EXERCISE 5.1 PAGE NO: 5.6

1. If a matrix has 8 elements, what are the possible orders it can have? What if it has 5 elements?

Solution:

If a matrix is of order m × n elements, it has m n elements. So, if the matrix has 8 elements, we will find the ordered pairs m and n.

m n = 8

Then, ordered pairs m and n will be

m × n be (8 × 1),(1 × 8),(4 × 2),(2 × 4)

Now, if it has 5 elements

Possible orders are (5 × 1), (1 × 5).

$$2. If\ A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}\ and\ B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}\ then\ find$$

$(i) a_{22} + b_{21}$

$(ii) a_{11} b_{11} + a_{22} b_{22}$

Solution:

(i)

We know that

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} .....(i)$$

$$And\ B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} .....(ii)$$

Also given that

$$A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}\ and\ B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$$

Now, Comparing with equation (1) and (2)

$a_{22} = 4$ and $b_{21} = -3$

$a_{22} + b_{21} = 4 + (-3) = 1$

(ii)

We know that

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \ldots..(i)$$

$$And\ B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} \ldots..(ii)$$

Also given that

$$A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}\ and\ B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$$

Now, Comparing with equation (1) and (2)

$a_{11} = 2$, $a_{22} = 4$, $b_{11} = 2$, $b_{22} = 4$

$a_{11}\, b_{11} + a_{22}\, b_{22} = 2 \times 2 + 4 \times 4 = 4 + 16 = 20$

3. Let A be a matrix of order 3 × 4. If R_1 denotes the first row of A and C_2 denotes its second column, then determine the orders of matrices R_1 and C_2.

Solution:

Given A be a matrix of order 3 × 4.

So, $A = [a_{ij}]_{3\times4}$

R_1 = first row of A = $[a_{11}, a_{12}, a_{13}, a_{14}]$

So, order of matrix $R_1 = 1 \times 4$

C_2 = second column of

$$A = \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}$$

Therefore order of $C_2 = 3 \times 1$

4. Construct a 2 ×3 matrix $A = [a_{jj}]$ whose elements a_{jj} are given by:

(i) $a_{ij} = i \times j$

(ii) $a_{ij} = 2i - j$

(iii) $a_{ij} = i + j$

(iv) $a_{ij} = (i + j)^2/2$

Solution:

(i) Given $a_{ij} = i \times j$

Let $A = [a_{ij}]_{2 \times 3}$

So, the elements in a 2 × 3 matrix are

$[a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}]$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = 1 \times 1 = 1$

$a_{12} = 1 \times 2 = 2$

$a_{13} = 1 \times 3 = 3$

$a_{21} = 2 \times 1 = 2$

$a_{22} = 2 \times 2 = 4$

$a_{23} = 2 \times 3 = 6$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

(ii) Given $a_{ij} = 2i - j$

Let $A = [a_{ij}]_{2\times3}$

So, the elements in a 2 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = 2 \times 1 - 1 = 2 - 1 = 1$

$a_{12} = 2 \times 1 - 2 = 2 - 2 = 0$

$a_{13} = 2 \times 1 - 3 = 2 - 3 = -1$

$a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$

$a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$

$a_{23} = 2 \times 2 - 3 = 4 - 3 = 1$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

(iii) Given $a_{ij} = i + j$

Let $A = [a_{ij}]_{2\times3}$

So, the elements in a 2 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = 1 + 1 = 2$

$a_{12} = 1 + 2 = 3$

$a_{13} = 1 + 3 = 4$

$a_{21} = 2 + 1 = 3$

$a_{22} = 2 + 2 = 4$

$a_{23} = 2 + 3 = 5$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

(iv) Given $a_{ij} = (i + j)^2/2$

Let $A = [a_{ij}]_{2\times3}$

So, the elements in a 2 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Let $A = [a_{ij}]_{2\times3}$

So, the elements in a 2 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = \frac{(1+1)^2}{2} = \frac{2^2}{2} = \frac{4}{2} = 2$

$a_{12} = \frac{(1+2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{13} = \frac{(1+3)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$

$a_{21} = \frac{(2+1)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{22} = \frac{(2+2)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$

$a_{23} = \frac{(2+3)^2}{2} = \frac{5^2}{2} = \frac{25}{2} = 12.5$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 2 & 4.5 & 8 \\ 4.5 & 8 & 12.5 \end{bmatrix}$$
$$A = \begin{bmatrix} 2 & \frac{9}{2} & 8 \\ \frac{9}{2} & 8 & \frac{25}{2} \end{bmatrix}$$

5. Construct a 2 × 2 matrix A = [a_{ij}] whose elements a_{ij} are given by:
(i) $(i + j)^2/2$
(ii) $a_{ij} = (i - j)^2/2$
(iii) $a_{ij} = (i - 2j)^2/2$
(iv) $a_{ij} = (2i + j)^2/2$
(v) $a_{ij} = |2i – 3j|/2$
(vi) $a_{ij} = |-3i + j|/2$
(vii) $a_{ij} = e^{2ix} \sin x j$

Solution:
(i) Given $(i + j)^2/2$
Let $A = [a_{ij}]_{2\times2}$
So, the elements in a 2 × 2 matrix are
a_{11}, a_{12}, a_{21}, a_{22}
$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$
$a_{11} = \frac{(1+1)^2}{2} = \frac{2^2}{2} = \frac{4}{2} = 2$
$a_{12} = \frac{(1+2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$
$a_{21} = \frac{(2+1)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$
$a_{22} = \frac{(2+2)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$
Substituting these values in matrix A we get,
$$A = \begin{bmatrix} 2 & 4.5 \\ 4.25 & 8 \end{bmatrix}$$
$$A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$$

(ii) Given $a_{ij} = (i - j)^2/2$
Let $A = [a_{ij}]_{2\times2}$
So, the elements in a 2 × 2 matrix are

a_{11}, a_{12}, a_{21}, a_{22}

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = \frac{(1-1)^2}{2} = \frac{0^2}{2} = 0$

$a_{12} = \frac{(1-2)^2}{2} = \frac{1^2}{2} = \frac{1}{2} = 0.5$

$a_{21} = \frac{(2-1)^2}{2} = \frac{1^2}{2} = \frac{1}{2} = 0.5$

$a_{22} = \frac{(2-2)^2}{2} = \frac{0^2}{2} = 0$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$$

(iii) Given $a_{ij} = (i - 2j)^2/2$

Let $A = [a_{ij}]_{2\times 2}$

So, the elements in a 2 × 2 matrix are

a_{11}, a_{12}, a_{21}, a_{22}

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = \frac{(1-2\times 1)^2}{2} = \frac{1^2}{2} = 0.5$

$a_{12} = \frac{(1-2\times 2)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{21} = \frac{(2-2\times 1)^2}{2} = \frac{0^2}{2} = 0$

$a_{22} = \frac{(2-2\times 2)^2}{2} = \frac{2^2}{2} = \frac{4}{2} = 2$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 0.5 & 4.5 \\ 0 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{bmatrix}$$

(iv) Given $a_{ij} = (2i + j)^2/2$

Let $A = [a_{ij}]_{2\times2}$

So, the elements in a 2 × 2 matrix are

a_{11}, a_{12}, a_{21}, a_{22}

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = \frac{(2\times1+1)^2}{2} = \frac{3^2}{2} = \frac{9}{2} = 4.5$

$a_{12} = \frac{(2\times1+2)^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$

$a_{21} = \frac{(2\times2+1)^2}{2} = \frac{5^2}{2} = \frac{25}{2} = 12.5$

$a_{22} = \frac{(2\times2+2)^2}{2} = \frac{6^2}{2} = \frac{36}{2} = 18$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 4.5 & 8 \\ 12.5 & 18 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{9}{2} & 8 \\ \frac{25}{2} & 18 \end{bmatrix}$$

(v) Given $a_{ij} = |2i - 3j|/2$

Let $A = [a_{ij}]_{2\times2}$

So, the elements in a 2×2 matrix are

a_{11}, a_{12}, a_{21}, a_{22}

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = \frac{|2\times1-3\times1|}{2} = \frac{1}{2} = 0.5$

$a_{12} = \frac{|2\times1-3\times2|}{2} = \frac{4}{2} = 2$

$a_{21} = \frac{|2\times2-3\times1|}{2} = \frac{4-3}{2} = \frac{1}{2} = 0.5$

$a_{22} = \frac{|2\times2-3\times2|}{2} = \frac{2}{2} = 1$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 0.5 & 2 \\ 0.5 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 1 \end{bmatrix}$$

(vi) Given $a_{ij} = |-3i + j|/2$

Let $A = [a_{ij}]_{2\times2}$

So, the elements in a 2 × 2 matrix are

$a_{11}, a_{12}, a_{21}, a_{22}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = \frac{|-3\times1+1|}{2} = \frac{2}{2} = 1$

$a_{12} = \frac{|-3\times1+2|}{2} = \frac{1}{2} = 0.5$

$a_{21} = \frac{|-3\times2+1|}{2} = \frac{5}{2} = 2.5$

$a_{22} = \frac{|-3\times2+2|}{2} = \frac{4}{2} = 2$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 1 & 0.5 \\ 2.5 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$$

(vii) Given $a_{ij} = e^{2ix} \sin x\, j$

Let $A = [a_{ij}]_{2\times2}$

So, the elements in a 2 × 2 matrix are

$a_{11}, a_{12}, a_{21}, a_{22},$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$a_{11} = e^{2\times1x} \sin x \times 1 = e^{2x} \sin x$

$a_{12} = e^{2\times1x} \sin x \times 2 = e^{2x} \sin 2x$

$a_{21} = e^{2\times2x} \sin x \times 1 = e^{4x} \sin x$

$a_{22} = e^{2\times2x} \sin x \times 2 = e^{4x} \sin 2x$

Substituting these values in matrix A we get,

$$A = \begin{bmatrix} e^{2x} sinx & e^{2x} sin2x \\ e^{4x} sinx & e^{4x} sin2x \end{bmatrix}$$

6. Construct a 3×4 matrix A = $[a_{ij}]$ whose elements a_{ij} are given by:

(i) $a_{ij} = i + j$
(ii) $a_{ij} = i - j$
(iii) $a_{ij} = 2i$
(iv) $a_{ij} = j$
(v) $a_{ij} = ½ \ |-3i + j|$

Solution:

(i) Given $a_{ij} = i + j$

Let $A = [a_{ij}]_{2\times3}$

So, the elements in a 3 × 4 matrix are

$a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

$A = \begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$

$a_{11} = 1 + 1 = 2$
$a_{12} = 1 + 2 = 3$
$a_{13} = 1 + 3 = 4$
$a_{14} = 1 + 4 = 5$
$a_{21} = 2 + 1 = 3$
$a_{22} = 2 + 2 = 4$
$a_{23} = 2 + 3 = 5$
$a_{24} = 2 + 4 = 6$
$a_{31} = 3 + 1 = 4$
$a_{32} = 3 + 2 = 5$
$a_{33} = 3 + 3 = 6$
$a_{34} = 3 + 4 = 7$

Substituting these values in matrix A we get,

$A = \begin{bmatrix} 2 & \cdots & 5 \\ \vdots & \ddots & \vdots \\ 4 & \cdots & 7 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$$

(ii) Given $a_{ij} = i - j$

Let $A = [a_{ij}]_{2\times3}$

So, the elements in a 3×4 matrix are

a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}

A = $\begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$

$a_{11} = 1 - 1 = 0$

$a_{12} = 1 - 2 = -1$

$a_{13} = 1 - 3 = -2$

$a_{14} = 1 - 4 = -3$

$a_{21} = 2 - 1 = 1$

$a_{22} = 2 - 2 = 0$

$a_{23} = 2 - 3 = -1$

$a_{24} = 2 - 4 = -2$

$a_{31} = 3 - 1 = 2$

$a_{32} = 3 - 2 = 1$

$a_{33} = 3 - 3 = 0$

$a_{34} = 3 - 4 = -1$

Substituting these values in matrix A we get,

A = $\begin{bmatrix} 0 & \cdots & -3 \\ \vdots & \ddots & \vdots \\ 2 & \cdots & -1 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \end{bmatrix}$$

(iii) Given $a_{ij} = 2i$

Let A = $[a_{ij}]_{2\times 3}$

So, the elements in a 3×4 matrix are

a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}

A = $\begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$

$a_{11} = 2\times 1 = 2$

$a_{12} = 2\times 1 = 2$

$a_{13} = 2\times 1 = 2$

$a_{14} = 2\times 1 = 2$

$a_{21} = 2\times 2 = 4$

$a_{22} = 2\times2 = 4$
$a_{23} = 2\times2 = 4$
$a_{24} = 2\times2 = 4$
$a_{31} = 2\times3 = 6$
$a_{32} = 2\times3 = 6$
$a_{33} = 2\times3 = 6$
$a_{34} = 2\times3 = 6$
Substituting these values in matrix A we get,

$$A = \begin{bmatrix} 2 & \cdots & 2 \\ \vdots & \ddots & \vdots \\ 6 & \cdots & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \\ 6 & 6 & 6 & 6 \end{bmatrix}$$

(iv) Given $a_{ij} = j$
Let $A = [a_{ij}]_{2\times3}$
So, the elements in a 3×4 matrix are
$a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$$

$a_{11} = 1$
$a_{12} = 2$
$a_{13} = 3$
$a_{14} = 4$
$a_{21} = 1$
$a_{22} = 2$
$a_{23} = 3$
$a_{24} = 4$
$a_{31} = 1$
$a_{32} = 2$
$a_{33} = 3$
$a_{34} = 4$
Substituting these values in matrix A we get,

A = $\begin{bmatrix} 1 & \cdots & 4 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 4 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

(vi) Given a_{ij} = ½ |-3i + j|

Let A = $[a_{ij}]_{2\times3}$

So, the elements in a 3×4 matrix are

$a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}, a_{31}, a_{32}, a_{33}, a_{34}$

A = $\begin{bmatrix} a_{11} & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{34} \end{bmatrix}$

$a_{11} = \frac{1}{2}(-3 \times 1 + 1) = \frac{1}{2}(-3 + 1) = \frac{1}{2}(-2) = -1$

$a_{12} = \frac{1}{2}(-3 \times 1 + 2) = \frac{1}{2}(-3 + 2) = \frac{1}{2}(-1) = -\frac{1}{2}$

$a_{13} = \frac{1}{2}(-3 \times 1 + 3) = \frac{1}{2}(-3 + 3) = \frac{1}{2}(0) = 0$

$a_{14} = \frac{1}{2}(-3 \times 1 + 4) = \frac{1}{2}(-3 + 4) = \frac{1}{2}(1) = \frac{1}{2}$

$a_{21} = \frac{1}{2}(-3 \times 2 + 1) = \frac{1}{2}(-6 + 1) = \frac{1}{2}(-5) = -\frac{5}{2}$

$a_{22} = \frac{1}{2}(-3 \times 2 + 2) = \frac{1}{2}(-6 + 2) = \frac{1}{2}(-4) = -2$

$a_{23} = \frac{1}{2}(-3 \times 2 + 3) = \frac{1}{2}(-6 + 3) = \frac{1}{2}(-3) = -\frac{3}{2}$

$a_{24} = \frac{1}{2}(-3 \times 2 + 4) = \frac{1}{2}(-6 + 4) = \frac{1}{2}(-2) = -1$

$a_{31} = \frac{1}{2}(-3 \times 3 + 1) = \frac{1}{2}(-9 + 1) = \frac{1}{2}(-8) = -4$

$a_{32} = \frac{1}{2}(-3 \times 3 + 2) = \frac{1}{2}(-9 + 2) = \frac{1}{2}(-7) = -\frac{7}{2}$

$a_{33} = \frac{1}{2}(-3 \times 3 + 3) = \frac{1}{2}(-9 + 3) = \frac{1}{2}(-6) = -3$

$a_{34} = \frac{1}{2}(-3 \times 3 + 4) = \frac{1}{2}(-9 + 4) = \frac{1}{2}(-5) = -\frac{5}{2}$

Substituting these values in matrix A we get,

A = $\begin{bmatrix} -1 & \cdots & \frac{1}{2} \\ \vdots & \ddots & \vdots \\ -4 & \cdots & -\frac{5}{2} \end{bmatrix}$

Multiplying by negative sign we get,

7. Construct a 4 × 3 matrix A = [a_{ij}] whose elements a_{ij} are given by:
(i) $a_{ij} = 2i + i/j$
(ii) $a_{ij} = (i - j)/ (i + j)$
(iii) $a_{ij} = i$

Solution:

(i) Given $a_{ij} = 2i + i/j$

Let A = $[a_{ij}]_{4\times3}$

So, the elements in a 4 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, a_{41}, a_{42}, a_{43}$

A = $\begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{41} & \cdots & a_{43} \end{bmatrix}$

$a_{11} = 2\times1 + \frac{1}{1} = 2 + 1 = 3$

$a_{12} = 2\times1 + \frac{1}{2} = 2 + \frac{1}{2} = \frac{5}{2}$

$a_{13} = 2\times1 + \frac{1}{3} = 2 + \frac{1}{3} = \frac{7}{3}$

$a_{21} = 2\times2 + \frac{2}{1} = 4 + 2 = 6$

$a_{22} = 2\times2 + \frac{2}{2} = 4 + 1 = 5$

$a_{23} = 2\times2 + \frac{2}{3} = 4 + \frac{2}{3} = \frac{14}{3}$

$a_{31} = 2\times3 + \frac{3}{1} = 6 + 3 = 9$

$a_{32} = 2\times3 + \frac{3}{2} = 6 + \frac{3}{2} = \frac{15}{2}$

$a_{33} = 2\times3 + \frac{3}{3} = 6 + 1 = 7$

$a_{41} = 2\times4 + \frac{4}{1} = 8 + 4 = 12$

$a_{42} = 2 \times 4 + \frac{4}{2} = 8 + 2 = 10$

$a_{43} = 2 \times 4 + \frac{4}{3} = 8 + \frac{4}{3} = \frac{28}{3}$

Substituting these values in matrix A we get,

$A = \begin{bmatrix} 3 & \cdots & \frac{7}{3} \\ \vdots & \ddots & \vdots \\ 12 & \cdots & \frac{28}{3} \end{bmatrix}$

$$A = \begin{bmatrix} 3 & \frac{5}{2} & \frac{7}{3} \\ 6 & 5 & \frac{14}{3} \\ 9 & \frac{15}{2} & 7 \\ 12 & 10 & \frac{28}{3} \end{bmatrix}$$

(ii) Given $a_{ij} = (i - j)/ (i + j)$

Let $A = [a_{ij}]_{4\times3}$

So, the elements in a 4 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, a_{41}, a_{42}, a_{43}$

$A = \begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{41} & \cdots & a_{43} \end{bmatrix}$

$a_{11} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

$a_{12} = \frac{1-2}{1+2} = \frac{-1}{3}$

$a_{13} = \frac{1-3}{1+3} = \frac{-2}{4} = -\frac{1}{2}$

$a_{21} = \frac{2-1}{2+1} = \frac{1}{3}$

$a_{22} = \frac{2-2}{2+2} = \frac{0}{4} = 0$

$a_{23} = \frac{2-3}{2+3} = \frac{-1}{5}$

$a_{31} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$

$a_{32} = \frac{3-2}{3+2} = \frac{1}{5}$

$a_{33} = \frac{3-3}{3+3} = \frac{0}{6} = 0$

$a_{41} = \frac{4-1}{4+1} = \frac{3}{5}$

$a_{42} = \frac{4-2}{4+2} = \frac{2}{6} = \frac{1}{3}$

$a_{43} = \frac{4-3}{4+3} = \frac{1}{7}$

Substituting these values in matrix A we get,

$A = \begin{bmatrix} 0 & \cdots & -\frac{1}{2} \\ \vdots & \ddots & \vdots \\ \frac{3}{5} & \cdots & \frac{1}{7} \end{bmatrix}$

$$A = \begin{bmatrix} 0 & \frac{-1}{3} & \frac{-1}{2} \\ \frac{1}{3} & 0 & \frac{-1}{5} \\ \frac{1}{2} & \frac{1}{5} & 0 \\ \frac{3}{5} & \frac{1}{3} & \frac{1}{7} \end{bmatrix}$$

(iii) Given $a_{ij} = i$

Let $A = [a_{ij}]_{4\times3}$

So, the elements in a 4 × 3 matrix are

$a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, a_{41}, a_{42}, a_{43}$

$A = \begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{41} & \cdots & a_{43} \end{bmatrix}$

$a_{11} = 1$

$a_{12} = 1$

$a_{13} = 1$

$a_{21} = 2$

$a_{22} = 2$

$a_{23} = 2$

$a_{31} = 3$

$a_{32} = 3$

$a_{33} = 3$

$a_{41} = 4$

$a_{42} = 4$

$a_{43} = 4$

Substituting these values in matrix A we get,

$A = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 4 & \cdots & 4 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$$

8. Find x, y, a and b if

$$\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$$

Given that two matrices are equal.

We know that if two matrices are equal then the elements of each matrices are also equal.

Therefore by equating them we get,

3x + 4y = 2 …… (1)

x – 2y = 4 …… (2)

a + b = 5 …… (3)

2a – b = – 5 …… (4)

Multiplying equation (2) by 2 and adding to equation (1), we get

3x + 4y + 2x – 4y = 2 + 8

⇒ 5x = 10

⇒ x = 2

Now, substituting the value of x in equation (1)

3 × 2 + 4y = 2

⇒ 6 + 4y = 2

⇒ 4y = 2 – 6

⇒ 4y = – 4

⇒ y = – 1

Now by adding equation (3) and (4)

a + b + 2a – b = 5 + (– 5)

⇒ 3a = 5 – 5 = 0

⇒ a = 0

Now, again by substituting the value of a in equation (3), we get

0 + b = 5

⇒ b = 5
∴ a = 0, b = 5, x = 2 and y = – 1

9. Find x, y, a and b if
$$\begin{bmatrix} 2x-3y & a-b & 3 \\ 1 & x+4y & 3a+4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$

Solution:
$$\begin{bmatrix} 2x-3y & a-b & 3 \\ 1 & x+4y & 3a+4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$
We know that if two matrices are equal then the elements of each matrices are also equal.
Given that two matrices are equal.
Therefore by equating them we get,
2x - 3y = 1 …… (1)
And a – b = – 2 …… (2)
And x + 4y = 6 …… (3)
3a + 4b = 29 …… (4)
Multiplying equation (3) by 2 and subtract equation (1) from equation (3)
2x + 8y - 2x + 3y = 12 - 1
⇒ 11y = 11
⇒ y = 1
Now, substituting the value of y in equation (1)
2x - 3 × 1 = 1
⇒ 2x - 3 = 1
⇒ 2x = 1 + 3
⇒ 2x = 4
⇒ x = 2
Multiplying equation (2) by 3 and subtract equation (2) from equation (4)
⇒ 3a + 4b - 3a + 3b = 29 - (-6)
⇒ 7b = 35
⇒ b = 5
Now, substituting the value of b in equation (2)
a - 5 = - 2
⇒ a = - 2 + 5
⇒ a = 3

∴ x = 2, y = 1, a = 3 and b = 5

10. Find the values of a, b, c and d from the following equations:

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

We know that if two matrices are equal then the elements of each matrices are also equal.

Given that two matrices are equal.

Therefore by equating them we get,

2a + b = 4 …… (1)

And a – 2b = – 3 …… (2)

And 5c – d = 11 …… (3)

4c + 3d = 24 …… (4)

Multiplying equation (1) by 2 and adding to equation (2)

4a + 2b + a – 2b = 8 – 3

⇒ 5a = 5

⇒ a = 1

Now, substituting the value of a in equation (1)

2 × 1 + b = 4

⇒ 2 + b = 4

⇒ b = 4 – 2

⇒ b = 2

Multiplying equation (3) by 3 and adding to equation (4)

15c – 3d + 4c + 3d = 33 + 24

⇒ 19c = 57

⇒ c = 3

Now, substituting the value of c in equation (4)

4 × 3 + 3d = 24

⇒ 12 + 3d = 24

⇒ 3d = 24 – 12

⇒ 3d = 12

⇒ d = 4

∴ $a = 1$, $b = 2$, $c = 3$ and $d = 4$

EXERCISE 5.2 PAGE NO: 5.18

1. Compute the following sums:

$(i)\ \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$

$(ii)\ \begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix}$

Solution:

(i) Given

$\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$

Corresponding elements of two matrices should be added

Therefore, we get

$= \begin{bmatrix} 3-2 & -2+4 \\ 1+1 & 4+3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$

Therefore, $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$

(ii) Given

$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2+1 & 1-2 & 3+3 \\ 0+2 & 3+6 & 5+1 \\ -1+0 & 2-3 & 5+1 \end{bmatrix}$

$= \begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$

Therefore,

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix}+\begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix}=\begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$$

$$2.Let\ \ A=\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\ \ B=\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\ \ and\ C=\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}.$$

Find each of the following:

(i) 2A – 3B

(ii) B – 4C

(iii) 3A – C

(iv) 3A – 2B + 3C

Solution:

(i) Given

$$A=\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\ \ B=\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\ \ and\ C=\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}.$$

First we have to compute 2A

$$2A=2\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}=\begin{bmatrix} 4 & 8 \\ 6 & 4 \end{bmatrix}$$

Now by computing 3B we get,

$$= 3B=3\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}=\begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix}$$

Now by we have to compute 2A – 3B we get

$$= 2A-3B=\begin{bmatrix} 4 & 8 \\ 6 & 4 \end{bmatrix}-\begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix}=\begin{bmatrix} 4-3 & 8-9 \\ 6+6 & 4-15 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$$

Therefore

$$2A-3B=\begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$$

(ii) Given $A=\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\ \ B=\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\ \ and\ C=\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}.$

First we have to compute 4C,

$4C=4\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}=\begin{bmatrix}-8 & 20\\ 12 & 16\end{bmatrix}$

Now,

$B-4C=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}-\begin{bmatrix}-8 & 20\\ 12 & 16\end{bmatrix}$

$=\begin{bmatrix}1+8 & 3-20\\ -2-12 & 5-16\end{bmatrix}=\begin{bmatrix}9 & -17\\ -14 & -11\end{bmatrix}$

Therefore we get,

$B-4C=\begin{bmatrix}9 & -17\\ -14 & -11\end{bmatrix}$

(iii) Given

$$A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}\ B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}\ and\ C=\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}.$$

First we have to compute 3A,

$3A=3\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}=\begin{bmatrix}6 & 12\\ 9 & 6\end{bmatrix}$

Now,

$=3A-C=\begin{bmatrix}6 & 12\\ 9 & 6\end{bmatrix}-\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}$

$=\begin{bmatrix}6+2 & 12-5\\ 9-3 & 6-4\end{bmatrix}=\begin{bmatrix}8 & 7\\ 6 & 2\end{bmatrix}$

Therefore,

$3A-C=\begin{bmatrix}8 & 7\\ 6 & 2\end{bmatrix}$

(iv) Given

$$A=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}\ B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}\ and\ C=\begin{bmatrix}-2 & 5\\ 3 & 4\end{bmatrix}.$$

First we have to compute 3A

$3A=3\begin{bmatrix}2 & 4\\3 & 2\end{bmatrix}=\begin{bmatrix}6 & 12\\9 & 6\end{bmatrix}$

Now we have to compute 2B

$= 2B=2\begin{bmatrix}1 & 3\\-2 & 5\end{bmatrix}=\begin{bmatrix}2 & 6\\-4 & 10\end{bmatrix}$

By computing 3C we get,

$= 3C=3\begin{bmatrix}-2 & 5\\3 & 4\end{bmatrix}=\begin{bmatrix}-6 & 15\\9 & 12\end{bmatrix}$

$= 3A-2B+3C=\begin{bmatrix}6 & 12\\9 & 6\end{bmatrix}-\begin{bmatrix}2 & 6\\-4 & 10\end{bmatrix}+\begin{bmatrix}-6 & 15\\9 & 12\end{bmatrix}$

$=\begin{bmatrix}6-2-6 & 12-6+15\\9+4+9 & 6-10+12\end{bmatrix}=\begin{bmatrix}-2 & 21\\22 & 8\end{bmatrix}$

Therefore,

$3A-2B+3C=\begin{bmatrix}-2 & 21\\22 & 8\end{bmatrix}$

$3. If\ A=\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix}, B=\begin{bmatrix}-1 & 0 & 2\\3 & 4 & 1\end{bmatrix}, C=\begin{bmatrix}-1 & 2 & 3\\2 & 1 & 0\end{bmatrix}, find$

(i) A + B and B + C

(ii) 2B + 3A and 3C – 4B

Solution:

(i) Consider A + B,

A + B is not possible because matrix A is an order of 2 x 2 and Matrix B is an order of 2 x 3, so the Sum of the matrix is only possible when their order is same.

Now consider B + C

$\Rightarrow B+C=\begin{bmatrix}-1 & 0 & 2\\3 & 4 & 1\end{bmatrix}+\begin{bmatrix}-1 & 2 & 3\\2 & 1 & 0\end{bmatrix}$

$\Rightarrow B+C=\begin{bmatrix}-1-1 & 0+2 & 2+3\\3+2 & 4+1 & 1+0\end{bmatrix}$

$\Rightarrow B+C=\begin{bmatrix}-2 & 2 & 5\\5 & 5 & 1\end{bmatrix}$

(ii) Consider 2B + 3A

2B + 3A also does not exist because the order of matrix B and matrix A is different, so we cannot find the sum of these matrix.

Now consider 3C – 4B,

$$\Rightarrow 3C-4B=3\begin{bmatrix}-1 & 2 & 3\\ 2 & 1 & 0\end{bmatrix}-4\begin{bmatrix}-1 & 0 & 2\\ 3 & 4 & 1\end{bmatrix}$$

$$\Rightarrow 3C-4B=\begin{bmatrix}-3 & 6 & 9\\ 6 & 3 & 0\end{bmatrix}-\begin{bmatrix}-4 & 0 & 8\\ 12 & 16 & 4\end{bmatrix}$$

$$\Rightarrow 3C-4B=\begin{bmatrix}-3+4 & 6-0 & 9-8\\ 6-12 & 3-16 & 0-4\end{bmatrix}$$

$$\Rightarrow 3C-4B=\begin{bmatrix}1 & 6 & 1\\ -6 & -13 & -4\end{bmatrix}$$

$4. Let\ A=\begin{bmatrix}-1 & 0 & 2\\ 3 & 1 & 4\end{bmatrix}, B=\begin{bmatrix}0 & -2 & 5\\ 1 & -3 & 1\end{bmatrix}\ and\ C=\begin{bmatrix}1 & -5 & 2\\ 6 & 0 & -4\end{bmatrix}. Compute\ 2A-3B+4C$

Solution:

Given

$$A=\begin{bmatrix}-1 & 0 & 2\\ 3 & 1 & 4\end{bmatrix}, B=\begin{bmatrix}0 & -2 & 5\\ 1 & -3 & 1\end{bmatrix}\ and\ C=\begin{bmatrix}1 & -5 & 2\\ 6 & 0 & -4\end{bmatrix}$$

Now we have to compute 2A – 3B + 4C

$$2A-3B+4C=2\begin{bmatrix}-1 & 0 & 2\\ 3 & 1 & 4\end{bmatrix}-3\begin{bmatrix}0 & -2 & 5\\ 1 & -3 & 1\end{bmatrix}+4\begin{bmatrix}1 & -5 & 2\\ 6 & 0 & -4\end{bmatrix}$$

$$\Rightarrow 2A-3B+4C=\begin{bmatrix}-2 & 0 & 4\\ 6 & 2 & 8\end{bmatrix}-\begin{bmatrix}0 & -6 & 15\\ 3 & -9 & 3\end{bmatrix}+\begin{bmatrix}4 & -20 & 8\\ 24 & 0 & -16\end{bmatrix}$$

$$\Rightarrow 2A-3B+4C=\begin{bmatrix}-2-0+4 & 0+6-20 & 4-15+8\\ 6-3+24 & 2+9+0 & 8-3-16\end{bmatrix}$$

$$\Rightarrow 2A-3B+4C=\begin{bmatrix}2 & -14 & -3\\ 27 & 11 & -11\end{bmatrix}$$

5. If A = diag (2 -5 9), B = diag (1 1 -4) and C = diag (-6 3 4), find
(i) A – 2B
(ii) B + C - 2A
(iii) 2A + 3B – 5C

Solution:
(i) Given A = diag (2 -5 9), B = diag (1 1 -4) and C = diag (-6 3 4)

Here,

$$A=\begin{bmatrix}2 & 0 & 0\\ 0 & -5 & 0\\ 0 & 0 & 9\end{bmatrix}$$

$$B=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -4\end{bmatrix}$$

$A-2B$

$$\Rightarrow A-2B=\begin{bmatrix}2 & 0 & 0\\ 0 & -5 & 0\\ 0 & 0 & 9\end{bmatrix}-2\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 4\end{bmatrix}$$

$$\Rightarrow A-2B=\begin{bmatrix}2 & 0 & 0\\ 0 & -5 & 0\\ 0 & 0 & 9\end{bmatrix}-\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & -8\end{bmatrix}$$

$$\Rightarrow A-2B=\begin{bmatrix}0 & 0 & 0\\ 0 & -7 & 0\\ 0 & 0 & 17\end{bmatrix}=\text{diag }(0\ -7\ 17)$$

(ii) Given A = diag (2 -5 9), B = diag (1 1 -4) and C = diag (-6 3 4)
We have to find B + C - 2A

Here,

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix} \quad C = \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Now we have to compute B + C - 2A

$$\Rightarrow B + C - 2A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix} + \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} - 2\begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow B + C - 2A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix} + \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & 18 \end{bmatrix}$$

$$\Rightarrow B + C - 2A = \begin{bmatrix} 1-6-4 & 0+0-0 & 0+0-0 \\ 0+0-0 & 1+3+10 & 0+0-0 \\ 0+0-0 & 0+0-0 & -4+4-18 \end{bmatrix}$$

$$\Rightarrow B + C - 2A = \begin{bmatrix} -9 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & -18 \end{bmatrix} = diag(-9\ 14 - 18)$$

(iii) Given A = diag (2 -5 9), B = diag (1 1 -4) and C = diag (-6 3 4)
Now we have to find 2A + 3B – 5C
Here,

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$\text{and } C = \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Now consider 2A + 3B – 5C

$$\Rightarrow 2A+3B-5C=2\begin{bmatrix}2&0&0\\0&-5&0\\0&0&9\end{bmatrix}+3\begin{bmatrix}1&0&0\\0&1&0\\0&0&-4\end{bmatrix}-5\begin{bmatrix}-6&0&0\\0&3&0\\0&0&4\end{bmatrix}$$

$$\Rightarrow 2A+3B-5C=\begin{bmatrix}4&0&0\\0&-10&0\\0&0&18\end{bmatrix}+\begin{bmatrix}3&0&0\\0&3&0\\0&0&-12\end{bmatrix}-\begin{bmatrix}-30&0&0\\0&15&0\\0&0&20\end{bmatrix}$$

$$\Rightarrow 2A+3B-5C=\begin{bmatrix}4+3+30&0+0-0&0+0-0\\0+0-0&-10+3-15&0+0-0\\0+0-0&0+0-0&18-12-20\end{bmatrix}$$

$$\Rightarrow 2A+3B-5C=\begin{bmatrix}37&0&0\\0&-22&0\\0&0&-14\end{bmatrix}$$

$=\text{diag}(37\ -22\ -14)$

6. Given the matrices

$$A=\begin{bmatrix}2&1&1\\3&-1&0\\0&2&4\end{bmatrix},B=\begin{bmatrix}9&7&-1\\3&5&4\\2&1&6\end{bmatrix}\text{ and } C=\begin{bmatrix}2&-4&3\\1&-1&0\\9&4&5\end{bmatrix}$$

Verify that (A + B) + C = A + (B + C)

Solution:

Given

$$A=\begin{bmatrix}2&1&1\\3&-1&0\\0&2&4\end{bmatrix},B=\begin{bmatrix}9&7&-1\\3&5&4\\2&1&6\end{bmatrix}\text{ and } C=\begin{bmatrix}2&-4&3\\1&-1&0\\9&4&5\end{bmatrix}$$

Now we have to verify (A + B) + C = A + (B + C)

First consider LHS, (A + B) + C,

$$=\left(\begin{bmatrix}2&1&1\\3&-1&0\\0&2&4\end{bmatrix}+\begin{bmatrix}9&7&-1\\3&5&4\\2&1&6\end{bmatrix}\right)+\begin{bmatrix}2&-4&3\\1&-1&0\\9&4&5\end{bmatrix}$$

$$=\left(\begin{bmatrix}2+9&1+7&1-1\\3+3&-1+5&0+4\\0+2&2+1&4+6\end{bmatrix}\right)+\begin{bmatrix}2&-4&3\\1&-1&0\\9&4&5\end{bmatrix}$$

$$= \begin{bmatrix} 11 & 8 & 0 \\ 6 & 4 & 4 \\ 2 & 3 & 10 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 11+2 & 8-4 & 0+3 \\ 6+1 & 4-1 & 4+0 \\ 2+9 & 3+4 & 10+5 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 4 & 3 \\ 7 & 3 & 4 \\ 11 & 7 & 15 \end{bmatrix}$$

Now consider RHS, that is A + (B + C)

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \left(\begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \left(\begin{bmatrix} 9+2 & 7-4 & -1+3 \\ 3+1 & 5-1 & 4+0 \\ 2+9 & 1+4 & 6+5 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 11 & 3 & 2 \\ 4 & 4 & 4 \\ 11 & 5 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 2+11 & 1+3 & 1+2 \\ 3+4 & -1+4 & 0+4 \\ 0+11 & 2+5 & 4+11 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 4 & 3 \\ 7 & 3 & 4 \\ 11 & 7 & 15 \end{bmatrix}$$

Therefore LHS = RHS

Hence (A + B) + C = A + (B + C)

7. Find the matrices X and Y,

$$\textbf{if } \mathbf{X} + \mathbf{Y} = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} \textbf{ and } \mathbf{X} - \mathbf{Y} = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

Solution:

Consider,

$$(X+Y)+(X-Y)=\begin{bmatrix}5 & 2\\0 & 9\end{bmatrix}+\begin{bmatrix}3 & 6\\0 & -1\end{bmatrix}$$

Now by simplifying we get,

$$\Rightarrow 2X=\begin{bmatrix}5+3 & 2+6\\0+0 & 9-1\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}8 & 8\\0 & 8\end{bmatrix}$$

$$\Rightarrow X=\frac{1}{2}\begin{bmatrix}8 & 8\\0 & 8\end{bmatrix}$$

Therefore,

$$\Rightarrow X=\begin{bmatrix}4 & 4\\0 & 4\end{bmatrix}$$

Again consider,

$$(X+Y)-(X-Y)=\begin{bmatrix}5 & 2\\0 & 9\end{bmatrix}-\begin{bmatrix}3 & 6\\0 & -1\end{bmatrix}$$

$$\Rightarrow X+Y-X+Y=\begin{bmatrix}5-3 & 2-6\\0-0 & 9+1\end{bmatrix}$$

Now by simplifying we get,

$$\Rightarrow 2Y=\begin{bmatrix}2 & -4\\0 & 10\end{bmatrix}$$

$$\Rightarrow Y=\frac{1}{2}\begin{bmatrix}2 & -4\\0 & 10\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}1 & -2\\0 & 5\end{bmatrix}$$

Therefore,

$$X=\begin{bmatrix}4 & 4\\0 & 4\end{bmatrix}\text{ and }Y=\begin{bmatrix}1 & -2\\0 & 5\end{bmatrix}$$

8.Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

Solution:

Given

$$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

Now by transposing, we get

$$\Rightarrow 2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

Therefore,

$$\Rightarrow X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

9.Find matrices X and Y, if $2X - Y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ and $X + 2Y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$.

Solution:

Given

$$(2X - Y) = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} \dots (1)$$

$$(X + 2Y) = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix} \dots (2)$$

Now by multiplying equation (1) and (2) we get,

$$2(2X - Y) = 2\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$$

$$\Rightarrow 4X-2Y=\begin{bmatrix}12 & -12 & 0\\ -8 & 4 & 2\end{bmatrix}\ldots(3)$$

Now by adding equation (2) and (3) we get,

$$(4X-2Y)+(X+2Y)=\begin{bmatrix}12 & -12 & 0\\ -8 & 4 & 2\end{bmatrix}+\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}$$

$$\Rightarrow 5X=\begin{bmatrix}12+3 & -12+2 & 0+5\\ -8-2 & 4+1 & 2-7\end{bmatrix}$$

$$\Rightarrow 5X=\begin{bmatrix}15 & -10 & 5\\ -10 & 5 & -5\end{bmatrix}$$

$$\Rightarrow X=\frac{1}{5}\begin{bmatrix}15 & -10 & 5\\ -10 & 5 & -5\end{bmatrix}$$

$$\Rightarrow X=\begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}$$

Now by substituting X in equation (2) we get,

$$(X+2Y)=\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}+2Y=\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}$$

$$\Rightarrow 2Y=\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}-\begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}$$

$$\Rightarrow 2Y=\begin{bmatrix}3-3 & 2+2 & 5-1\\ -2+2 & 1-1 & -7+1\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}0 & 2 & 2\\ 0 & 0 & -3\end{bmatrix}$$

10.If $X-Y=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}$ and $X+Y=\begin{bmatrix}3 & 5 & 1\\ -1 & 1 & 4\\ 11 & 8 & 0\end{bmatrix}$ find X and Y.

Solution:

Consider

$$X-Y+X+Y= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1+3 & 1+5 & 1+1 \\ 1-1 & 1+1 & 0+4 \\ 1+11 & 0+8 & 0+0 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 2 & 4 \\ 12 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2}\begin{bmatrix} 4 & 6 & 2 \\ 0 & 2 & 4 \\ 12 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$$

Now, again consider

$$(X-Y)-(X+Y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow X-Y-X-Y= \begin{bmatrix} 1-3 & 1-5 & 1-1 \\ 1+1 & 1-1 & 0-4 \\ 1-11 & 0-8 & 0-0 \end{bmatrix}$$

$$\Rightarrow -2Y= \begin{bmatrix} -2 & -4 & 0 \\ 2 & 0 & -4 \\ -10 & -8 & 0 \end{bmatrix}$$

$$\Rightarrow Y=-\frac{1}{2}\begin{bmatrix} -2 & -4 & 0 \\ 2 & 0 & -4 \\ -10 & -8 & 0 \end{bmatrix}$$

$$\Rightarrow Y= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

Therefore,

$$X = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$$

And

$$Y = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

EXERCISE 5.3 PAGE NO: 5.41

1. Compute the indicated products:

$(i)\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$(ii)\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & 1 \end{bmatrix}$

$(iii)\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

Solution:

(i) Consider

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a \times a \times +b+b & a \times (-b) + b \times a \\ (-b) \times a + a \times b & (-b) \times (-b) + a \times a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2 + b^2 & -ab \ +ab \\ -ab + ab & b^2 + a^2 \end{bmatrix}$$

On simplification we get,

$$\Rightarrow \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}$$

(ii) Consider

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 \times 1 + (-2 \times (-3) & 1 \times 2 + (-2) \times 2 & 1 \times 3 + (-2) \times (-1)] \\ 2 \times 1 + 3 \times (-3) & 2 \times 2 + 3 \times 2 & 2 \times 3 + 3 \times (-1) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+6 & 2-4 & 3+2 \\ 2-9 & 4+6 & 6-3 \end{bmatrix}$$

On simplification we get,

$$\Rightarrow \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

(iii) Consider

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\times 1+3\times 0+4\times 3 & 2\times(-3)+3\times 2+4\times 0 & 2\times 5+3\times 4+4\times 5 \\ 3\times 1+4\times 0+5\times 3 & 3\times(-3)+4\times 2+5\times 0 & 3\times 5+4\times 4+5\times 5 \\ 4\times 1+5\times 0+6\times 3 & 4\times(-3)+5\times 2+6\times 0 & 4\times 5+5\times 4+6\times 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix}$$

On simplification we get,

$$\Rightarrow \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

2. Show that AB ≠ BA in each of the following cases:

$$(i) A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} and B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$(ii) A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} and B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$(iii) A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix} and B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

Solution:

(i) Consider,

$$AB = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \ldots\ldots\ldots\ldots(1)$$

Again consider,

$$BA = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \ldots\ldots\ldots\ldots(2)$$

From equation (1) and (2), it is clear that
AB ≠ BA

(ii) Consider,

$$AB = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ 0+0+1 & 0-1+1 & 0+0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix} \ldots\ldots\ldots\ldots (1)$$

Now again consider,

$$BA = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \ldots\ldots\ldots\ldots(2)$$

From equation (1) and (2), it is clear that
AB ≠ BA

(iii) Consider,

$$AB = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 0+3+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 4+0+0 & 0+0+0 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix} \text{.............................(1)}$$

Now again consider,

$$BA = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 0+1+0 & 0+1+0 & 0+0+0 \\ 1+0+0 & 3+0+0 & 0+0+0 \\ 0+5+4 & 0+5+1 & 0+0+0 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 0 \\ 9 & 6 & 0 \end{bmatrix} \text{.......................(2)}$$

From equation (1) and (2), it is clear that
AB ≠ BA

3. Compute the products AB and BA whichever exists in each of the following cases:

$(i) A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} and B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

$(ii) A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix} and B = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

$(iii) A = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix} and B = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$

$$(iv)\ \begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c & d \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Solution:

(i) Consider,

$$AB = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

BA does not exist

Because the number of columns in B is greater than the rows in A

(ii) Consider,

$$AB = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 12+0 & 15+2 & 18+4 \\ -4+0 & -5+0 & -6+0 \\ -4+0 & -5+1 & -6+2 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix}$$

Again consider,

$$BA = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 12-5-6 & 8+0+6 \\ 0-1-2 & 0+0+2 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix}$$

(iii) Consider,

$$AB = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

AB = [0 + (-1) + 6 + 6]

AB = 11

Again consider,

$$BA = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 2 & 3 \\ 3 & -3 & 6 & 9 \\ 2 & -2 & 4 & 6 \end{bmatrix}$$

(iv) Consider,

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\Rightarrow [ac + bd] + [a^2 + b^2 + c^2 + d^2]$$

$$[a^2 + b^2 + c^2 + d^2 + ac + bd]$$

4. Show that AB ≠ BA in each of the following cases:

$$(i) A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix} \ and\ B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

$$(ii) A = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \ and\ B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

Solution:

(i) Consider,

$$AB = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4+1+6 & 6-2-9 & -2+1+4 \\ -6-0+6 & 9+0-9 & -3-0+4 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -5 & 3 \\ 0 & 0 & 1 \end{bmatrix} \text{..........................(1)}$$

Again consider,

$$BA = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} -2+6-3 & -6-3+0 & 2-3+1 \\ -1+4-3 & -3-2+0 & 1-2+1 \\ -6+18-12 & -18-9+0 & 6-9+4 \end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 1 & -9 & 0 \\ 0 & -5 & 0 \\ 0 & -27 & 1 \end{bmatrix} \text{.................(2)}$$

From equation (1) and (2), it is clear that

AB ≠ BA

(ii) Consider,

$$AB = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10-12-1 & 20-16-3 & 10-8-2 \\ -11+15+0 & -22+20+0 & -11+10+0 \\ 9-15+1 & 18-20+3 & 9-10+2 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \ldots\ldots(1)$$

Again consider,

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10-22+9 & -4+10-5 & -9+0+1 \\ 30-44+10 & -12+20-10 & -3+0+2 \\ 10-33+18 & -4+15-10 & -1+0+2 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \ldots\ldots(2)$$

From equation (1) and (2) it is clear that,
AB ≠ BA

5. Evaluate the following:

(i) $\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}\right)$

Solution:

(i) Given

$$\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

First we have to add first two matrix,

$$\Rightarrow \left(\begin{bmatrix} 1+3 & 3-2 \\ -1-1 & -4+1 \end{bmatrix}\right)\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4+2 & 12+4 & 20+6 \\ -2-6 & -6-12 & -10-18 \end{bmatrix}$$

On simplifying, we get

$$\Rightarrow \begin{bmatrix} 6 & 16 & 26 \\ -8 & -18 & -28 \end{bmatrix}$$

(ii) Given,

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

First we have to multiply first two given matrix,

$$\Rightarrow \begin{bmatrix} 1+4+0 & 0+0+3 & 2+2+6 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 3 & 10 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10+12+60 \end{bmatrix}$$

= 82

(iii) Given

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}\right)$$

First we have subtract the matrix which is inside the bracket,

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1-0 & 0-1 & 2-2 \\ 2-1 & 0-0 & 1-2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-1 & -1+0 & 0+1 \\ 0+2 & 0+0 & 0-2 \\ 2+3 & -2+0 & 0-3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -2 \\ 5 & -2 & -3 \end{bmatrix}$$

6. If $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, show that $\mathbf{A}^2 = \mathbf{B}^2 = \mathbf{C}^2 = \mathbf{I}_2$

Solution:

Given

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

We know that,

$$A^2 = AA$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1+0 & 0+1 \\ 0+0 & 0+1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{.............(1)}$$

Again we know that,

$$B^2 = BB$$

$$\Rightarrow B^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow B^2 = \begin{bmatrix} 1+0 & 0-0 \\ 0-0 & 0+1 \end{bmatrix}$$

$$\Rightarrow B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{.............(2)}$$

Now, consider,

$C^2 = C\,C$

$$\Rightarrow B^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow B^2 = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}$$

$$\Rightarrow B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{.............(3)}$$

We have,

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{..........................(4)}$$

Now, from equation (1), (2), (3) and (4), it is clear that $A^2 = B^2 = C^2 = I_2$

7.If $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, **find** $\mathbf{3A^2 - 2B + I}$

Solution:

Given

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$$

Consider,

$A^2 = A\,A$

$$\Rightarrow A^2 = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}$$

Now we have to find,

$3A^2 - 2B + I$

$$\Rightarrow 3A^2 - 2B + I = 3\begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - 2\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 3A^2 - 2B + I = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 3A^2 - 2B + I = \begin{bmatrix} 3-0+1 & -12-8+0 \\ 36+2+0 & 3-14+1 \end{bmatrix}$$

$$\Rightarrow 3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$$

8. If $\mathbf{A} = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, prove that $(\mathbf{A} - 2\mathbf{I})(\mathbf{A} - 3\mathbf{I}) = \mathbf{0}$.

Solution:

Given

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

Consider,

$$\Rightarrow (A-2I)(A-3I) = \left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\right)\left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$\Rightarrow (A-2I)(A-3I) = \left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right)\left(\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right)$$

$$\Rightarrow (A-2I)(A-3I) = \begin{bmatrix} 4-2 & 2-0 \\ -1-0 & 1-2 \end{bmatrix}\begin{bmatrix} 4-3 & 2-0 \\ -1-0 & 1-3 \end{bmatrix}$$

$$\Rightarrow (A-2I)(A-3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$\Rightarrow (A-2I)(A-3I) = \begin{bmatrix} 2-2 & 4-4 \\ -1+1 & -2+2 \end{bmatrix}$$

$$\Rightarrow (A-2I)\ A-3I) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow (A-2I)(A-3I) = 0$

Hence the proof.

9. If $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, show that $\mathbf{A}^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{A}^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

Solution:

Given,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Consider,

$$A^2 = A\,A$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Again consider,

$$A^3 = A^2 A$$

$$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 1+0 & 1+2 \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

Hence the proof.

10.If $\mathbf{A} = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, show that $\mathbf{A^2 = 0}$

Solution:

Given,

$$A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

Consider,

$$A^2 = A\ \ A$$

$$\Rightarrow A^2 = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = 0$$

Hence the proof.

11.If $\mathbf{A} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -sin 2\theta & \cos 2\theta \end{bmatrix}$, **find** $\mathbf{A^2}$

Solution:

Given,

$$A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -sin 2\theta & \cos 2\theta \end{bmatrix}$$

Consider,

$$A^2 = A\,A$$

$$\Rightarrow A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} \cos^2(2\theta) - \sin^2(2\theta) & \cos(2\theta)\sin 2\theta + \cos(2\theta)\sin 2\theta \\ -\cos(2\theta)\sin 2\theta - \sin 2\theta\cos 2\theta & -\sin^2(2\theta) + \cos^2(2\theta) \end{bmatrix}$$

We know that,

$$\cos^2\theta - \sin^2\theta = \cos^2(2\theta)$$

$$\Rightarrow A^2 = \begin{bmatrix} \cos(2\times 2\theta) & 2\sin 2\theta\cos 2\theta \\ -2\sin 2\theta\cos(2\theta) & \cos(2\times 2\theta) \end{bmatrix}$$

Again we have,

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\Rightarrow A^2 = \begin{bmatrix} \cos 4\theta & \sin(2\times 2\theta) \\ -\sin(2\times 2\theta) & \cos 4\theta \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

12.If $\mathbf{A} = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ **and** $\mathbf{B} = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ **show that** $\mathbf{AB = BA = 0_{3\times 3}}$

Solution:

Given,

$$A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \ and \ B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

Consider,

$$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+5 & 6+9-15 & 5+15-20 \\ 1+4-5 & -3-12+15 & -5-15+20 \\ -1-3+4 & 3+9-12 & 5+15-20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$AB = 0_{3\times3}$ ......(1)

Again consider,

$$BA = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+5 & 3+12-15 & 5+15-20 \\ 2+3-5 & -3-12+15 & -5-15+20 \\ -2-3+5 & 3+9-12 & 5+15-20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$BA = 0_{3\times3}$ ......(2)

From equation (1) and (2) AB = BA = 0_{3x3}

$$13.If\ A=\begin{bmatrix}0 & c & -b\\ -c & 0 & a\\ b & -a & 0\end{bmatrix}\ and\ B=\begin{bmatrix}a^2 & ab & ac\\ ab & b^2 & bc\\ ac & bc & c^2\end{bmatrix}\ show\ that\ AB=BA=0_{3\times 3}$$

Solution:

Given

$$A=\begin{bmatrix}0 & c & -b\\ -c & 0 & a\\ b & -a & 0\end{bmatrix}\ and\ B=\begin{bmatrix}a^2 & ab & ac\\ ab & b^2 & bc\\ ac & bc & c^2\end{bmatrix}$$

Consider,

$$AB=\begin{bmatrix}0 & c & -b\\ -c & 0 & a\\ b & -a & 0\end{bmatrix}\begin{bmatrix}a^2 & ab & ac\\ ab & b^2 & bc\\ ac & bc & c^2\end{bmatrix}$$

$$\Rightarrow AB=\begin{bmatrix}0+abc-abc & 0+b^2c-b^2c & 0+bc^2-bc^2\\ -a^2c+0+a^2c & -abc+0+abc & -ac^2+0+ac^2\\ a^2b-a^2b+0 & ab^2-ab^2+0 & abc-abc+0\end{bmatrix}$$

$$\Rightarrow AB=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\Rightarrow AB=O_{3\times 3}\ldots(1)$$

Again consider,

$$BA=\begin{bmatrix}a^2 & ab & ac\\ ab & b^2 & bc\\ ac & bc & c^2\end{bmatrix}\begin{bmatrix}0 & c & -b\\ -c & 0 & a\\ b & -a & 0\end{bmatrix}$$

$$\Rightarrow BA=\begin{bmatrix}0-abc+abc & a^2c+0-a^2c & -a^2b+a^2b+0\\ 0-b^2c+b^2c & abc+0-abc & -ab^2+ab^2+0\\ 0-bc^2+bc^2 & ac^2+0-ac^2 & -abc+abc+0\end{bmatrix}$$

$$\Rightarrow BA = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow BA = O_{3\times 3} \ldots (2)$$

From equation (1) and (2) AB = BA = 0_{3x3}

$$\mathbf{14.} If\ A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\ and\ B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\ show\ that\ AB = A\ and\ BA = B.$$

Solution:

Given

$$A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\ and\ B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

Now consider,

$$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & 2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+18 & -4-12+12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

Therefore AB = A

Again consider, BA we get,

$$BA = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2-4 & -6-8+12 & -10-10+16 \\ -2-3+4 & 3+12-12 & 5+15-16 \\ 2+2-3 & -3-8+9 & -5-10+12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

Hence BA = B

Hence the proof.

$$\mathbf{15.}\boldsymbol{Let\ A} = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix} \boldsymbol{and\ B} = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}, \boldsymbol{compute\ A^2 - B^2.}$$

Solution:

Given,

$$A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix} \ and\ B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$$

Consider,

$$A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3+5 & -1-3-5 & 1+3-5 \\ -3-9+15 & 3+9+15 & -3-9+15 \\ -5+15+25 & 5-15+25 & -5+15+25 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} \text{......(1)}$$

Now again consider, B^2

$$B^2 = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}\begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0+4-3 & 0-12+12 & 0-12+12 \\ 0-3+3 & 4+9-12 & 3+9-12 \\ 0+4-4 & -4-12+16 & -3-12+16 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \ldots\ldots(2)$$

Now by subtracting equation (2) from equation (1) we get,

$$A^2 - B^2 = \begin{bmatrix} -1 & 9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

16. For the following matrices verify the associativity of matrix multiplication i.e. (AB) C = A (BC)

$$(i) A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix},\ and\ C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$(ii) A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix},\ and\ C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution:

(i) Given

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix},\ and\ C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Consider,

$$(AB)C = \left(\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}\right)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+0 & 0+4+0 \\ -1+0+0 & 0+0+3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4 \\ -1-3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \text{......(1)}$$

Now consider RHS,

$$A(BC) = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1+0 \\ -1-2 \\ 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6+0 \\ -1+0-3 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \text{......(2)}$$

From equation (1) and (2), it is clear that (AB) C = A (BC)

(ii) Given,

$$A=\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix},\ B=\begin{bmatrix}1&-1&1\\0&1&2\\2&-1&1\end{bmatrix},\ and\ C=\begin{bmatrix}1&2&-1\\3&0&1\\0&0&1\end{bmatrix}$$

Consider the LHS,

$$(AB)C=\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix}\begin{bmatrix}1&-1&1\\0&1&2\\2&-1&1\end{bmatrix}\begin{bmatrix}1&2&-1\\3&0&1\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}4+0+6&-4+2-3&4+4+3\\1+0+4&-1+1-2&1+2+2\\3+0+2&-3+0-1&3+0+1\end{bmatrix}\begin{bmatrix}1&2&-1\\3&0&1\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}10&-5&11\\5&-2&5\\5&-4&4\end{bmatrix}\begin{bmatrix}1&2&-1\\3&0&1\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}10-15+0&20+0+0&-10+5+11\\5-6+0&10+0+0&-5-2+5\\5-12+0&10+0+0&-5-4+4\end{bmatrix}$$

$$(AB)C=\begin{bmatrix}-5&20&-4\\-1&10&-2\\-7&10&-5\end{bmatrix}\ldots\ldots(1)$$

Now consider RHS,

$$A(BC)=\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix}\left(\begin{bmatrix}1&-1&1\\0&1&2\\2&-1&1\end{bmatrix}\begin{bmatrix}1&2&-1\\3&0&1\\0&0&1\end{bmatrix}\right)$$

$$=\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix}\begin{bmatrix}1-3+0&2+0+0&-1-1+1\\0+3+0&0+0+0&0+1+2\\2-3+0&4+0+0&-2-1+1\end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & 1 \\ 3 & 0 & 3 \\ -1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -8 + 6 - 3 & 8 + 0 + 12 & -4 + 6 - 6 \\ -2 + 3 - 2 & 2 + 0 + 8 & -1 + 3 - 4 \\ -6 + 0 - 1 & 6 + 0 + 4 & -3 + 0 - 2 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \ldots\ldots(2)$$

From equation (1) and (2), it is clear that (AB) C = A (BC)

17. For the following matrices verify the distributivity of matrix multiplication over matrix addition i.e. A (B + C) = AB + AC.

$$(i) A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$(ii) A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

Solution:

(i) Given

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

Consider LHS,

$$A(B + C) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \left(\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 + 0 & 0 + 1 \\ 2 + 1 & 1 - 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1-3 & 1+0 \\ 0+6 & 0+0 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \text{......(1)}$$

Now consider RHS,

$$AB = AC = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-2 & 0-1 \\ 0+4 & 0+2 \end{bmatrix} + \begin{bmatrix} 0+-1 & 1+1 \\ 0+2 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -3-1 & -1+2 \\ 4+2 & 2-2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \text{......(2)}$$

From equation (1) and (2), it is clear that A (B + C) = AB + AC

(ii) Given,

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix},\ B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix},\ and\ C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

Consider the LHS

$$A(B + C) = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0+1 & 1-1 \\ 1+0 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 \\ 1+1 & 0+2 \\ -1+2 & 0+4 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \ldots\ldots(1)$$

Now consider RHS,

$$AB + AC = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 2-1 \\ 0+1 & 1+1 \\ 0+2 & -1+2 \end{bmatrix} + \begin{bmatrix} 2+0 & -2-1 \\ 1+0 & -1+1 \\ -1+0 & 1+2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 1-3 \\ 1+1 & 2+0 \\ 2-1 & 1+3 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \ldots\ldots(2)$$

$$18. If\, A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix},\ and\ C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix},$$

$verify\ that\ A(B-C)=AB-AC.$

Solution:

Given,

$$A=\begin{bmatrix}1 & 0 & -2\\ 3 & -1 & 0\\ -2 & 1 & 1\end{bmatrix},\ B=\begin{bmatrix}0 & 5 & -4\\ -2 & 1 & 3\\ -1 & 0 & 2\end{bmatrix}$$

$$C=\begin{bmatrix}1 & 5 & 2\\ -1 & 1 & 0\\ 0 & -1 & 1\end{bmatrix}$$

Consider the LHS,

$$A(B-C)=\begin{bmatrix}1 & 0 & -2\\ 3 & -1 & 0\\ -2 & 1 & 1\end{bmatrix}\left(\begin{bmatrix}0 & 5 & -4\\ -2 & 1 & 3\\ -1 & 0 & 2\end{bmatrix}-\begin{bmatrix}1 & 5 & 2\\ -1 & 1 & 0\\ 0 & -1 & 1\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 0 & -2\\ 3 & -1 & 0\\ -2 & 1 & 1\end{bmatrix}\begin{bmatrix}-1 & 0 & 6\\ -1 & 0 & 3\\ -1 & 1 & 1\end{bmatrix}$$

$$A(B-C)=\begin{bmatrix}1 & -2 & -8\\ -2 & 0 & -21\\ 0 & 1 & 16\end{bmatrix}$$

Now consider RHS

$$AB-AC=\begin{bmatrix}1 & 0 & -2\\ 3 & -1 & 0\\ -2 & 1 & 1\end{bmatrix}\begin{bmatrix}0 & 5 & -4\\ -2 & 1 & 3\\ -1 & 0 & 2\end{bmatrix}-\begin{bmatrix}1 & 0 & -2\\ 3 & -1 & 0\\ -2 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 5 & 2\\ -1 & 1 & 0\\ 0 & -1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}2 & 5 & -8\\ 2 & 14 & -15\\ -3 & -9 & 13\end{bmatrix}-\begin{bmatrix}1 & 7 & 0\\ 4 & 14 & 6\\ -3 & -10 & -3\end{bmatrix}$$

$$=\begin{bmatrix}1 & -2 & -8\\ -2 & 0 & -21\\ 0 & 1 & 16\end{bmatrix}$$

From the above equations LHS = RHS

Therefore, A (B – C) = AB – AC.

19. Compute the elements a_{43} and a_{22} of the matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

Solution:

Given

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & 2 \\ 12 & 4 \\ -1 & 12 \\ 24 & 8 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & -9 & 11 & -14 & 6 \\ 12 & 0 & 4 & 8 & -24 \\ 36 & -37 & 49 & -50 & 2 \\ 24 & 0 & 8 & 16 & -48 \end{bmatrix}$$

From the above matrix, a_{43} = 8and a_{22} = 0

$$20. If A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} \text{ and } I \text{ is the identity matrix of order } 3, \text{that } A^3 = pI + qA + rA^2$$

Solution:

Given

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

Consider,

$A^2 = A.A$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix}$$

Again consider,

$A^3 = A^2.A$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+0+pq+pr^2 & pr+0+q^2+qr^2 & 0+p+qr+qr+r^2 \end{bmatrix}$$

$$= \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix}$$

Now, consider the RHS

$pI + qA + rA^2$

$$= p\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + q\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} + r\begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix}$$

$$= \begin{bmatrix} p & q & r \\ pr & p + qr & q + r^2 \\ pq + pr^2 & pr + q^2 + qr^2 & p + 2qr + r^2 \end{bmatrix}$$

Therefore, $A^3 = p\,I + q\,A + rA^2$

Hence the proof.

21. If ω is a complex cube root of unity, show that

$$\left(\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix} + \begin{bmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix}\right) \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution:

Given

$$\left(\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix} + \begin{bmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix}\right) \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

It is also given that ω is a complex cube root of unity,

Consider the LHS,

$$= \begin{bmatrix} 1+\omega & \omega+\omega^2 & \omega^2+1 \\ \omega+\omega^2 & \omega^2+1 & 1+\omega \\ \omega^2+\omega & 1+\omega^2 & \omega+1 \end{bmatrix} \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix}$$

We know that $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$

$$= \begin{bmatrix} -\omega^2 & -1 & -\omega \\ -1 & -\omega & -\omega^2 \\ -1 & -\omega & -\omega^2 \end{bmatrix} \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix}$$

Now by simplifying we get,

$$= \begin{bmatrix} -\omega^2 & -\omega & -\omega^3 \\ -1 & -\omega^2 & -\omega^4 \\ -1 & -\omega^2 & -\omega^4 \end{bmatrix}$$

Again by substituting $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$ in above matrix we get,

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore LHS = RHS

Hence the proof.

$$22. If\ A=\begin{bmatrix}2&-3&-5\\-1&4&5\\1&-3&-4\end{bmatrix},\ show\ that\ A^2=A$$

Solution:

Given,

$$A=\begin{bmatrix}2&-3&-5\\-1&4&5\\1&-3&-4\end{bmatrix}$$

Consider A^2

$$A^2=A.A$$

$$=\begin{bmatrix}2&-3&-5\\-1&4&5\\1&-3&-4\end{bmatrix}\begin{bmatrix}2&-3&-5\\-1&4&5\\1&-3&-4\end{bmatrix}$$

$$=\begin{bmatrix}4+3-5&-6-12+15&-10-15+20\\-2-4+5&3+16-15&5+20-20\\2+3-4&-3-12+12&-5-15+16\end{bmatrix}$$

$$=\begin{bmatrix}2&-3&-5\\-1&4&5\\1&-3&-4\end{bmatrix}=A$$

Therefore $A^2 = A$

$$23. If\ A=\begin{bmatrix}4&-1&-4\\3&0&-4\\3&-1&-3\end{bmatrix},\ show\ that\ A^2=I_3$$

Solution:

Given

$$A=\begin{bmatrix}4&-1&-4\\3&0&-4\\3&-1&-3\end{bmatrix}$$

Consider A^2,

$A^2 = A.A$

$$= \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 16-3-12 & -4+0+4 & 16+4+12 \\ 12+0-12 & -3+0+4 & -12+0+12 \\ 12-3-9 & -3+0+3 & -12+4+9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

Hence $A^2 = I_3$

24. (i) If $\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$, find x.

(ii) If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, find x.

Solution:

(i) Given

$$\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+2x+0 & x+0+2 & 2+1+0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x+4 & x+2 & 2x+4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

= [2x + 1 + 2 + x + 3] = 0

= [3x + 6] = 0

= 3x = -6

x = -6/3
x = -2

(ii) Given,

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2-6 & -6+12 \\ 5-14 & -15+28 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

On comparing the above matrix we get,
x = 13

25. $\boldsymbol{If\ \begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0,\ find\ x.}$

Solution:

Given

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix}$$

$$\Rightarrow [2x + 4 + 0 \quad x + 0 + 2 \quad 2x + 8 - 4] \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow [2x + 4 \quad x + 2 \quad 2x + 4] \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

⇒ [(2x + 4) x + 4 (x + 2) - 1(2x + 4)] = 0
⇒ $2x^2 + 4x + 4x + 8 - 2x - 4 = 0$
⇒ $2x^2 + 6x + 4 = 0$
⇒ $2x^2 + 2x + 4x + 4 = 0$
⇒ 2x (x + 1) + 4 (x + 1) = 0
⇒ (x + 1) (2x + 4) = 0
⇒ x = -1 or x = -2

Hence, x = -1 or x = -2

26. $If \begin{bmatrix} 1 & -1 & x \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0,\ find\ x.$

Solution:

Given

$$\begin{bmatrix} 1 & -1 & x \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

By multiplying we get,

$$\Rightarrow \begin{bmatrix} 0-2+x & x & (-1)-3+x \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x-2 & x & x-4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$[(x-2)\times 0 + x\times 1 + (x-4)\times 1] = 0$$

$$\Rightarrow x + x - 4 = 0$$

$$\Rightarrow 2x = 4 \Rightarrow x = 2$$

27. $If\ A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\ and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ then\ prove\ that\ A^2 - A + 2I = 0.$

Solution:

Given

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\ and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now we have to prove $A^2 - A + 2I = 0$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (-2 \times 4) & 3 \times (-2) + (-2 \times -2) \\ 4 \times 3 + (-2 \times 4) & 4 \times (-2) + (-2 \times -2) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 - 8 & -6 + 4 \\ 12 - 8 & -8 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \ldots\ldots\ldots\ldots(i)$$

Now, we will find the matrix for 2I, we get

$$2I = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 2I = \begin{bmatrix} 2 \times 1 & 2 \times 0 \\ 2 \times 0 & 2 \times 1 \end{bmatrix}$$

$$\Rightarrow 2I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \ldots\ldots\ldots\ldots(ii)$$

$A^2 - A + 2I$

Substitute corresponding values from eqn (i) and eqn (ii), we get

$$\Rightarrow = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 1 - 3 + 2 & -2 - (-2) + 0 \\ 4 - 4 + 0 & -4 - (-2) + 2 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Therefore,

$A^2 - A + 2I = 0$

Hence proved

28. $If\ A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\ and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ then\ prove\ that\ A^2 = 5A + \lambda I.$

Solution:

Given

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \; and \; I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now, we have to find A^2,

$$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (1 \times -1) & 3 \times 1 + 1 \times 2 \\ (-1 \times 3) + 2 \times (-1) & (-1 \times 1) + 2 \times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \ldots\ldots\ldots.(i)$$

Now, we will find the matrix for 5A, we get

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times 1 \\ 5 \times (-1) & 5 \times 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} \ldots\ldots\ldots\ldots.(ii)$$

So,

$$A^2 = 5A + \lambda I$$

Substitute corresponding values from eqn (i) and eqn (ii), we get

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 + \lambda & 5 + 0 \\ -5 + 0 & 10 + \lambda \end{bmatrix}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal,

Hence,

$$8 = 15 + \lambda \Rightarrow \lambda = -7$$

$$3 = 10 + \lambda \Rightarrow \lambda = -7$$

29. $If\ A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ *show that* $A^2 - 5A + 7I_2 = 0.$

Solution:

Given

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

I_2 is an identity matrix of size 2, so

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

To show that

$$A^2 - 5A + 7I_2 = 0$$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (1 \times -1) & 3 \times 1 + 1 \times 2 \\ (-1 \times 3) + 2 \times (-1) & (-1 \times 1) + 2 \times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \ldots\ldots\ldots(i)$$

Now, we will find the matrix for 5A, we get

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times 1 \\ 5 \times (-1) & 5 \times 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} \ldots\ldots\ldots\ldots (ii)$$

Now,

$$7I_2 = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \ldots\ldots (iii)$$

So,

$$A^2 - 5A + 7I_2$$

Substitute corresponding values from eqn (i), (ii) and (iii), we get

$$\Rightarrow = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 8 - 15 + 7 & 5 - 5 + 0 \\ -5 - (-5) + 0 & 3 - 10 + 7 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence the proof.

30. $If\ A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}\ show\ that\ A^2 - 2A + 3I_2 = 0.$

Solution:

Given

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

I_2 is an identity matrix of size 2, so

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now we have to show,

$A^2 - 2A + 3I_2 = 0$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 2 \times 2 + (3 \times -1) & 2 \times 3 + 3 \times 0 \\ (-1 \times 2) + 0 \times (-1) & (-1 \times 3) + 0 \times 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4 - 3 & 6 + 0 \\ -2 + 0 & -3 + 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} \ldots\ldots\ldots. (i)$$

Now, we will find the matrix for 2A, we get

$$2A = 2\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow 2A = \begin{bmatrix} 2 \times 2 & 2 \times 3 \\ 2 \times (-1) & 2 \times 0 \end{bmatrix}$$

$$\Rightarrow 2A = \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} \ldots\ldots\ldots\ldots. (ii)$$

Now,

$$3I_2 = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \ldots\ldots..(iii)$$

So,

$A^2 - 2A + 3I_2$

Substitute corresponding values from eqn (i), (ii) and (iii), we get

$$\Rightarrow = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 1 - 4 + 3 & 6 - 6 + 0 \\ -2 - (-2) + 0 & -3 - 0 + 3 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence the proof.

$31.\ Show\ that\ the\ matrix\ A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\ satiesfies\ the\ equation\ A^3 - 4A^2 + A = 0.$

Solution:

Given

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

To show that $A^3 - 4A^2 + A = 0$

Now, we will find the matrix for A^2, we get

$$A^2 = (A \times A) = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 2 \times 2 + (3 \times 1) & 2 \times 3 + 3 \times 2 \\ 1 \times 2 + 2 \times 1 & 1 \times 3 + 2 \times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4 + 3 & 6 + 6 \\ 2 + 2 & 3 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \ldots\ldots\ldots\ldots (i)$$

Now, we will find the matrix for A^3, we get

$$A^3 = A^2 \times A = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 7 \times 2 + 12 \times 1 & 7 \times 3 + 12 \times 2 \\ 4 \times 2 + 7 \times 1 & 4 \times 3 + 7 \times 2 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 14 + 12 & 21 + 24 \\ 8 + 7 & 12 + 14 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} \ldots\ldots\ldots\ldots (ii)$$

So,

$A^3 - 4A^2 + A$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4 \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 4\times 7 & 4\times 12 \\ 4\times 4 & 4\times 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Therefore,

$A^3 - 4A^2 + A = 0$

Hence matrix A satisfies the given equation.

32. *Show that the matrix* $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ *satiesfies the equation* $A^2 - 12A - I = 0.$

Solution:

Given

$$A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$$

I is an identity matrix so $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

To show that $A^2 - 12A - I = 0$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 5\times 5 + 3\times 12 & 5\times 3 + 3\times 7 \\ 12\times 5 + 7\times 12 & 12\times 3 + 7\times 7 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 25+36 & 15+21 \\ 60+84 & 36+49 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} \ldots\ldots\ldots (i)$$

Now, we will find the matrix for 12A, we get

$$12A = 12\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$$

$$\Rightarrow 12A = \begin{bmatrix} 12 \times 5 & 12 \times 3 \\ 12 \times 12 & 12 \times 7 \end{bmatrix}$$

$$\Rightarrow 12A = \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} \ldots\ldots\ldots\ldots (ii)$$

So,

$A^2 - 12A - I$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow = \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 61 - 60 - 1 & 36 - 36 - 0 \\ 144 - 144 - 0 & 85 - 84 - 1 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Therefore,
$A^2 - 12A - I = 0$

Hence matrix A is the root of the given equation.

33. $If\ A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\ find\ A^2 - 5A - 14I.$

Solution:
Given

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

I is identity matrix so

$$14I = 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

To find $A^2 - 5A - 14I$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (-5 \times -4) & 3 \times (-5) + (-5 \times 2) \\ (-4 \times 3) + (2 \times -4) & (-4 \times -5) + 2 \times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 + 20 & -15 - 10 \\ -12 - 8 & 20 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} \ldots\ldots\ldots. (i)$$

Now, we will find the matrix for 5A, we get

$$5A = 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times (-5) \\ 5 \times (-4) & 5 \times 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} \ldots\ldots\ldots\ldots. (ii)$$

So,

$A^2 - 5A - 14I$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 29 - 15 - 14 & -25 + 25 - 0 \\ -20 + 20 - 0 & 24 - 10 - 14 \end{bmatrix}$$

$$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

34. $\boldsymbol{If\ A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\ show\ that\ A^2 - 5A + 7I = 0.\ Use\ this\ to\ find\ A^4.}$

Solution:

Given

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

I is identity matrix so

$$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

To show that $A^2 - 5A + 7I = 0$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (1 \times -1) & 3 \times 1 + 1 \times 2 \\ (-1 \times 3) + (2 \times -1) & (-1 \times 1) + 2 \times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \ldots\ldots\ldots.(i)$$

Now, we will find the matrix for 5A, we get

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 5 \times 3 & 5 \times 1 \\ 5 \times (-1) & 5 \times 2 \end{bmatrix}$$

$$\Rightarrow 5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} \ldots\ldots\ldots\ldots.(ii)$$

So,

$A^2 - 5A + 7I$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$\Rightarrow = \begin{bmatrix} 8-15-7 & 5-5-0 \\ -5+5-0 & 3-10-7 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Therefore

$A^2 - 5A + 7I = 0$

Hence proved

We will find A^4

$A^2 - 5A + 7I = 0$

Multiply both sides by A^2, we get

$A^2(A^2 - 5A + 7I) = A^2(0)$

$\Rightarrow A^4 - 5A^2.A + 7I.A^2$

$\Rightarrow A^4 = 5A^2.A - 7I.A^2$

$\Rightarrow A^4 = 5A^2A - 7A^2$

As multiplying by the identity matrix, I don't change anything. Now will substitute the corresponding values we get

$\Rightarrow A^4 = 5\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 7\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\Rightarrow A^4 = 5\begin{bmatrix} 24-5 & 8+10 \\ -15-3 & -5+6 \end{bmatrix} - 7\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\Rightarrow A^4 = 5\begin{bmatrix} 19 & 18 \\ -18 & 1 \end{bmatrix} - 7\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\Rightarrow A^4 = \begin{bmatrix} 5\times 19 & 5\times 18 \\ 5\times(-18) & 5\times 1 \end{bmatrix} - \begin{bmatrix} 7\times 8 & 7\times 5 \\ 7\times(-5) & 7\times 3 \end{bmatrix}$

$\Rightarrow A^4 = \begin{bmatrix} 95 & 90 \\ -90 & 5 \end{bmatrix} - \begin{bmatrix} 56 & 35 \\ -35 & 21 \end{bmatrix}$

$\Rightarrow A^4 = \begin{bmatrix} 95-56 & 90-35 \\ -90+35 & 5-21 \end{bmatrix}$

$$\Rightarrow A^4 = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$$

35. $\boldsymbol{If\ A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\ find\ k\ such\ that\ A^2 = kA - 2I_2.}$

Solution:

Given

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

I_2 is an identity matrix of size 2, so

$$2I_2 = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Also given,

$$A^2 = kA - 2I_2$$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 \times 3 + (-2 \times 4) & 3 \times (-2) + (-2 \times -2) \\ (4 \times 3) + (-2 \times 4) & (4 \times -2) + (-2 \times -2) \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 9 - 8 & -6 + 4 \\ 12 - 8 & -8 + 4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \ldots \ldots \ldots (i)$$

Now, we will find the matrix for kA, we get

$$kA = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow kA = \begin{bmatrix} k \times 3 & k \times (-2) \\ k \times 4 & k \times (-2) \end{bmatrix}$$

So,

$A^2 = kA - 2I_2$

Substitute corresponding values from eqn (i) and (ii), we get

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k-0 \\ 4k-0 & -2k-2 \end{bmatrix}$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

Hence, $3k - 2 = 1 \Rightarrow k = 1$

Therefore, the value of k is 1

36. $\boldsymbol{If\ A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\ find\ k\ such\ that\ A^2 - 8A + kI = 0.}$

Solution:
Given

$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

I is identity matrix, so

$kI = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

Also given, $A^2 - 8A + kI = 0$

Now, we have to find A^2, we get

$A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 \times 1 + 0 & 0 + 0 \\ (-1 \times 1) + 7 \times (-1) & 0 + 7 \times 7 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} \ldots\ldots\ldots.(i)$

Now, we will find the matrix for 8A, we get

$8A = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

$$\Rightarrow 8A = \begin{bmatrix} 8\times 1 & 8\times 0 \\ 8\times(-1) & 8\times 7 \end{bmatrix}$$

$$\Rightarrow 8A = \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} \ldots\ldots\ldots\ldots (ii)$$

So,

$A^2 - 8A + kI = 0$

Substitute corresponding values from eqn (i) and (ii), we get

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1-8+k & 0-0+0 \\ -8+8+0 & 49-56+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

Hence,

$1 - 8 + k = 0 \Rightarrow k = 7$

Therefore, the value of k is 7

37. $\boldsymbol{If\ A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\ and\ f(x) = x^2 - 2x - 3\ show\ that\ f(A) = 0}$

Solution:

Given

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

To show that f (A) = 0

Substitute x = A in f(x), we get

$f(A) = A^2 - 2A - 3I \ldots\ldots\ldots (i)$

I is identity matrix, so

$$3I = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Now, we will find the matrix for A^2, we get

$$A^2 = A\times A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1\times 1 + 2\times 2 & 1\times 2 + 2\times 1 \\ 2\times 1 + 1\times 2 & 2\times 2 + 1\times 1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} \ldots\ldots\ldots (ii)$$

Now, we will find the matrix for 2A, we get

$$2A = 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\Rightarrow 2A = \begin{bmatrix} 2\times 1 & 2\times 2 \\ 2\times 2 & 2\times 1 \end{bmatrix}$$

$$\Rightarrow 2A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} \ldots\ldots\ldots\ldots (iii)$$

Substitute corresponding values from eqn (ii) and (iii) in eqn (i), we get

$$f(A) = A^2 - 2A - 3I$$

$$\Rightarrow f(A) = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So,

$$\Rightarrow f(A) = 0$$

Hence Proved

38. $If\ A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\ and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\ then\ find\ \lambda, \mu\ so\ that\ A^2 = \lambda A + \mu I$

Solution:

Given

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\ and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So

$$\mu I = \mu\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$$

Now, we will find the matrix for A^2, we get

$$A^2 = A \times A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 2\times 2 + 3\times 1 & 2\times 3 + 3\times 2 \\ 1\times 2 + 2\times 1 & 1\times 3 + 2\times 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \ldots\ldots\ldots (i)$$

Now, we will find the matrix for λ A, we get

$$\lambda A = \lambda\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$$
$$\Rightarrow \lambda A = \begin{bmatrix}\lambda\times 2 & \lambda\times 3\\\lambda\times 1 & \lambda\times 2\end{bmatrix}$$
$$\Rightarrow \lambda A = \begin{bmatrix}2\lambda & 3\lambda\\\lambda & 2\lambda\end{bmatrix}\ldots\ldots\ldots\ldots(ii)$$

But given, $A^2 = \lambda A + \mu I$

Substitute corresponding values from equation (i) and (ii), we get

$$\Rightarrow \begin{bmatrix}7 & 12\\4 & 7\end{bmatrix} = \begin{bmatrix}2\lambda & 3\lambda\\\lambda & 2\lambda\end{bmatrix} + \begin{bmatrix}\mu & 0\\0 & \mu\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}7 & 12\\4 & 7\end{bmatrix} = \begin{bmatrix}2\lambda+\mu & 3\lambda+0\\\lambda+0 & 2\lambda+\mu\end{bmatrix}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

Hence, $\lambda + 0 = 4 \Rightarrow \lambda = 4$

And also, $2\lambda + \mu = 7$

Substituting the obtained value of λ in the above equation, we get

$2(4) + \mu = 7 \Rightarrow 8 + \mu = 7 \Rightarrow \mu = -1$

Therefore, the value of λ and μ are 4 and – 1 respectively

39. Find the value of x for which the matrix product

$$\begin{bmatrix}2 & 0 & 7\\0 & 1 & 0\\1 & -2 & 1\end{bmatrix}\begin{bmatrix}-x & 14x & 7x\\0 & 1 & 0\\x & -4x & -2x\end{bmatrix} \textit{ equal to an identity matrix.}$$

Solution:

We know,

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

is identity matrix of size 3.

So according to the given criteria

$$\begin{bmatrix}2 & 0 & 7\\0 & 1 & 0\\1 & -2 & 1\end{bmatrix}\begin{bmatrix}-x & 14x & 7x\\0 & 1 & 0\\x & -4x & -2x\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Now we will multiply the two matrices on LHS using the formula $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$, we get

$$\begin{bmatrix} 2\times(-x) + 0 + 7\times x & 2\times 14x + 0 + 7\times(-4x) & 2\times 7x + 0 + 7\times(-2x) \\ 0 + 0 + 0 & 0 + 1\times 1 + 0 & 0 + 0 + 0 \\ 1\times(-x) + 0 + 1\times x & 1\times 14x + (-2\times 1) + (1\times -4x) & 1\times 7x + 0 + 1\times(-2x) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And to satisfy the above condition of equality, the corresponding entries of the matrices should be equal

So we get

$5x = 1 \Rightarrow x = \frac{1}{5}$

So the value of x is $\frac{1}{5}$

EXERCISE 5.4 PAGE NO: 5.54

1. $\boldsymbol{Let\ A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}\ and\ B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\ verify\ that}$

(i) $(2A)^T = 2\,A^T$
(ii) $(A + B)^T = A^T + B^T$
(iii) $(A - B)^T = A^T - B^T$
(iv) $(AB)^T = B^T A^T$

Solution:

(i) Given

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}\ and\ B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

Consider,

$(2A)^T = 2A^T$

Put the value of A

$$\Rightarrow \left(2\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}\right)^T = 2\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 4 & -6 \\ -14 & 10 \end{bmatrix}^T = 2\begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix} = \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix}$$

L.H.S = R.H.S

(ii) Given

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}\ and\ B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

Consider,

$(A + B)^T = A^T + B^T$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2+1 & -3+0 \\ -7+2 & 5-4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -3 \\ -5 & 1 \end{bmatrix}^T = \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix}$$

L.H.S = R.H.S
Hence proved.

(iii) Given

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \ and \ B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

Consider,

$(A - B)^T = A^T - B^T$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2-1 & -3-0 \\ -7-2 & 5+4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -3 \\ -9 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$$

L.H.S = R.H.S

(iv) Given

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \ and \ B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$(AB)^T = B^T A^T$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$$

$$\begin{bmatrix} 2-6 & 0+12 \\ -7+10 & 0-20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 12 \\ 3 & -20 \end{bmatrix}^T = \begin{bmatrix} 2-6 & -7+10 \\ 0+12 & 0-20 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix}$$

So,

$(AB)^T = B^T A^T$

$$2. \ A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \ and \ B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix} \ verify \ that \ (AB)^T = B^T A^T$$

Solution:

Given

$$A=\begin{bmatrix}3\\5\\2\end{bmatrix}\ and\ B=\begin{bmatrix}1 & 0 & 4\end{bmatrix}$$

$(AB)^T = B^TA^T$

$$\Rightarrow \left(\begin{bmatrix}3\\5\\2\end{bmatrix}\begin{bmatrix}1 & 0 & 4\end{bmatrix}\right)^T = \begin{bmatrix}1 & 0 & 4\end{bmatrix}^T\begin{bmatrix}3\\5\\2\end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix}3 & 0 & 12\\5 & 0 & 20\\2 & 0 & 8\end{bmatrix}^T = \begin{bmatrix}1\\0\\4\end{bmatrix}\begin{bmatrix}3 & 5 & 2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}3 & 5 & 2\\0 & 0 & 0\\12 & 20 & 8\end{bmatrix} = \begin{bmatrix}3 & 5 & 2\\0 & 0 & 0\\12 & 20 & 8\end{bmatrix}$$

L.H.S = R.H.S

So, $(AB)^T = B^TA^T$

3. $\boldsymbol{Let\ A=\begin{bmatrix}1 & -1 & 0\\2 & 1 & 3\\1 & 2 & 1\end{bmatrix}\ and\ B=\begin{bmatrix}1 & 2 & 3\\2 & 1 & 3\\0 & 1 & 1\end{bmatrix}\ find\ A^T, B^T\ and\ verify\ that}$

(i) A + B)^T = A^T + B^T

(ii) (AB)^T = B^T A^T

(iii) (2A)^T = 2 A^T

Solution:

(i) Given

$$A=\begin{bmatrix}1 & -1 & 0\\2 & 1 & 3\\1 & 2 & 1\end{bmatrix}\ and\ B=\begin{bmatrix}1 & 2 & 3\\2 & 1 & 3\\0 & 1 & 1\end{bmatrix}$$

Consider,

$(A + B)^T = A^T + B^T$

$$\left(\begin{bmatrix}1 & -1 & 0\\2 & 1 & 3\\1 & 2 & 1\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 1 & 3\\0 & 1 & 1\end{bmatrix}\right)^T = \begin{bmatrix}1 & -1 & 0\\2 & 1 & 3\\1 & 2 & 1\end{bmatrix}^T + \begin{bmatrix}1 & 2 & 3\\2 & 1 & 3\\0 & 1 & 1\end{bmatrix}^T$$

$$\left(\begin{bmatrix}1+1 & -1+2 & 0+3\\2+2 & 1+1 & 3+3\\1+0 & 2+1 & 1+1\end{bmatrix}\right)^T = \begin{bmatrix}1 & 2 & 1\\-1 & 1 & 2\\0 & 3 & 1\end{bmatrix} + \begin{bmatrix}1 & 2 & 0\\2 & 1 & 1\\3 & 3 & 1\end{bmatrix}$$

$$\begin{bmatrix}2 & 1 & 3\\4 & 2 & 6\\1 & 3 & 2\end{bmatrix}^T = \begin{bmatrix}1+1 & 2+2 & 1+0\\-1+2 & 1+1 & 2+1\\0+3 & 3+3 & 1+1\end{bmatrix}$$

$$\begin{bmatrix}2 & 4 & 1\\1 & 2 & 3\\3 & 6 & 2\end{bmatrix} = \begin{bmatrix}2 & 4 & 1\\1 & 2 & 3\\3 & 6 & 2\end{bmatrix}$$

L.H.S = R.H.S

So, $(A+B)^T = A^T + B^T$

(ii) Given

$$A = \begin{bmatrix}1 & -1 & 0\\2 & 1 & 3\\1 & 2 & 1\end{bmatrix} \text{ and } B = \begin{bmatrix}1 & 2 & 3\\2 & 1 & 3\\0 & 1 & 1\end{bmatrix}$$

Consider,

$(AB)^T = B^T A^T$

$$\left(\begin{bmatrix}1 & -1 & 0\\2 & 1 & 3\\1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\2 & 1 & 3\\0 & 1 & 1\end{bmatrix}\right)^T = \begin{bmatrix}1 & 2 & 3\\2 & 1 & 3\\0 & 1 & 1\end{bmatrix}^T \begin{bmatrix}1 & -1 & 0\\2 & 1 & 3\\1 & 2 & 1\end{bmatrix}^T$$

$$\begin{bmatrix}1-2+0 & 2-1+0 & 3-3+0\\2+2+0 & 4+1+3 & 6+3+3\\1+4+0 & 2+2+1 & 3+6+1\end{bmatrix}^T = \begin{bmatrix}1 & 2 & 0\\2 & 1 & 1\\3 & 3 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 1\\-1 & 1 & 2\\0 & 3 & 1\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & 0\\4 & 8 & 12\\5 & 5 & 10\end{bmatrix}^T = \begin{bmatrix}1-2+0 & 2+2+0 & 1+4+0\\2-1+0 & 4+1+3 & 2+2+1\\3-3+0 & 6+3+3 & 3+6+1\end{bmatrix}$$

$$\begin{bmatrix}-1 & 4 & 5\\1 & 8 & 5\\0 & 12 & 10\end{bmatrix} = \begin{bmatrix}-1 & 4 & 5\\1 & 8 & 5\\0 & 12 & 10\end{bmatrix}$$

L.H.S = R.H.S

So, $(AB)^T = B^T A^T$

(iii) Given

$$A = \begin{bmatrix}1 & -1 & 0\\2 & 1 & 3\\1 & 2 & 1\end{bmatrix} \text{ and } B = \begin{bmatrix}1 & 2 & 3\\2 & 1 & 3\\0 & 1 & 1\end{bmatrix}$$

Consider,

$(2A)^T = 2A^T$

$$\Rightarrow \left(2\begin{bmatrix}1 & -1 & 0\\ 2 & 1 & 3\\ 1 & 2 & 1\end{bmatrix}\right)^{T} = 2\begin{bmatrix}1 & -1 & 0\\ 2 & 1 & 3\\ 1 & 2 & 1\end{bmatrix}^{T}$$

$$\Rightarrow \begin{bmatrix}2 & -2 & 0\\ 4 & 2 & 6\\ 2 & 4 & 2\end{bmatrix}^{T} = 2\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 2\\ 0 & 3 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2 & 4 & 2\\ -2 & 2 & 4\\ 0 & 6 & 2\end{bmatrix} = \begin{bmatrix}2 & 4 & 2\\ -2 & 2 & 4\\ 0 & 6 & 2\end{bmatrix}$$

L.H.S = R.H.S

So,

$(2A)^{T} = 2A^{T}$

4. $\boldsymbol{If\ A = \begin{bmatrix}-2\\ 4\\ 5\end{bmatrix}, B = \begin{bmatrix}1 & 3 & -6\end{bmatrix},\ verify\ that\ (AB)^T = B^T A^T}$

Solution:

Given

$$A = \begin{bmatrix}-2\\ 4\\ 5\end{bmatrix}, B = \begin{bmatrix}1 & 3 & -6\end{bmatrix}$$

Consider,

$(AB)^{T} = B^{T}A^{T}$

$$\Rightarrow \left(\begin{bmatrix}-2\\ 4\\ 5\end{bmatrix}\begin{bmatrix}1 & 3 & -6\end{bmatrix}\right)^{T} = \begin{bmatrix}1 & 3 & -6\end{bmatrix}^{T}\begin{bmatrix}-2\\ 4\\ 5\end{bmatrix}^{T}$$

$$\Rightarrow \begin{bmatrix}-2 & -6 & -12\\ 4 & 12 & -24\\ 5 & 15 & -30\end{bmatrix}^{T} = \begin{bmatrix}1\\ 3\\ -6\end{bmatrix}\begin{bmatrix}-2 & 4 & 5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}-2 & 4 & 5\\ -6 & 12 & 15\\ -12 & -24 & -30\end{bmatrix} = \begin{bmatrix}-2 & 4 & 5\\ -6 & 12 & 15\\ -12 & -24 & -30\end{bmatrix}$$

L.H.S = R.H.S

So,

$(AB)^{T} = B^{T}A^{T}$

5. $If\ A=\begin{bmatrix}2 & 4 & -1\\-1 & 0 & 2\end{bmatrix},\begin{bmatrix}3 & 4\\-1 & 2\\2 & 1\end{bmatrix},\ find\ (AB)^T$

Solution:

Given

$A=\begin{bmatrix}2 & 4 & -1\\-1 & 0 & 2\end{bmatrix},\begin{bmatrix}3 & 4\\-1 & 2\\2 & 1\end{bmatrix}$

Now we have to find $(AB)^T$

$$\Rightarrow \left(\begin{bmatrix}2 & 4 & -1\\-1 & 0 & 2\end{bmatrix}\begin{bmatrix}3 & 4\\-1 & 2\\2 & 1\end{bmatrix}\right)^T$$

$$\Rightarrow \begin{bmatrix}6-4-2 & 8+8-1\\-3-0+4 & -4+0+2\end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix}0 & 15\\1 & -2\end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix}0 & 1\\15 & -2\end{bmatrix}$$

So,

$$(AB)^T=\begin{bmatrix}0 & 1\\15 & -2\end{bmatrix}$$

EXERCISE 5.5 PAGE NO: 5.60

1. $If\ A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, \ prove\ that\ A - A^T\ is\ a\ skew - symmetric\ matrix.$

Solution:

Given

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

Consider,

$$(A - A^T) = \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^T\right)$$

$$= \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2-2 & 3-4 \\ 4-3 & 5-5 \end{bmatrix}$$

$$(A - A^T) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \ldots (i)$$

$$-(A - A^T)^T = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^T$$

$$= -\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$-(A - A^T) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \ldots (ii)$$

From (i) and (ii) we can see that

A skew-symmetric matrix is a square matrix whose transpose equal to its negative, that is,

$X = -X^T$

So, $A - A^T$ is a skew-symmetric.

2. $If\ A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}, \ show\ that\ A - A^T\ is\ a\ skew - symmetric\ matrix.$

Solution:

Given

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

Consider,

$(A - A^T) = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$ ... (i)

$-(A - A^T)^T = -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}^T$

$= -\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$

$-(A - A^T) = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$ ... (ii)

From (i) and (ii) we can see that

A skew-symmetric matrix is a square matrix whose transpose equals its negative, that is, $X = -X^T$

So, $A - A^T$ is a skew-symmetric matrix.

$3.\ If\ the\ matrix\ A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix},\ is\ a\ symmetric\ matrix\ matrix\ find\ x, y, z\ and\ t$

Solution:

Given,

$A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$ is a symmetric matrix.

We know that $A = [a_{ij}]_{m \times n}$ is a symmetric matrix if $a_{ij} = a_{ji}$

So,

$x = a_{13} = a_{31} = 4$

$y = a_{21} = a_{12} = 2$

$z = a_{22} = a_{22} = z$

$t = a_{32} = a_{23} = -3$

Hence, x = 4, y = 2, t = -3 and z can have any value.

4. Let $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$**. Find matrices X and Y such that X + Y = A, where X is a symmetric and y is a skew-symmetric matrix.**

Solution:

Given, $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$ Then $A^T = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}$

$$X = \frac{1}{2}(A + A^T)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 3+3 & 2+1 & 7-2 \\ 1+2 & 4+4 & 3+5 \\ -2+7 & 5+3 & 8+8 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 3 & 5 \\ 3 & 8 & 8 \\ 5 & 8 & 16 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}$$

Now,

$$Y = \frac{1}{2}(A - A^T)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 3-3 & 2-1 & 7+2 \\ 1-2 & 4-4 & 3-5 \\ -2-7 & 5-3 & 8-8 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 1 & 9 \\ -1 & 0 & -2 \\ -9 & 2 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ -\frac{1}{2} & 0 & -1 \\ -\frac{9}{2} & 1 & 0 \end{bmatrix}$$

Now,

$$X^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} = X$$

$\Rightarrow$ X is a symmetric matrix.

Now,

$$-Y^T = -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ -\frac{1}{2} & 0 & -1 \\ -\frac{9}{2} & 1 & 0 \end{bmatrix}^T = -\begin{bmatrix} 0 & -\frac{1}{2} & -\frac{9}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{9}{2} & -1 & 0 \end{bmatrix}$$

$$-Y^T = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ -\frac{1}{2} & 0 & -1 \\ -\frac{9}{2} & 1 & 0 \end{bmatrix}$$

$-Y^T = Y$

Y is a skew symmetric matrix.

And,

$$X + Y = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ -\frac{1}{2} & 0 & -1 \\ -\frac{9}{2} & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & \frac{3}{2}+\frac{1}{2} & \frac{5}{2}+\frac{9}{2} \\ \frac{3}{2}-\frac{1}{2} & 4+0 & 4-1 \\ \frac{5}{2}-\frac{9}{2} & 4+1 & 8+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} = A$$

Hence, X + Y = A