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1. Find the area of the triangle with vertices at the points:
(i) (3, 8), (-4, 2) and (5, -1)
(ii) (2, 7), (1, 1) and (10, 8)
(iii) (-1, -8), (-2, -3) and (3, 2)
(iv) (0, 0), (6, 0) and (4, 3)

Solution:

(i) Given (3, 8), (-4, 2) and (5, -1) are the vertices of the triangle.

We know that, if vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[3\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} - 8\begin{vmatrix} -4 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} -4 & 2 \\ 5 & -1 \end{vmatrix}\right]$$

$$= \frac{1}{2}\left[3(3) - 8(-9) + 1(-6)\right]$$

$$= \frac{1}{2}\left[9 + 72 - 6\right]$$

$= \frac{75}{2}$ Square units

Thus area of triangle is $\frac{75}{2}$ square units

(ii) Given (2, 7), (1, 1) and (10, 8) are the vertices of the triangle.

We know that if vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[2\begin{vmatrix} 1 & 1 \\ 8 & 1 \end{vmatrix} - 7\begin{vmatrix} 1 & 1 \\ 10 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 10 & 8 \end{vmatrix}\right]$$

$$= \frac{1}{2}[2(-7) - 7(-9) + 1(-2)]$$

$$= \frac{1}{2}[-14 + 63 - 2]$$

$$= \frac{47}{2} \text{ Square units}$$

Thus area of triangle is $\frac{47}{2}$ square units

(iii) Given (-1, -8), (-2, -3) and (3, 2) are the vertices of the triangle.
We know that if vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} -1 & -8 & 1 \\ -2 & -3 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[-1\begin{vmatrix}-3 & 1\\ 2 & 1\end{vmatrix} - 8\begin{vmatrix}-2 & 1\\ 3 & 1\end{vmatrix} + 1\begin{vmatrix}-2 & -3\\ 3 & 2\end{vmatrix}\right]$$

$$= \frac{1}{2}[-1(-5) - 8(-5) + 1(5)]$$

$$= \frac{1}{2}[5 - 40 + 5]$$

$= \frac{-30}{2}$ Square units

As we know area cannot be negative. Therefore, 15 square unit is the area

Thus area of triangle is 15 square units

(iv) Given (-1, -8), (-2, -3) and (3, 2) are the vertices of the triangle.

We know that if vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 6 & 0 & 1\\ 4 & 3 & 1\end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[0\begin{vmatrix}0 & 1\\ 3 & 1\end{vmatrix} - 0\begin{vmatrix}6 & 1\\ 4 & 1\end{vmatrix} + 1\begin{vmatrix}6 & 0\\ 4 & 3\end{vmatrix}\right]$$

$$= \frac{1}{2}[0 - 0 + 1(18)]$$

$$= \frac{1}{2}[18]$$

= 9 square units

Thus area of triangle is 9 square units

2. Using the determinants show that the following points are collinear:

(i) (5, 5), (-5, 1) and (10, 7)

(ii) (1, -1), (2, 1) and (10, 8)
(iii) (3, -2), (8, 8) and (5, 2)
(iv) (2, 3), (-1, -2) and (5, 8)

Solution:
(i) Given (5, 5), (-5, 1) and (10, 7)
We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix} = 0$$

$$\frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[5\begin{vmatrix} 1 & 1 \\ 7 & 1 \end{vmatrix} - 5\begin{vmatrix} -5 & 1 \\ 10 & 1 \end{vmatrix} + 1\begin{vmatrix} -5 & 1 \\ 10 & 7 \end{vmatrix}\right]$$

$$= \frac{1}{2}[5(-6) - 5(-15) + 1(-45)]$$

$$= \frac{1}{2}[-35 + 75 - 45]$$

= 0

Since, Area of triangle is zero

Hence, points are collinear

(ii) Given (1, -1), (2, 1) and (10, 8)
We have the condition that three points to be collinear, the area of the triangle formed

by these points will be zero. Now, we know that, vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now, by substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix} = 0$$

$$\frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[1\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix}\right]$$

$$= \frac{1}{2}[1 - 5 + 2 - 4 + 10 - 4]$$

$$= \frac{1}{2}[0]$$

= 0

Since, Area of triangle is zero.

Hence, points are collinear.

(iii) Given (3, -2), (8, 8) and (5, 2)

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now, by substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ 5 & 2 & 1 \end{vmatrix} = 0$$

$$\frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ 5 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[3\begin{vmatrix} 8 & 1 \\ 2 & 1 \end{vmatrix} + 2\begin{vmatrix} 8 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 8 & 8 \\ 5 & 2 \end{vmatrix}\right]$$

$$= \frac{1}{2}[3(6) + 2(3) + 1(-24)]$$

$$= \frac{1}{2}[0]$$

$= 0$

Since, Area of triangle is zero

Hence, points are collinear.

(iv) Given (2, 3), (-1, -2) and (5, 8)

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now, by substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix} = 0$$

$$\frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

Expanding along R_1

$= \frac{1}{2}\left[2\begin{vmatrix}-2 & 1\\ 8 & 1\end{vmatrix} - 3\begin{vmatrix}-1 & 1\\ 5 & 1\end{vmatrix} + 1\begin{vmatrix}-1 & -2\\ 5 & 8\end{vmatrix}\right]$

$= \frac{1}{2}[2(-10) - 3(-1-5) + 1(-8+10)]$

$= \frac{1}{2}[-20 + 18 + 2]$

= 0

Since, Area of triangle is zero

Hence, points are collinear.

3. If the points (a, 0), (0, b) and (1, 1) are collinear, prove that a + b = ab

Solution:

Given (a, 0), (0, b) and (1, 1) are collinear

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix} = 0$$

Thus

$$\frac{1}{2}\begin{vmatrix}a & 0 & 1\\ 0 & b & 1\\ 1 & 1 & 1\end{vmatrix} = 0$$

Expanding along R_1

$\Rightarrow 0 = \frac{1}{2}\left[a\begin{vmatrix}b & 1\\ 1 & 1\end{vmatrix} - 0\begin{vmatrix}0 & 1\\ 1 & 1\end{vmatrix} + 1\begin{vmatrix}0 & b\\ 1 & 1\end{vmatrix}\right]$

$\Rightarrow \frac{1}{2}[a(b-1) - 0(-1) + 1(-b)] = 0$

$\Rightarrow \frac{1}{2}[ab - a - b] = 0$

$\Rightarrow$ a + b = ab

Hence Proved

4. Using the determinants prove that the points (a, b), (a', b') and (a - a', b - b) are

collinear if a b' = a' b.

Solution:

Given (a, b), (a', b') and (a - a', b - b) are collinear

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Thus

$$\frac{1}{2}\begin{vmatrix} a & b & 1 \\ a' & b' & 1 \\ a-a' & b-b' & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow 0 = \frac{1}{2}\left[a\begin{vmatrix} b' & 1 \\ b-b' & 1 \end{vmatrix} - b\begin{vmatrix} a' & 1 \\ a-a' & 1 \end{vmatrix} + 1\begin{vmatrix} a' & b' \\ a-a' & b-b' \end{vmatrix}\right]$$

$$\Rightarrow \frac{1}{2}[a(b' - b + b') - b(a' - a + a') + 1(a'b - a'b' - ab' + a'b')] = 0$$

$$\Rightarrow \frac{1}{2}[a'b - ab + ab' - a'b + ab + a'b + a'b - a'b' - ab' + a'b'] = 0$$

$\Rightarrow ab' - a'b = 0$

$\Rightarrow$ a b' = a' b

Hence, the proof.

5. Find the value of λ so that the points (1, -5), (-4, 5) and (λ, 7) are collinear.

Solution:

Given (1, -5), (-4, 5) and (λ, 7) are collinear

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now, by substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & -5 & 1 \\ -4 & 5 & 1 \\ \lambda & 7 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow \frac{1}{2}\left[1\begin{vmatrix} 5 & 1 \\ 7 & 1 \end{vmatrix} + 5\begin{vmatrix} -4 & 1 \\ \lambda & 1 \end{vmatrix} + 1\begin{vmatrix} -4 & 5 \\ \lambda & 7 \end{vmatrix}\right] = 0$$

$$\Rightarrow \frac{1}{2}[1(-2) + 5(-4-\lambda) + 1(-28-5\lambda)] = 0$$

$$\Rightarrow \frac{1}{2}[-2-20-5\lambda-28-5\lambda] = 0$$

$\Rightarrow -50 - 10\lambda = 0$

$\Rightarrow \lambda = -5$

6. Find the value of x if the area of Δ is 35 square cms with vertices (x, 4), (2, -6) and (5, 4).

Solution:

Given (x, 4), (2, -6) and (5, 4) are the vertices of a triangle.

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, by substituting given value in above formula

$$\Rightarrow 35 = \left|\frac{1}{2}\begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}\right|$$

Removing modulus

$$\Rightarrow \pm 2 \times 35 = \begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

Expanding along R_1

$$\Rightarrow \left[x\begin{vmatrix} -6 & 1 \\ 4 & 1 \end{vmatrix} - 4\begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & -6 \\ 5 & 4 \end{vmatrix}\right] = \pm 70$$

⇒ [x (– 10) – 4(– 3) + 1(8 – 30)] = ± 70

⇒ [– 10x + 12 + 38] = ± 70

⇒ ±70 = – 10x + 50

Taking positive sign, we get

⇒ + 70 = – 10x + 50

⇒ 10x = – 20

⇒ x = – 2

Taking –negative sign, we get

⇒ – 70 = – 10x + 50

⇒ 10x = 120

⇒ x = 12

Thus x = – 2, 12