=\mathrm{k}$
Hence, $6 x=2-4 k$ or $9 x=3-6 k$
$X=\frac{1-2 k}{3}$
Hence, $X=\frac{1-2 k}{3}, Y=k$
(ii) Given $2 x+3 y=5$
$6 x+9 y=15$
The above system of equations can be written as
$\left[\begin{array}{ll}2 & 3 \\ 6 & 9\end{array}\right]\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{c}5 \\ 15\end{array}\right]$ Or $\mathrm{AX}=\mathrm{B}$
Where $A=\left[\begin{array}{ll}2 & 3 \\ 6 & 9\end{array}\right] B=\left[\begin{array}{c}5 \\ 15\end{array}\right]$ and $X=\left[\begin{array}{c}X \\ Y\end{array}\right]$
$|A|=18-18=0$
So, A is singular,
Now $X$ will be consistence if $(\operatorname{Adj} A) \times B=0$
$C_{11}=(-1)^{1+1} 9=9$
$C_{12}=(-1)^{1+2} 6=-6$
$C_{21}=(-1)^{2+1} 3=-3$
$C_{22}=(-1)^{2+2} 2=2$
Also, adj $A=\left[\begin{array}{cc}9 & -6 \\ -3 & 2\end{array}\right]^{T}$
$=\left[\begin{array}{cc}9 & -3 \\ -6 & 2\end{array}\right]$
$(\operatorname{Adj} A) \cdot B=\left[\left[\begin{array}{cc}9 & -3 \\ -6 & 2\end{array}\right]\left[\begin{array}{c}5 \\ 15\end{array}\right]\right.$
$=\left[\begin{array}{c}45-45 \\ -30+30\end{array}\right]=\left[\begin{array}{l}0 \\ 0\end{array}\right]$
Thus, $A X=B$ will be infinite solution,

## Let $\mathrm{y}=\mathrm{k}$

Hence,

$$
\begin{aligned}
& 2 x=5-3 k \text { or } X=\frac{5-3 k}{2} \\
& x=15-9 k \text { or } X=\frac{5-3 k}{2}
\end{aligned}
$$

Hence, $X=\frac{5-3 k}{2}, Y=k$
(iii) Given $5 x+3 y+7 z=4$
$3 x+26 y+2 z=9$
$7 x+2 y+10 z=5$
This can be written as:

$$
\left[\begin{array}{ccc}
5 & 3 & 7 \\
3 & 26 & 2 \\
7 & 2 & 10
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{l}
4 \\
9 \\
5
\end{array}\right]
$$

$$
|A|=5(260-4)-3(30-14)+7(6-182)
$$

$$
=5(256)-3(16)+7(176)
$$

$|A|=0$
So, $A$ is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:
$(\operatorname{Adj} A) \times B \neq 0$ or $(\operatorname{Adj} A) \times B=0$
Cofactors of $A$ are
$\mathrm{C}_{11}=(-1)^{1+1} 260-4=256$
$C_{21}=(-1)^{2+1} 30-14=-16$
$C_{31}=(-1)^{3+1} 6-182=-176$
$\mathrm{C}_{12}=(-1)^{1+2} 30-14=-16$
$C_{22}=(-1)^{2+1} 50-49=1$
$C_{32}=(-1)^{3+1} 10-21=11$
$\mathrm{C}_{13}=(-1)^{1+2} 6-182=-176$
$\mathrm{C}_{23}=(-1)^{2+1} 10-21=11$
$C_{33}=(-1)^{3+1} 130-9=121$

Adj $A=\left[\begin{array}{ccc}256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121\end{array}\right]$
Adj $\mathrm{A} \times \mathrm{B}=\left[\begin{array}{ccc}256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121\end{array}\right]\left[\begin{array}{l}4 \\ 9 \\ 5\end{array}\right]=\left[\begin{array}{l}0 \\ 0 \\ 0\end{array}\right]$
Now, $A X=B$ has infinite many solution
Let $\mathrm{z}=\mathrm{k}$
Then, $5 x+3 y=4-7 k$
$3 x+26 y=9-2 k$
This can be written as
$\left[\begin{array}{cc}5 & 3 \\ 3 & 26\end{array}\right]\left[\begin{array}{l}\mathrm{x} \\ \mathrm{y}\end{array}\right]=\left[\begin{array}{c}4-7 \mathrm{k} \\ 9-2 \mathrm{k}\end{array}\right]$
$|A|=121$
$\operatorname{Adj} A=\left[\begin{array}{cc}26 & -3 \\ -3 & 5\end{array}\right]$
Now, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{|\mathrm{~A}|} \operatorname{Adj} \mathrm{A} \times \mathrm{B}$

$$
\begin{aligned}
& \frac{1}{121}\left[\begin{array}{cc}
26 & -3 \\
-3 & 5
\end{array}\right]\left[\begin{array}{c}
4-7 \mathrm{k} \\
9-2 \mathrm{k}
\end{array}\right] \\
= & \frac{1}{121}\left[\begin{array}{c}
77-176 \mathrm{k} \\
11 \mathrm{k}+33
\end{array}\right]
\end{aligned}
$$

$$
\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]=\left[\begin{array}{c}
\frac{7-16 \mathrm{k}}{11} \\
\frac{\mathrm{k}+3}{11}
\end{array}\right]
$$

There values of $x, y$ and $z$ satisfy the third equation
(v) Given $x+y+z=6$
$x+2 y+3 z=14$
$x+4 y+7 z=30$
This can be written as:

$$
\left[\begin{array}{lll}
1 & 1 & 1 \\
1 & 2 & 3 \\
1 & 4 & 7
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{c}
6 \\
14 \\
30
\end{array}\right]
$$

$|A|=1(2)-1(4)+1(2)$
$=2-4+2$
$|A|=0$
So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:
( $\operatorname{Adj} \mathrm{A}) \times \mathrm{B} \neq 0$ or $(\operatorname{Adj} \mathrm{A}) \times B=0$
Cofactors of $A$ are
$\mathrm{C}_{11}=(-1)^{1+1} 14-12=2$
$\mathrm{C}_{21}=(-1)^{2+1} 7-4=-3$
$\mathrm{C}_{31}=(-1)^{3+1} 3-2=1$
$\mathrm{C}_{12}=(-1)^{1+2} 7-3=-4$
$C_{22}=(-1)^{2+1} 7-1=6$
$\mathrm{C}_{32}=(-1)^{3+1} 3-1=2$
$\mathrm{C}_{13}=(-1)^{1+2} 4-2=2$
$\mathrm{C}_{23}=(-1)^{2+1} 4-1=-3$
$\mathrm{C}_{33}=(-1)^{3+1} 2-1=1$
$\operatorname{Adj} \mathrm{A}=\left[\begin{array}{ccc}2 & -4 & 2 \\ -3 & 6 & -3 \\ 1 & -2 & 1\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}2 & -3 & 1 \\ -4 & 1 & -2 \\ 2 & -3 & 1\end{array}\right]$
$\operatorname{Adj} \mathrm{A} \times \mathrm{B}=\left[\begin{array}{ccc}2 & -3 & 1 \\ -4 & 1 & -2 \\ 2 & -3 & 1\end{array}\right]\left[\begin{array}{c}6 \\ 14 \\ 30\end{array}\right]=\left[\begin{array}{l}0 \\ 0 \\ 0\end{array}\right]$
Now, $A X=B$ has infinite many solution
Let $\mathrm{z}=\mathrm{k}$
Then, $x+y=6-k$
$x+2 y=14-3 k$
This can be written as:

$$
\left[\begin{array}{ll}
1 & 1 \\
1 & 2
\end{array}\right]\left[\begin{array}{l}
x \\
y
\end{array}\right]=\left[\begin{array}{c}
6-\mathrm{k} \\
14-3 \mathrm{k}
\end{array}\right]
$$

$$
|A|=1
$$

$\operatorname{Adj} \mathrm{A}=\left[\begin{array}{cc}2 & -1 \\ -1 & 1\end{array}\right]$
Now, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{|\mathrm{~A}|} \operatorname{Adj} \mathrm{A} \times \mathrm{B}$

$$
\begin{aligned}
& \frac{1}{1}\left[\begin{array}{cc}
2 & -1 \\
-1 & 1
\end{array}\right]\left[\begin{array}{c}
6-\mathrm{k} \\
14-3 \mathrm{k}
\end{array}\right] \\
&= \frac{1}{1}\left[\begin{array}{c}
12-2 \mathrm{k}-14+3 \mathrm{k} \\
-6+\mathrm{k}+14-3 \mathrm{k}
\end{array}\right] \\
& {\left[\begin{array}{c}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]=\left[\begin{array}{c}
-2+\mathrm{k} \\
8-2 \mathrm{k}
\end{array}\right] }
\end{aligned}
$$

There values of $x, y$ and $z$ satisfy the third equation
Hence, $x=k-2, y=8-2 k, z=k$
(vi) Given $x+y+z=6$
$x+2 y+3 z=14$
$x+4 y+7 z=30$
This can be written as

$$
\left[\begin{array}{ccc}
2 & 2 & -2 \\
4 & 4 & -1 \\
6 & 6 & 2
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{l}
1 \\
2 \\
3
\end{array}\right]
$$

$|A|=2(14)-2(14)-2(0)$
$|A|=0$
So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:
$(\operatorname{Adj} \mathrm{A}) \times \mathrm{B} \neq 0$ or $(\operatorname{Adj} \mathrm{A}) \times \mathrm{B}=0$
Cofactors of $A$ are:
$\mathrm{C}_{11}=(-1)^{1+1} 8+6=14$
$C_{21}=(-1)^{2+1} 4+12=-16$
$C_{31}=(-1)^{3+1}-2+8=6$
$C_{12}=(-1)^{1+2} 8+6=-14$
$C_{22}=(-1)^{2+1} 4+12=16$
$C_{32}=(-1)^{3+1}-2+8=-6$
$C_{13}=(-1)^{1+2} 24-24=0$
$C_{23}=(-1)^{2+1} 12-12=0$
$C_{33}=(-1)^{3+1} 8-8=0$
Adj $A=\left[\begin{array}{ccc}14 & -14 & 0 \\ -16 & 16 & 0 \\ 6 & -6 & 0\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0\end{array}\right]$
Adj $A \times B=\left[\begin{array}{ccc}14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0\end{array}\right]\left[\begin{array}{l}1 \\ 2 \\ 3\end{array}\right]=\left[\begin{array}{l}0 \\ 0 \\ 0\end{array}\right]$
Now, $A X=B$ has infinite many solution
Let $\mathrm{z}=\mathrm{k}$
Then, $2 x+2 y=1+2 k$
$4 x+4 y=2+k$
This can be written as:
$\left[\begin{array}{ll}2 & 2 \\ 4 & 4\end{array}\right]\left[\begin{array}{l}x \\ y\end{array}\right]=\left[\begin{array}{c}1+2 k \\ 2+k\end{array}\right]$
Hence, $|A|=0 \angle=0$
Hence, the given equation doesn't satisfy.
4. Show that each one of the following systems of linear equations is consistent:
(i) $2 x+5 y=7$
$6 x+15 y=13$
(ii) $2 x+3 y=5$
$6 x+9 y=10$
(iii) $4 x-2 y=3$
$6 x-3 y=5$
(iv) $4 x-5 y-2 z=2$
$5 x-4 y+2 z=-2$
$2 x+2 y+8 z=-1$
(v) $3 x-y-2 z=2$
$2 y-z=-1$
$3 x-5 y=3$
(vi) $x+y-2 z=5$
$x-2 y+z=-2$
$-2 x+y+z=4$

## Solution:

(i) Given $2 x+5 y=7$
$6 x+15 y=13$
The above system of equations can be written as
$\left[\begin{array}{cc}2 & 5 \\ 6 & 15\end{array}\right]\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{c}7 \\ 13\end{array}\right]$ Or $\mathrm{AX}=\mathrm{B}$
Where $\mathrm{A}=\left[\begin{array}{cc}2 & 5 \\ 6 & 15\end{array}\right] \mathrm{B}=\left[\begin{array}{c}7 \\ 13\end{array}\right]$ and $\mathrm{X}=\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]$
$|A|=30-30=0$
So, A is singular,
Now $X$ will be consistence if $(\operatorname{Adj} A) \times B=0$
$\mathrm{C}_{11}=(-1)^{1+1} 15=15$
$\mathrm{C}_{12}=(-1)^{1+2} 6=-6$
$\mathrm{C}_{21}=(-1)^{2+1} 5=-5$
$\mathrm{C}_{22}=(-1)^{2+2} 2=2$
Also, $\operatorname{adj} A=\left[\begin{array}{cc}15 & -6 \\ -5 & 2\end{array}\right]^{T}$
$=\left[\begin{array}{cc}15 & -5 \\ -5 & 2\end{array}\right]$
$(\operatorname{Adj} A) \cdot B=\left[\begin{array}{cc}15 & -5 \\ -5 & 2\end{array}\right]\left[\begin{array}{c}7 \\ 13\end{array}\right]$
$=\left[\begin{array}{c}105-65 \\ -35+26\end{array}\right]=\left[\begin{array}{c}40 \\ -16\end{array}\right]$
$\neq 0$
Hence, the given system is inconsistent.
(ii) Given $2 x+3 y=5$
$6 x+9 y=10$
The above system of equations can be written as
$\left[\begin{array}{ll}2 & 3 \\ 6 & 9\end{array}\right]\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{c}5 \\ 10\end{array}\right]$ Or $\mathrm{AX}=\mathrm{B}$
Where $A=\left[\begin{array}{ll}2 & 3 \\ 6 & 9\end{array}\right] B=\left[\begin{array}{c}5 \\ 10\end{array}\right]$ and $X=\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]$
$|A|=18-18=0$
So, A is singular,
Now $X$ will be consistence if $(\operatorname{Adj} A) \times B=0$
$C_{11}=(-1)^{1+1} 9=9$
$C_{12}=(-1)^{1+2} 6=-6$
$C_{21}=(-1)^{2+1} 3=-3$
$C_{22}=(-1)^{2+2} 2=2$
Also, adj $A=\left[\begin{array}{cc}9 & -6 \\ -3 & 2\end{array}\right]^{T}$
$=\left[\begin{array}{cc}9 & -3 \\ -6 & 2\end{array}\right]$
$(\operatorname{Adj} A) \cdot B=\left[\begin{array}{cc}9 & -3 \\ -6 & 2\end{array}\right]\left[\begin{array}{c}5 \\ 10\end{array}\right]$
$=\left[\begin{array}{c}45-30 \\ -30+20\end{array}\right]=\left[\begin{array}{c}15 \\ -10\end{array}\right] \neq 0$
Hence, the given system is inconsistent.
(iii) Given $4 x-2 y=3$
$6 x-3 y=5$
The above system of equations can be written as
$\left[\begin{array}{ll}4 & -2 \\ 6 & -3\end{array}\right]\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]=\left[\begin{array}{l}3 \\ 5\end{array}\right]$ Or AX $=\mathrm{B}$
Where $\mathrm{A}=\left[\begin{array}{ll}4 & -2 \\ 6 & -3\end{array}\right] \mathrm{B}=\left[\begin{array}{l}3 \\ 5\end{array}\right]$ and $\mathrm{X}=\left[\begin{array}{l}\mathrm{X} \\ \mathrm{Y}\end{array}\right]$
$|A|=-12+12=0$
So, A is singular,
Now $X$ will be consistence if $(\operatorname{Adj} A) \times B=0$
$\mathrm{C}_{11}=(-1)^{1+1}-3=-3$
$\mathrm{C}_{12}=(-1)^{1+2} 6=-6$
$\mathrm{C}_{21}=(-1)^{2+1}-2=2$
$C_{22}=(-1)^{2+2} 4=4$
Also, $\operatorname{adj} A=$
$=\left[\begin{array}{ll}-3 & 2 \\ -6 & 4\end{array}\right]$
(Adj $A) . B=\left[\begin{array}{ll}-3 & 2 \\ -6 & 4\end{array}\right]\left[\begin{array}{l}3 \\ 5\end{array}\right]$
$=\left[\begin{array}{c}-9+10 \\ -18+20\end{array}\right]=\left[\begin{array}{l}1 \\ 2\end{array}\right]$
Hence, the given system is inconsistent.
(iv) Given $4 x-5 y-2 z=2$
$5 x-4 y+2 z=-2$
$2 x+2 y+8 z=-1$

$$
\begin{aligned}
& {\left[\begin{array}{ccc}
4 & -5 & -2 \\
5 & -4 & 2 \\
2 & 2 & 8
\end{array}\right]\left[\begin{array}{c}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{c}
2 \\
-2 \\
-1
\end{array}\right]} \\
& |A|=4(-36)+5(36)-2(18) \\
& |A|=0
\end{aligned}
$$

Cofactors of A are:
$C_{11}=(-1)^{1+1}-32-4=-36$
$C_{21}=(-1)^{2+1}-40+4=-36$
$C_{31}=(-1)^{3+1}-10-8=-18$
$C_{12}=(-1)^{1+2} 40-4=-36$
$C_{22}=(-1)^{2+1} 32+4=36$
$C_{32}=(-1)^{3+1} 8+10=-18$
$C_{13}=(-1)^{1+2} 10+8=18$
$C_{23}=(-1)^{2+1} 8+10=-18$
$C_{33}=(-1)^{3+1}-16+25=9$
Adj $A=\left[\begin{array}{ccc}-36 & -36 & 18 \\ 36 & 36 & -18 \\ -18 & -18 & 9\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}-36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9\end{array}\right]$
Adj $\mathrm{A} \times \mathrm{B}=\left[\begin{array}{ccc}-36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9\end{array}\right]\left[\begin{array}{c}2 \\ -2 \\ -1\end{array}\right]$
$=\left[\begin{array}{c}-72-72+18 \\ -72-72+18 \\ 36+36-9\end{array}\right] \neq\left[\begin{array}{l}0 \\ 0 \\ 0\end{array}\right]$
Hence, the above system is inconsistent.
(v) Given $3 x-y-2 z=2$
$2 y-z=-1$
$3 x-5 y=3$

$$
\begin{aligned}
& {\left[\begin{array}{ccc}
3 & -1 & -2 \\
0 & 2 & -1 \\
3 & -5 & 0
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{c}
2 \\
-1 \\
3
\end{array}\right]} \\
& |A|=3(-5)+1(3)-2(-6) \\
& |A|=0
\end{aligned}
$$

Cofactors of A are
$\mathrm{C}_{11}=(-1)^{1+1} 0-5=-5$
$C_{21}=(-1)^{2+1} 0-10=10$
$C_{31}=(-1)^{3+1} 1+4=5$
$\mathrm{C}_{12}=(-1)^{1+2} 0+3=-3$
$C_{22}=(-1)^{2+1} 0+6=6$
$C_{32}=(-1)^{3+1}-3-0=3$
$C_{13}=(-1)^{1+2} 0-6=-6$
$C_{23}=(-1)^{2+1}-15+3=12$
$C_{33}=(-1)^{3+1} 6-0=6$
Adj $A=\left[\begin{array}{ccc}-5 & 3 & -6 \\ 10 & 6 & 12 \\ 5 & 3 & 6\end{array}\right]^{\mathrm{T}}$
$=\left[\begin{array}{ccc}-5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6\end{array}\right]$
$\operatorname{Adj} A \times B=\left[\begin{array}{ccc}-5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6\end{array}\right]\left[\begin{array}{c}2 \\ -1 \\ 3\end{array}\right]$
$=\left[\begin{array}{c}-10-10+15 \\ 6-6+9 \\ -12-12+18\end{array}\right] \neq\left[\begin{array}{l}0 \\ 0 \\ 0\end{array}\right]$
Hence, the above system is inconsistent.
(vi) Given $x+y-2 z=5$
$x-2 y+z=-2$
$-2 x+y+z=4$
$\left[\begin{array}{ccc}1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{c}5 \\ -2 \\ 4\end{array}\right]$
$|A|=1(-3)-1(3)-2(-3)=-3-3+6$
$|A|=0$
Cofactors of A are:
$\mathrm{C}_{11}=(-1)^{1+1}-2-1=-3$
$C_{21}=(-1)^{2+1} 1+2=-3$
$C_{31}=(-1)^{3+1} 1-4=-3$
$\mathrm{C}_{12}=(-1)^{1+2} 1+2=-3$
$C_{22}=(-1)^{2+1} 1-4=-3$
$C_{32}=(-1)^{3+1} 1+2=-3$
$C_{13}=(-1)^{1+2} 1-4=-3$
$C_{23}=(-1)^{2+1} 1+2=-3$
$C_{33}=(-1)^{3+1}-2-1=-3$
Adj $A=\left[\begin{array}{lll}-3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3\end{array}\right]^{\mathrm{T}}$

$$
=\left[\begin{array}{lll}
-3 & -3 & -3 \\
-3 & -3 & -3 \\
-3 & -3 & -3
\end{array}\right]
$$

$\operatorname{Adj} A \times B=\left[\begin{array}{lll}-3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3\end{array}\right]\left[\begin{array}{c}5 \\ -2 \\ 4\end{array}\right]$

$$
=\left[\begin{array}{l}
-15+6-12 \\
-15+6-12 \\
-15+6-12
\end{array}\right] \neq\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right]
$$

Hence, the above system is inconsistent.
5. If $A=\left[\begin{array}{ccc}1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2\end{array}\right]$ and $B=\left[\begin{array}{ccc}2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & 1 & -5\end{array}\right]$ are two square matrices.

Find $A B$ and hence solve the system of linear equations:
$x-y=3,2 x+3 y+4 z=17, y+2 z=7$

## Solution:

$$
\begin{aligned}
& A=\left[\begin{array}{ccc}
1 & -1 & 0 \\
2 & 3 & 4 \\
0 & 1 & 2
\end{array}\right]_{\mathrm{B}}^{\mathrm{A}} \mathrm{=}=\left[\begin{array}{ccc}
2 & 2 & -4 \\
-4 & 2 & -4 \\
2 & -1 & 5
\end{array}\right] \\
& A B=\left[\begin{array}{ccc}
2+4+0 & 2-2+0 & -4+4+0 \\
4-12+8 & 4+6-4 & -8-12+20 \\
0-4+4 & 0+2-2 & 0-4+10
\end{array}\right]
\end{aligned}
$$

$$
A B=\left[\begin{array}{lll}
6 & 0 & 0 \\
0 & 6 & 0 \\
0 & 0 & 6
\end{array}\right]
$$

Now, we can see that it is $A B=6 I$. Where $I$ is the unit Matrix
Or, $A^{-1}=\frac{1}{6}\left[\begin{array}{ccc}2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5\end{array}\right]$
Now the given equation can be written as:
$\left[\begin{array}{ccc}1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{c}3 \\ 17 \\ 7\end{array}\right]$
$A X=B$
Or, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}$
$=\frac{1}{6}\left[\begin{array}{ccc}2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5\end{array}\right]\left[\begin{array}{c}3 \\ 17 \\ 7\end{array}\right]$
$\left[\begin{array}{l}\mathrm{x} \\ \mathrm{y} \\ \mathrm{z}\end{array}\right]=\frac{1}{6}\left[\begin{array}{c}6+34-28 \\ -12+34-28 \\ 6-17+35\end{array}\right]$
$=\frac{1}{6}\left[\begin{array}{l}12 \\ -6 \\ 24\end{array}\right]$
$X=\left[\begin{array}{c}2 \\ -1 \\ 4\end{array}\right]$
Hence, $x=2, y=-1$ and $z=4$
6. If $A=\left[\begin{array}{ccc}2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2\end{array}\right]$, find $A^{-1}$ and hence solve the system of linear equations :
$2 x-3 y+5 z=11,3 x+2 y-4 z=-5, x+y-2 z=-3$.
Solution:
$\mathrm{A}=\left[\begin{array}{ccc}2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2\end{array}\right]$
$|A|=2(0)+3(-2)+5(1)$
$=-1$
Now, the cofactors of $A$
$\mathrm{C}_{11}=(-1)^{1+1}-4+4=0$
$C_{21}=(-1)^{2+1} 6-5=-1$
$C_{31}=(-1)^{3+1} 12-10=2$
$\mathrm{C}_{12}=(-1)^{1+2}-6+4=2$
$C_{22}=(-1)^{2+1}-4-5=-9$
$C_{32}=(-1)^{3+1}-8-15=23$
$\mathrm{C}_{13}=(-1)^{1+2} 3-2=1$
$C_{23}=(-1)^{2+1} 2+3=-5$
$C_{33}=(-1)^{3+1} 4+9=13$
Adj $A=\left[\begin{array}{ccc}0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13\end{array}\right]^{T}=\left[\begin{array}{ccc}0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13\end{array}\right]$
$A^{-1}=\frac{1}{|A|} \operatorname{adj} A$
$\mathrm{A}^{-1}=\frac{1}{-1}\left[\begin{array}{ccc}0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13\end{array}\right]$
$A^{-1}=\left[\begin{array}{ccc}0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13\end{array}\right]$
Now the given equation can be written as:
$\left[\begin{array}{ccc}2 & -3 & 5 \\ 3 & 2 & -4 \\ -1 & 1 & -2\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{c}11 \\ -5 \\ -3\end{array}\right]$
$A X=B$

$$
\text { Or, } X=A^{-1} B
$$

$$
=\left[\begin{array}{ccc}
0 & 1 & -2 \\
-2 & 9 & -23 \\
-1 & 5 & -13
\end{array}\right]\left[\begin{array}{c}
11 \\
-5 \\
-3
\end{array}\right]
$$

$\left[\begin{array}{l}\mathrm{x} \\ \mathrm{y} \\ \mathrm{z}\end{array}\right]=\left[\begin{array}{c}0-5+6 \\ -22+45+69 \\ -11-25+39\end{array}\right]$
$X=\left[\begin{array}{l}1 \\ 2 \\ 3\end{array}\right]$
Hence, $x=1, y=2$ and $z=3$
7. Find $A^{-2}$, if $A=\left[\begin{array}{ccc}1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1\end{array}\right]$. Hence solve the following system of linear equations : $x+2 y+5 z=10, x-y-z=-2,2 x+3 y-z=-11$.

## Solution:

Given

$$
\begin{aligned}
& A=\left[\begin{array}{ccc}
1 & 2 & 5 \\
1 & -1 & -1 \\
2 & 3 & -1
\end{array}\right] \\
& |A|=1(1+3)+2(-1+2)+5(3+2) \\
& =4+2+25 \\
& =27
\end{aligned}
$$

Now, the cofactors of $A$
$\mathrm{C}_{11}=(-1)^{1+1} 1+3=4$
$C_{21}=(-1)^{2+1}-2-15=17$
$C_{31}=(-1)^{3+1}-2+5=3$
$C_{12}=(-1)^{1+2}-1+2=-1$
$C_{22}=(-1)^{2+1}-1-10=-11$
$C_{32}=(-1)^{3+1}-1-5=6$
$C_{13}=(-1)^{1+2} 3+2=5$
$C_{23}=(-1)^{2+1} 3-4=1$
$C_{33}=(-1)^{3+1}-1-2=-3$
Adj $A=\left[\begin{array}{ccc}4 & -1 & 5 \\ 17 & -11 & 1 \\ 3 & 6 & -3\end{array}\right]^{\mathrm{T}}=\left[\begin{array}{ccc}4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3\end{array}\right]$
$A^{-1}=\frac{1}{|A|}$ adj $A$

$$
A^{-1}=\frac{1}{27}\left[\begin{array}{ccc}
4 & 17 & 3 \\
-1 & -11 & 6 \\
5 & 1 & -3
\end{array}\right]
$$

Now the given equation can be written as:

$$
\left[\begin{array}{ccc}
1 & 2 & 5 \\
1 & -1 & -1 \\
2 & 3 & -1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{c}
10 \\
-2 \\
-11
\end{array}\right]
$$

$$
\mathrm{AX}=\mathrm{B}
$$

$$
\text { Or, } \mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}
$$

$$
=\frac{1}{27}\left[\begin{array}{ccc}
4 & 17 & 3 \\
-1 & -11 & 6 \\
5 & 1 & -3
\end{array}\right]\left[\begin{array}{c}
10 \\
-2 \\
-11
\end{array}\right]
$$

$$
\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\frac{1}{27}\left[\begin{array}{c}
40-34-33 \\
-10+22-66 \\
50-2+33
\end{array}\right]
$$

$$
\mathrm{X}=\frac{1}{27}\left[\begin{array}{c}
-27 \\
-54 \\
81
\end{array}\right]
$$

$$
X=\left[\begin{array}{c}
-1 \\
-2 \\
3
\end{array}\right]
$$

Hence, $x=-1, y=-2$ and $z=3$
8. (i) If $A=\left[\begin{array}{ccc}1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1\end{array}\right]$, find $A^{-1} \cdot U \operatorname{sing} A^{-1}$, solve thesy stem of linear equations :
$x-2 y=10,2 x+y+3 z=8,-2 y+z=7$

## Solution:

Given
$A=\left[\begin{array}{ccc}1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1\end{array}\right]$
$|A|=1(1+6)+2(2-0)+0$
$=7+4$
$=11$
Now, the cofactors of $A$
$C_{11}=(-1)^{1+1} 1+6=7$
$C_{21}=(-1)^{2+1}-2-0=2$
$C_{31}=(-1)^{3+1}-6-0=-6$
$\mathrm{C}_{12}=(-1)^{1+2} 2-0=-2$
$C_{22}=(-1)^{2+1} 1-0=1$
$C_{32}=(-1)^{3+1} 3-0=-3$
$C_{13}=(-1)^{1+2}-4-0=-4$
$C_{23}=(-1)^{2+1}-2-0=2$
$C_{33}=(-1)^{3+1} 1+4=5$
Adj $A=\left[\begin{array}{ccc}7 & 2 & -4 \\ -2 & 1 & -3 \\ -6 & 2 & 5\end{array}\right]^{\mathrm{T}}=\left[\begin{array}{ccc}7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5\end{array}\right]$
$A^{-1}=\frac{1}{|\mathrm{~A}|} \operatorname{adj} \mathrm{A}$
$\mathrm{A}^{-1}=\frac{1}{11}\left[\begin{array}{ccc}7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5\end{array}\right]$
Now the given equation can be written as:

$$
\left[\begin{array}{lll}
1 & -2 & 0 \\
2 & -1 & 3 \\
0 & -2 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{c}
10 \\
8 \\
7
\end{array}\right]
$$

$A X=B$
Or, $X=A^{-1} B$
$=\frac{1}{11}\left[\begin{array}{ccc}7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5\end{array}\right]\left[\begin{array}{c}10 \\ 8 \\ 7\end{array}\right]$
$\left[\begin{array}{l}\mathrm{x} \\ \mathrm{y} \\ \mathrm{z}\end{array}\right]=\frac{1}{11}\left[\begin{array}{c}70+16-42 \\ -20+8-21 \\ -40+16+35\end{array}\right]$
$X=\frac{1}{11}\left[\begin{array}{c}44 \\ -33 \\ 11\end{array}\right]$
$X=\left[\begin{array}{c}4 \\ -3 \\ 1\end{array}\right]$
Hence, $x=4, y=-3$ and $z=1$
(ii) $A=\left[\begin{array}{ccc}3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1\end{array}\right]$, find $A^{-1}$ and hence solve thesystem of linear equations :
$3 x-4 y+2 z=-1,2 x+3 y+5 z=7, x+z=2$

## Solution:

Given

$$
\begin{aligned}
& \quad\left[\begin{array}{ccc}
3 & -4 & 2 \\
2 & 3 & 5 \\
1 & 0 & 1
\end{array}\right] \\
& |A|=3(3-0)+4(2-5)+2(0-3) \\
& =9-12-6 \\
& =-9
\end{aligned}
$$

Now, the cofactors of $A$
$\mathrm{C}_{11}=(-1)^{1+1} 3-0=3$
$C_{21}=(-1)^{2+1}-4-0=4$
$C_{31}=(-1)^{3+1}-20-6=-26$
$C_{12}=(-1)^{1+2} 2-5=3$
$C_{22}=(-1)^{2+1} 3-2=1$
$C_{32}=(-1)^{3+1} 15-4=-11$
$C_{13}=(-1)^{1+2} 0-3=-3$
$C_{23}=(-1)^{2+1} 0+4=-4$
$C_{33}=(-1)^{3+1} 9+8=17$
Adj $A=\left[\begin{array}{ccc}3 & 3 & -3 \\ 4 & 1 & -4 \\ -26 & -4 & 27\end{array}\right]^{\mathrm{T}}=\left[\begin{array}{ccc}3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17\end{array}\right]$
$A^{-1}=\frac{1}{|A|} \operatorname{adj} \mathrm{A}$
$A^{-1}=\frac{1}{-9}\left[\begin{array}{ccc}3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17\end{array}\right]$
Now the given equation can be written as:
$\left[\begin{array}{ccc}3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{c}-1 \\ 7 \\ 2\end{array}\right]$
$A X=B$
Or, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}$
$=\frac{1}{-9}\left[\begin{array}{ccc}3 & 4 & -26 \\ 3 & 1 & 11 \\ -3 & -4 & 17\end{array}\right]\left[\begin{array}{c}-1 \\ 7 \\ 2\end{array}\right]$
$\left[\begin{array}{l}\mathrm{x} \\ \mathrm{y} \\ \mathrm{z}\end{array}\right]=\frac{1}{-9}\left[\begin{array}{c}-3+28-52 \\ 21+7+22 \\ 3-28+34\end{array}\right]$
$X=\frac{1}{-9}\left[\begin{array}{c}-27 \\ -18 \\ 9\end{array}\right]$
$X=\left[\begin{array}{c}3 \\ 2 \\ -1\end{array}\right]$
Hence, $x=3, y=2$ and $z=-1$
(iii) $A=\left[\begin{array}{ccc}1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1\end{array}\right]$, and $B=\left[\begin{array}{ccc}7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5\end{array}\right]$ find $A B$. Hence solve thesystem of linear equations : $x-2 y=10,2 x+y+3 z=8$ and $-2 y+z=7$

## Solution:

$$
\begin{aligned}
& A=\left[\begin{array}{ccc}
1 & -2 & 0 \\
2 & 1 & 3 \\
0 & -2 & 1
\end{array}\right] B=\left[\begin{array}{ccc}
7 & 2 & -6 \\
-2 & 1 & -3 \\
-4 & 2 & 5
\end{array}\right] \\
& A B=\left[\begin{array}{ccc}
7+4-0 & 2-2+0 & -6+6+0 \\
14-2-12 & 4+1+6 & -12-3+15 \\
0-4+4 & 0-2+2 & 0+6+5
\end{array}\right]
\end{aligned}
$$

$A B=\left[\begin{array}{ccc}11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11\end{array}\right]$
Now, we can see that it is $\mathrm{AB}=11 \mathrm{I}$. Where I is the unit Matrix
Or, $\mathrm{A}^{-1}=\frac{1}{11}\left[\begin{array}{ccc}7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5\end{array}\right]$
Now the given equation can be written as:

$$
\left[\begin{array}{ccc}
1 & -2 & 0 \\
2 & 1 & 3 \\
0 & -2 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{c}
10 \\
8 \\
7
\end{array}\right]
$$

$A X=B$
Or, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}$
$=\frac{1}{11}\left[\begin{array}{ccc}7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5\end{array}\right]\left[\begin{array}{c}10 \\ 8 \\ 7\end{array}\right]$
$\left[\begin{array}{l}\mathrm{x} \\ \mathrm{y} \\ \mathrm{z}\end{array}\right]=\frac{1}{11}\left[\begin{array}{c}70+16-42 \\ -20+8-21 \\ -40+16+35\end{array}\right]$
$=\frac{1}{11}\left[\begin{array}{c}44 \\ -33 \\ 11\end{array}\right]$
$X=\left[\begin{array}{c}4 \\ -3 \\ 1\end{array}\right]$
Hence, $x=4, y=-3$ and $z=1$
(iv) If $A=\left[\begin{array}{ccc}1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1\end{array}\right]$, find $A^{-1} . U \operatorname{sing} A^{-1}$ solve the system of linearequations :
$x-2 y=10,2 x-y-z=8,-2 y+z=7$

## Solution:

Given

$$
\begin{aligned}
& A=\left[\begin{array}{ccc}
1 & 2 & 0 \\
-2 & -1 & -1 \\
0 & -1 & 1
\end{array}\right] \\
& |A|=1(-1-1)-2(-2-0)+0 \\
& =-2+4 \\
& =2
\end{aligned}
$$

Now, the cofactors of $A$
$\mathrm{C}_{11}=(-1)^{1+1}-1-1=-2$
$C_{21}=(-1)^{2+1} 2-0=2$
$C_{31}=(-1)^{3+1}-2-0=-2$
$C_{12}=(-1)^{1+2} 2-0=-2$
$C_{22}=(-1)^{2+1} 1-0=1$
$C_{32}=(-1)^{3+1}-1-0=1$
$\mathrm{C}_{13}=(-1)^{1+2}-2-0=-2$
$C_{23}=(-1)^{2+1}-1-0=1$
$C_{33}=(-1)^{3+1}-1+4=3$
Adj $A=\left[\begin{array}{ccc}-2 & -2 & -2 \\ 2 & 1 & 1 \\ -2 & 1 & 3\end{array}\right]^{\mathrm{T}}=\left[\begin{array}{ccc}-2 & 2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 3\end{array}\right]$
$A^{-1}=\frac{1}{|A|} \operatorname{adj} A$
$A^{-1}=\frac{1}{2}\left[\begin{array}{ccc}-2 & 2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 3\end{array}\right]$
Now the given equation can be written as:
$\left[\begin{array}{ccc}1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{c}10 \\ 8 \\ 7\end{array}\right]$
$A X=B$
Or, $\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}$
$=\frac{1}{2}\left[\begin{array}{ccc}1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1\end{array}\right]\left[\begin{array}{c}10 \\ 8 \\ 7\end{array}\right]$
$\left[\begin{array}{l}\mathrm{x} \\ \mathrm{y} \\ \mathrm{z}\end{array}\right]=\frac{1}{2}\left[\begin{array}{c}10-16+0 \\ 20-8-7 \\ 0-16+7\end{array}\right]$
$X=\frac{1}{2}\left[\begin{array}{c}-6 \\ 5 \\ -9\end{array}\right]$

$\mathrm{BA}=\left[\begin{array}{ccc}2+4-0 & 2-2+0 & -4+4+0 \\ -4-12+16 & 4+6-4 & -8-12+20 \\ 0-4+8 & 0-2+2 & 0-4+10\end{array}\right]$
$B A=\left[\begin{array}{lll}6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6\end{array}\right]$
Now, we can see that it is $\mathrm{BA}=6 \mathrm{I}$. Where I is the unit Matrix
Or, $\mathrm{B}^{-1}=\frac{1}{6}\left[\begin{array}{ccc}2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5\end{array}\right]$
Now the given equation can be written as:

$$
\left[\begin{array}{ccc}
0 & 1 & 2 \\
1 & -1 & 0 \\
2 & 3 & 4
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{c}
7 \\
3 \\
17
\end{array}\right]
$$

$A X=B$
Or, $X=B^{-1} A$
$=\frac{1}{6}\left[\begin{array}{ccc}2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5\end{array}\right]\left[\begin{array}{c}7 \\ 3 \\ 17\end{array}\right]$
$\left[\begin{array}{l}\mathrm{x} \\ \mathrm{y} \\ \mathrm{z}\end{array}\right]=\frac{1}{6}\left[\begin{array}{c}14+6-68 \\ -28+6-68 \\ 28-3+85\end{array}\right]$
$=\begin{array}{r}\frac{1}{6}\left[\begin{array}{c}-48 \\ -90 \\ 110\end{array}\right], ~\end{array}$
$X=\left[\begin{array}{c}-8 \\ -15 \\ \frac{110}{6}\end{array}\right]$
Hence, $x=-8, y=-15$ and $z=\frac{110}{6}$
9. The sum of three numbers is $\mathbf{2}$. If twice the second number is added to the sum of first and third, the sum is 1 . By adding second and third number to five times the first number, we get 6 . Find the three numbers by using matrices.

## Solution:

Let the numbers are $x, y, z$
$x+y+z=2$...... (i)
Also, $2 y+(x+z)+1$
$x+2 y+z=1$
Again,
$x+z+5(x)=6$
$5 x+y+z=6$

$$
\left[\begin{array}{lll}
1 & 1 & 1  \tag{iii}\\
1 & 2 & 1 \\
5 & 1 & 1
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{l}
2 \\
1 \\
6
\end{array}\right]
$$

$\mathrm{AX}=\mathrm{B}$
$|A|=1(1)-1(-4)+1(-9)$
$=1+4-9$
$=-4$
Hence, the unique solution given by $x=A^{-1} B$
$\mathrm{C}_{11}=(-1)^{1+1}(2-1)=1$
$\mathrm{C}_{12}=(-1)^{1+2}(1-5)=4$
$\mathrm{C}_{13}=(-1)^{1+3}(1-10)=-9$
$\mathrm{C}_{21}=(-1)^{2+1}(1-1)=0$
$\mathrm{C}_{22}=(-1)^{2+2}(1-5)=-4$
$\mathrm{C}_{23}=(-1)^{2+3}(1-5)=4$
$\mathrm{C}_{31}=(-1)^{3+1}(1-2)=-1$
$\mathrm{C}_{32}=(-1)^{3+2}(1-1)=0$
$\mathrm{C}_{33}=(-1)^{3+3}(2-1)=1$
$\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{|\mathrm{~A}|}(\operatorname{adj} \mathrm{A}) \mathrm{B}$
$\operatorname{Adj} \mathrm{A}=\left[\begin{array}{ccc}1 & 4 & -9 \\ 0 & -4 & 4 \\ -1 & 0 & 1\end{array}\right]^{\mathrm{T}}=\left[\begin{array}{ccc}1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1\end{array}\right]$

$$
\begin{aligned}
& X=\frac{1}{-4}\left[\begin{array}{ccc}
1 & 0 & -1 \\
4 & -4 & 0 \\
-9 & 4 & 1
\end{array}\right]\left[\begin{array}{l}
2 \\
1 \\
6
\end{array}\right] \\
& X=\frac{1}{-4}\left[\begin{array}{c}
2-6 \\
8-4 \\
-18+4+6
\end{array}\right] \\
& =\frac{1}{-4}\left[\begin{array}{c}
-4 \\
4 \\
-8
\end{array}\right] \\
& \text { Hence, }\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]=\left[\begin{array}{c}
1 \\
-2 \\
2
\end{array}\right]
\end{aligned}
$$

10. An amount of $₹ 10,000$ is put into three investments at the rate of 10,12 and $15 \%$ per annum. The combined incomes are ₹ 1310 and the combined income of first and second investment is ₹ $\mathbf{1 9 0}$ short of the income from the third. Find the investment in each using matrix method.

## Solution:

Let the numbers are $x, y$, and $z$
$x+y+z=10,000$
Also,
$0.1 x+0.12 y+0.15 z=1310$
Again,
$0.1 x+0.12 y-0.15 z=-190 \ldots . .$. (iii)

$$
\left[\begin{array}{ccc}
1 & 1 & 1 \\
0.1 & 0.12 & 0.15 \\
0.1 & 0.12 & -0.15
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{c}
10000 \\
1310 \\
-190
\end{array}\right]
$$

A $\mathrm{X}=\mathrm{B}$
$|A|=1(-0.036)-1(-0.03)+1(0)$
$=-0.006$
Hence, the unique solution given by $x=A^{-1} B$
$\mathrm{C}_{11}=-0.036$
$\mathrm{C}_{12}=0.27$
$\mathrm{C}_{13}=0$
$\mathrm{C}_{21}=0.27$
$\mathrm{C}_{22}=-0.25$
$\mathrm{C}_{23}=-0.02$
$\mathrm{C}_{31}=0.03$
$\mathrm{C}_{32}=-0.05$
$\mathrm{C}_{33}=0.02$
$\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{|\mathrm{~A}|}(\operatorname{adj} \mathrm{A}) \mathrm{B}$
$\operatorname{Adj} \mathrm{A}=\left[\begin{array}{ccc}-0.036 & 0.27 & 0.03 \\ 0.27 & -0.25 & -0.05 \\ 0.03 & -0.02 & 0.02\end{array}\right]^{\mathrm{T}}=\left[\begin{array}{ccc}-0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02\end{array}\right]$
$X=\frac{1}{-0.006}\left[\begin{array}{ccc}-0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02\end{array}\right]\left[\begin{array}{c}10000 \\ 1310 \\ -190\end{array}\right]$
$X=\frac{1}{-0.006}\left[\begin{array}{l}-12 \\ -18 \\ -30\end{array}\right]$
$\left[\begin{array}{l}\mathrm{x} \\ \mathrm{y} \\ \mathrm{z}\end{array}\right]=\left[\begin{array}{l}2000 \\ 3000 \\ 5000\end{array}\right]$
Hence, $x=$ Rs 2000, $y=$ Rs 3000 and $z=$ Rs 5000

Solve the following systems of homogeneous linear equations by matrix method:

1. $2 x-y+z=0$
$3 x+2 y-z=0$
$x+4 y+3 z=0$

## Solution:

Given
$2 x-y+z=0$
$3 x+2 y-z=0$
$X+4 y+3 z=0$
The system can be written as

$$
\left[\begin{array}{ccc}
2 & -1 & 1 \\
3 & 2 & -1 \\
1 & 4 & 3
\end{array}\right]\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right]
$$

$A X=0$
Now, $|A|=2(6+4)+1(9+1)+1(12-2)$
$|A|=2(10)+10+10$
$|A|=40 \neq 0$
Since, $|A| \neq 0$, hence $x=y=z=0$ is the only solution of this homogeneous equation.
2. $2 x-y+2 z=0$
$5 x+3 y-z=0$
$x+5 y-5 z=0$

## Solution:

Given $2 x-y+2 z=0$
$5 x+3 y-z=0$
$X+5 y-5 z=0$
The system can be written as

$$
\left[\begin{array}{ccc}
2 & -1 & 2 \\
5 & 3 & -1 \\
1 & 5 & -5
\end{array}\right]\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right]
$$

A $X=0$
Now, $|A|=2(-15+5)+1(-25+1)+2(25-3)$
$|A|=-20-24+44$
$|A|=0$
Hence, the system has infinite solutions
Let $\mathrm{z}=\mathrm{k}$
$2 x-y=-2 k$
$5 x+3 y=k$

$$
\left[\begin{array}{cc}
2 & -1 \\
5 & 3
\end{array}\right]\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y}
\end{array}\right]=\left[\begin{array}{c}
-2 \mathrm{k} \\
\mathrm{k}
\end{array}\right]
$$

$$
A X=B
$$

$$
|A|=6+5=11 \neq 0 \text { So, } A^{-1} \text { exist }
$$

$$
\text { Now adj } A=\left[\begin{array}{cc}
3 & -5 \\
1 & 2
\end{array}\right]^{\mathrm{T}}=\left[\begin{array}{cc}
3 & 1 \\
-5 & 2
\end{array}\right]
$$

$$
\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{|\mathrm{~A}|}(\operatorname{adj} \mathrm{A}) \mathrm{B}=\frac{1}{11}\left[\begin{array}{cc}
3 & 1 \\
-5 & 2
\end{array}\right]\left[\begin{array}{c}
-2 \mathrm{k} \\
\mathrm{k}
\end{array}\right]
$$

$$
X=\left[\begin{array}{c}
\frac{-5 \mathrm{k}}{11} \\
\frac{12 \mathrm{k}}{11}
\end{array}\right]
$$

Hence, $x=\frac{-5 k}{11}, y=\frac{12 k}{11}$ and $z=k$
3. $3 x-y+2 z=0$
$4 x+3 y+3 z=0$
$5 x+7 y+4 z=0$

## Solution:

Given $3 x-y+2 z=0$
$4 x+3 y+3 z=0$
$5 x+7 y+4 z=0$
The system can be written as

$$
\left[\begin{array}{ccc}
3 & -1 & 2 \\
4 & 3 & 3 \\
5 & 7 & 4
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right]
$$

$A X=0$

Now, $|A|=3(12-21)+1(16-15)+2(28-15)$
$|A|=-27+1+26$
$|A|=0$
Hence, the system has infinite solutions
Let $\mathrm{z}=\mathrm{k}$
$3 x-y=-2 k$
$4 x+3 y=-3 k$
$\left[\begin{array}{cc}3 & -1 \\ 4 & 3\end{array}\right]\left[\begin{array}{l}x \\ y\end{array}\right]=\left[\begin{array}{c}-2 \mathrm{k} \\ -3 \mathrm{k}\end{array}\right]$
$A X=B$
$|A|=9+4=13 \neq 0$ So, $A^{-1}$ exist
Now $\operatorname{adj} A=\left[\begin{array}{cc}3 & -1 \\ 4 & 3\end{array}\right]^{T}=\left[\begin{array}{cc}3 & 1 \\ -4 & 3\end{array}\right]$
$\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{|\mathrm{~A}|}(\operatorname{adj} \mathrm{A}) \mathrm{B}=\frac{1}{13}\left[\begin{array}{cc}3 & 1 \\ -4 & 3\end{array}\right]\left[\begin{array}{l}-2 \mathrm{k} \\ -3 \mathrm{k}\end{array}\right]$
$X=\left[\begin{array}{c}\frac{-9 \mathrm{k}}{13} \\ \frac{-\mathrm{k}}{13}\end{array}\right]$
Hence, $x=\frac{-9 k}{13}, y=\frac{-k}{13}$ and $z=k$
4. $x+y-6 z=0$
$x-y+2 z=0$
$-3 x+y+2 z=0$

## Solution:

Given $x+y-6 z=0$
$x-y+2 z=0$
$-3 x+y+2 z=0$
The system can be written as

$$
\left[\begin{array}{ccc}
1 & 1 & -6 \\
1 & -1 & 2 \\
-3 & 1 & 2
\end{array}\right]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right]
$$

$A X=0$
Now, $|A|=1(-2-2)-1(2+6)-6(1-3)$
$|A|=-4-8+12$
$|A|=0$
Hence, the system has infinite solutions
Let $\mathrm{z}=\mathrm{k}$
$x+y=6 k$
$x-y=-2 k$
$\left[\begin{array}{cc}1 & 1 \\ 1 & -1\end{array}\right]\left[\begin{array}{l}\mathrm{x} \\ \mathrm{y}\end{array}\right]=\left[\begin{array}{c}6 \mathrm{k} \\ -2 \mathrm{k}\end{array}\right]$
$A X=B$
$|A|=-1-1=-2 \neq 0$ So, $A^{-1}$ exist
Now $\operatorname{adj} \mathrm{A}=\left[\begin{array}{cc}-1 & -1 \\ -1 & 1\end{array}\right]^{\mathrm{T}}=\left[\begin{array}{cc}-1 & -1 \\ -1 & 1\end{array}\right]$
$\mathrm{X}=\mathrm{A}^{-1} \mathrm{~B}=\frac{1}{|\mathrm{~A}|}(\operatorname{adj} \mathrm{A}) \mathrm{B}=\frac{1}{-2}\left[\begin{array}{cc}-1 & -1 \\ -1 & 1\end{array}\right]\left[\begin{array}{c}6 \mathrm{k} \\ -2 \mathrm{k}\end{array}\right]$
$X=\frac{1}{-2}\left[\begin{array}{c}-6 k+2 k \\ -6 k-2 k\end{array}\right]$
$X=\left[\begin{array}{c}-4 \mathrm{k} \\ -8 \mathrm{k}\end{array}\right]$
Hence, $x=2 k, y=4 k$ and $z=k$

