EXERCISE 6.4

Solve the following system of linear equations by Cramer's rule:

1. x – 2y = 4

-3x + 5y = -7

Solution:

Given x – 2y = 4

-3x + 5y = -7

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\vdots \quad \vdots \quad \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

x – 2y = 4

– 3x + 5y = – 7

So by comparing with the theorem, let's find D, D_1 and D_2

$\Rightarrow \ D = \begin{vmatrix} 1 & -2 \\ -3 & 5 \end{vmatrix}$

Solving determinant, expanding along 1st row

⇒ D = 5(1) – (– 3) (– 2)

⇒ D = 5 – 6

⇒ D = – 1

Again,

$\Rightarrow \ D_1 = \begin{vmatrix} 4 & -2 \\ -7 & 5 \end{vmatrix}$

Solving determinant, expanding along 1st row

⇒ D_1 = 5(4) – (– 7) (– 2)

⇒ D_1 = 20 – 14

⇒ D_1 = 6

And

$\Rightarrow \ D_2 = \begin{vmatrix} 1 & 4 \\ -3 & -7 \end{vmatrix}$

Solving determinant, expanding along 1st row

⇒ D_2 = 1(– 7) – (– 3) (4)

⇒ D_2 = – 7 + 12

⇒ D_2 = 5

Thus by Cramer’s Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{6}{-1}$

⇒ x = – 6

And

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{5}{-1}$

⇒ y = – 5

2. 2x – y = 1
7x – 2y = -7

Solution:

Given 2x – y = 1 and

7x – 2y = -7

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\vdots \ \vdots \ \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

2x – y = 1

7x – 2y = – 7

So by comparing with the theorem, let's find D, D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 2 & -1 \\ 7 & -2 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$\Rightarrow D_1 = 1(-2) - (-7)(-1)$

$\Rightarrow D_1 = -2 - 7$

$\Rightarrow D_1 = -9$

And

$$\Rightarrow D_2 = \begin{vmatrix} 2 & 1 \\ 7 & -7 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D_2 = 2(-7) - (7)(1)$

$\Rightarrow D_2 = -14 - 7$

$\Rightarrow D_2 = -21$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{-9}{3}$

$\Rightarrow x = -3$

And $\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-21}{3}$

$\Rightarrow y = -7$

3. 2x – y = 17
3x + 5y = 6

Solution:

Given 2x – y = 17 and

3x + 5y = 6

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\vdots \; \vdots \; \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let D} = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

$2x - y = 17$

$3x + 5y = 6$

So by comparing with the theorem, let's find D, D_1 and D_2

$\Rightarrow D = \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D_1 = 17(5) - (6)(-1)$

$\Rightarrow D_1 = 85 + 6$

$\Rightarrow D_1 = 91$

$\Rightarrow D_2 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D_2 = 2(6) - (17)(3)$

$\Rightarrow D_2 = 12 - 51$

$\Rightarrow D_2 = -39$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{91}{13}$

$\Rightarrow x = 7$

And $\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-39}{13}$

$\Rightarrow y = -3$

4. 3x + y = 19
3x – y = 23

Solution:
Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$

$$\vdots \; \vdots \; \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

3x + y = 19

3x – y = 23

So by comparing with the theorem, let's find D, D_1 and D_2

$$\Rightarrow \; D = \begin{vmatrix} 3 & 1 \\ 3 & -1 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row
⇒ D = 3(– 1) – (3) (1)
⇒ D = – 3 – 3

⇒ D = – 6

Again,

$\Rightarrow \quad D_1 = \begin{vmatrix} 19 & 1 \\ 23 & -1 \end{vmatrix}$

Solving determinant, expanding along 1st row

⇒ D_1 = 19(– 1) – (23) (1)

⇒ D_1 = – 19 – 23

⇒ D_1 = – 42

$\Rightarrow \quad D_2 = \begin{vmatrix} 3 & 19 \\ 3 & 23 \end{vmatrix}$

Solving determinant, expanding along 1st row

⇒ D_2 = 3(23) – (19) (3)

⇒ D_2 = 69 – 57

⇒ D_2 = 12

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{-42}{-6}$

⇒ x = 7

And $\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{12}{-6}$

⇒ y = – 2

5. 2x – y = -2

3x + 4y = 3

Solution:

Given 2x – y = -2 and

3x + 4y = 3

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

⋮ ⋮ ⋮

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

$2x - y = -2$

$3x + 4y = 3$

So by comparing with the theorem, let's find D, D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$\Rightarrow D = 2(4) - (3)(-1)$

$\Rightarrow D = 8 + 3$

$\Rightarrow D = 11$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} -2 & -1 \\ 3 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$\Rightarrow D_1 = -2(4) - (3)(-1)$

$\Rightarrow D_1 = -8 + 3$

$\Rightarrow D_1 = -5$

$$\Rightarrow D_2 = \begin{vmatrix} 2 & -2 \\ 3 & 3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

⇒ $D_2 = 3(2) - (-2)(3)$

⇒ $D_2 = 6 + 6$

⇒ $D_2 = 12$

Thus by Cramer's Rule, we have

⇒ $x = \frac{D_1}{D}$

⇒ $x = \frac{-5}{11}$

And ⇒ $y = \frac{D_2}{D}$

⇒ $y = \frac{12}{11}$

6. 3x + ay = 4
2x + ay = 2, a ≠ 0

Solution:

Given 3x + ay = 4 and

2x + ay = 2, a ≠ 0

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$

$$\vdots \;\vdots\; \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

3x + ay = 4

2x + ay = 2, a≠0

So by comparing with the theorem, let's find D, D_1 and D_2

$\Rightarrow D = \begin{vmatrix} 3 & a \\ 2 & a \end{vmatrix}$

Solving determinant, expanding along 1st row

⇒ D = 3(a) – (2) (a)

⇒ D = 3a – 2a

⇒ D = a

Again,

$\Rightarrow D_1 = \begin{vmatrix} 4 & a \\ 2 & a \end{vmatrix}$

Solving determinant, expanding along 1st row

⇒ D_1 = 4(a) – (2) (a)

⇒ D = 4a – 2a

⇒ D = 2a

$\Rightarrow D_2 = \begin{vmatrix} 3 & 4 \\ 2 & 2 \end{vmatrix}$

Solving determinant, expanding along 1st row

⇒ D_2 = 3(2) – (2) (4)

⇒ D = 6 – 8

⇒ D = – 2

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{2a}{a}$

⇒ x = 2

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-2}{a}$

7. 2x + 3y = 10

x + 6y = 4

Solution:

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \vdots \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

Let $D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

2x + 3y = 10

x + 6y = 4

So by comparing with the theorem, let's find D, D_1 and D_2

$\Rightarrow D = \begin{vmatrix} 2 & 3 \\ 1 & 6 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D = 2(6) - (3)(1)$

$\Rightarrow D = 12 - 3$

$\Rightarrow D = 9$

Again,

$\Rightarrow \ D_1 = \begin{vmatrix} 10 & 3 \\ 4 & 6 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D_1 = 10 (6) – (3) (4)$

$\Rightarrow D = 60 – 12$

$\Rightarrow D = 48$

$\Rightarrow \ D_2 = \begin{vmatrix} 2 & 10 \\ 1 & 4 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D_2 = 2 (4) – (10) (1)$

$\Rightarrow D_2 = 8 – 10$

$\Rightarrow D_2 = – 2$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{48}{9}$

$\Rightarrow x = \frac{16}{3}$

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-2}{9}$

$\Rightarrow y = \frac{-2}{9}$

8. 5x + 7y = -2
4x + 6y = -3

Solution:

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \ \vdots \ \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

$5x + 7y = -2$

$4x + 6y = -3$

So by comparing with the theorem, let's find D, D_1 and D_2

$\Rightarrow D = \begin{vmatrix} 5 & 7 \\ 4 & 6 \end{vmatrix}$

Solving determinant, expanding along 1^{st} row

$\Rightarrow D = 5(6) - (7)(4)$

$\Rightarrow D = 30 - 28$

$\Rightarrow D = 2$

Again,

$\Rightarrow D_1 = \begin{vmatrix} -2 & 7 \\ -3 & 6 \end{vmatrix}$

Solving determinant, expanding along 1^{st} row

$\Rightarrow D_1 = -2(6) - (7)(-3)$

$\Rightarrow D_1 = -12 + 21$

$\Rightarrow D_1 = 9$

$\Rightarrow D_2 = \begin{vmatrix} 5 & -2 \\ 4 & -3 \end{vmatrix}$

Solving determinant, expanding along 1^{st} row

$\Rightarrow D_2 = -3(5) - (-2)(4)$

$\Rightarrow D_2 = -15 + 8$

$\Rightarrow D_2 = -7$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{9}{2}$

$\Rightarrow x = \frac{9}{2}$

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-7}{2}$

$\Rightarrow y = \frac{-7}{2}$

9. 9x + 5y = 10
3y – 2x = 8

Solution:
Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \; \vdots \; \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

9x + 5y = 10

3y – 2x = 8

So by comparing with the theorem, let's find D, D_1 and D_2

$\Rightarrow \ D = \begin{vmatrix} 9 & 5 \\ -2 & 3 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D = 3(9) - (5)(-2)$

$\Rightarrow D = 27 + 10$

$\Rightarrow D = 37$

Again,

$\Rightarrow \ D_1 = \begin{vmatrix} 10 & 5 \\ 8 & 3 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D_1 = 10(3) - (8)(5)$

$\Rightarrow D_1 = 30 - 40$

$\Rightarrow D_1 = -10$

$\Rightarrow \ D_2 = \begin{vmatrix} 9 & 10 \\ -2 & 8 \end{vmatrix}$

Solving determinant, expanding along 1st row

$\Rightarrow D_2 = 9(8) - (10)(-2)$

$\Rightarrow D_2 = 72 + 20$

$\Rightarrow D_2 = 92$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{-10}{37}$

$\Rightarrow x = \frac{-10}{37}$

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{92}{37}$

$\Rightarrow y = \frac{92}{37}$

10. x + 2y = 1
3x + y = 4

Solution:

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$

$\vdots \; \vdots \; \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$

Let $D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

x + 2y = 1

3x + y = 4

So by comparing with theorem, now we have to find D, D_1 and D_2

$\Rightarrow \quad D = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$

Solving determinant, expanding along 1^{st} row

$\Rightarrow D = 1(1) – (3) (2)$

$\Rightarrow D = 1 – 6$

$\Rightarrow D = – 5$

Again,

$\Rightarrow \quad D_1 = \begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix}$

Solving determinant, expanding along 1^{st} row

$\Rightarrow D_1 = 1(1) – (2) (4)$

$\Rightarrow D_1 = 1 - 8$
$\Rightarrow D_1 = -7$
$\Rightarrow D_2 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix}$
Solving determinant, expanding along 1st row
$\Rightarrow D_2 = 1(4) - (1)(3)$
$\Rightarrow D_2 = 4 - 3$
$\Rightarrow D_2 = 1$
Thus by Cramer's Rule, we have
$\Rightarrow x = \frac{D_1}{D}$
$\Rightarrow x = \frac{-7}{-5}$
$\Rightarrow x = \frac{7}{5}$
$\Rightarrow y = \frac{D_2}{D}$
$\Rightarrow y = \frac{1}{-5}$
$\Rightarrow y = -\frac{1}{5}$

Solve the following system of linear equations by Cramer's rule:
11. 3x + y + z = 2
2x – 4y + 3z = -1
4x + y – 3z = -11

Solution:
Let there be a system of n simultaneous linear equations and with n unknown given by
$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$
$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$
$\vdots \; \vdots \; \vdots$
$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

$3x + y + z = 2$

$2x - 4y + 3z = -1$

$4x + y - 3z = -11$

So by comparing with the theorem, let's find D, D_1, D_2 and D_3

$$\Rightarrow D = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -4 & 3 \\ 4 & 1 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D = 3[(-4)(-3) - (3)(1)] - 1[(2)(-3) - 12] + 1[2 - 4(-4)]$

$\Rightarrow D = 3[12 - 3] - [-6 - 12] + [2 + 16]$

$\Rightarrow D = 27 + 18 + 18$

$\Rightarrow D = 63$

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 2 & 1 & 1 \\ -1 & -4 & 3 \\ -11 & 1 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D_1 = 2[(-4)(-3) - (3)(1)] - 1[(-1)(-3) - (-11)(3)] + 1[(-1) - (-4)(-11)]$

$\Rightarrow D_1 = 2[12 - 3] - 1[3 + 33] + 1[-1 - 44]$

$\Rightarrow D_1 = 2[9] - 36 - 45$

$\Rightarrow D_1 = 18 - 36 - 45$

$\Rightarrow D_1 = -63$

Again

$$\Rightarrow D_2 = \begin{vmatrix} 3 & 2 & 1 \\ 2 & -1 & 3 \\ 4 & -11 & -3 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D_2 = 3[3 + 33] - 2[-6 - 12] + 1[-22 + 4]$

$\Rightarrow D_2 = 3[36] - 2(-18) - 18$

$\Rightarrow D_2 = 126$

$$\Rightarrow D_3 = \begin{vmatrix} 3 & 1 & 2 \\ 2 & -4 & -1 \\ 4 & 1 & -11 \end{vmatrix}$$

Solving determinant, expanding along 1st row

$\Rightarrow D_3 = 3[44 + 1] – 1[– 22 + 4] + 2[2 + 16]$

$\Rightarrow D_3 = 3[45] – 1(– 18) + 2(18)$

$\Rightarrow D_3 = 135 + 18 + 36$

$\Rightarrow D_3 = 189$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{-63}{63}$

$\Rightarrow x = -1$

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{126}{63}$

$\Rightarrow y = 2$

$\Rightarrow z = \frac{D_3}{D}$

$\Rightarrow z = \frac{189}{63}$

$\Rightarrow z = 3$

12. x – 4y – z = 11
2x – 5y + 2z = 39
-3x + 2y + z = 1

Solution:

Given,

$x – 4y – z = 11$

$2x – 5y + 2z = 39$

$-3x + 2y + z = 1$

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$

$\vdots \; \vdots \; \vdots$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \ldots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

x – 4y – z = 11

2x – 5y + 2z = 39

– 3x + 2y + z = 1

So by comparing with theorem, now we have to find D, D_1 and D_2

$$\Rightarrow D = \begin{vmatrix} 1 & -4 & -1 \\ 2 & -5 & 2 \\ -3 & 2 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

⇒ D = 1[(– 5) (1) – (2) (2)] + 4[(2) (1) + 6] – 1[4 + 5(– 3)]

⇒ D = 1[– 5 – 4] + 4[8] – [– 11]

⇒ D = – 9 + 32 + 11

⇒ D = 34

Again,

$$\Rightarrow D_1 = \begin{vmatrix} 11 & -4 & -1 \\ 39 & -5 & 2 \\ 1 & 2 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} row

$\Rightarrow D_1 = 11[(-5)(1) - (2)(2)] + 4[(39)(1) - (2)(1)] - 1[2(39) - (-5)(1)]$
$\Rightarrow D_1 = 11[-5 - 4] + 4[39 - 2] - 1[78 + 5]$
$\Rightarrow D_1 = 11[-9] + 4(37) - 83$
$\Rightarrow D_1 = -99 - 148 - 45$
$\Rightarrow D_1 = -34$
Again

$$\Rightarrow D_2 = \begin{vmatrix} 1 & 11 & -1 \\ 2 & 39 & 2 \\ -3 & 1 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1st row
$\Rightarrow D_2 = 1[39 - 2] - 11[2 + 6] - 1[2 + 117]$
$\Rightarrow D_2 = 1[37] - 11(8) - 119$
$\Rightarrow D_2 = -170$
And,

$$\Rightarrow D_3 = \begin{vmatrix} 1 & -4 & 11 \\ 2 & -5 & 39 \\ -3 & 2 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1st row
$\Rightarrow D_3 = 1[-5 - (39)(2)] - (-4)[2 - (39)(-3)] + 11[4 - (-5)(-3)]$
$\Rightarrow D_3 = 1[-5 - 78] + 4(2 + 117) + 11(4 - 15)$
$\Rightarrow D_3 = -83 + 4(119) + 11(-11)$
$\Rightarrow D_3 = 272$
Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{-34}{34}$

$\Rightarrow x = -1$
Again,

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-170}{34}$

$\Rightarrow y = -5$

$\Rightarrow z = \frac{D_3}{D}$ = (272/34) = 8

13. 6x + y – 3z = 5
x + 3y – 2z = 5

2x + y + 4z = 8

Solution:

Given

6x + y – 3z = 5

x + 3y – 2z = 5

2x + y + 4z = 8

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\vdots \; \vdots \; \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

6x + y – 3z = 5

x + 3y – 2z = 5

2x + y + 4z = 8

So by comparing with theorem, now we have to find D , D_1 and D_2

$$\Rightarrow \; D = \begin{vmatrix} 6 & 1 & -3 \\ 1 & 3 & -2 \\ 2 & 1 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

⇒ D = 6[(4) (3) – (1) (– 2)] – 1[(4) (1) + 4] – 3[1 – 3(2)]
⇒ D = 6[12 + 2] – [8] – 3[– 5]
⇒ D = 84 – 8 + 15
⇒ D = 91
Again, Solve D_1 formed by replacing 1st column by B matrices
Here

$$B = \begin{vmatrix} 5 \\ 5 \\ 8 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 5 & 1 & -3 \\ 5 & 3 & -2 \\ 8 & 1 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1st Row
⇒ D_1 = 5[(4) (3) – (– 2) (1)] – 1[(5) (4) – (– 2) (8)] – 3[(5) – (3) (8)]
⇒ D_1 = 5[12 + 2] – 1[20 + 16] – 3[5 – 24]
⇒ D_1 = 5[14] – 36 – 3(– 19)
⇒ D_1 = 70 – 36 + 57
⇒ D_1 = 91
Again, Solve D_2 formed by replacing 1st column by B matrices
Here

$$B = \begin{vmatrix} 5 \\ 5 \\ 8 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 6 & 5 & -3 \\ 1 & 5 & -2 \\ 2 & 8 & 4 \end{vmatrix}$$

Solving determinant
⇒ D_2 = 6[20 + 16] – 5[4 – 2(– 2)] + (– 3)[8 – 10]
⇒ D_2 = 6[36] – 5(8) + (– 3) (– 2)
⇒ D_2 = 182
And, Solve D_3 formed by replacing 1st column by B matrices
Here

$$B = \begin{vmatrix} 5 \\ 5 \\ 8 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 6 & 1 & 5 \\ 1 & 3 & 5 \\ 2 & 1 & 8 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

⇒ $D_3 = 6[24 - 5] - 1[8 - 10] + 5[1 - 6]$

⇒ $D_3 = 6[19] - 1(-2) + 5(-5)$

⇒ $D_3 = 114 + 2 - 25$

⇒ $D_3 = 91$

Thus by Cramer's Rule, we have

⇒ $x = \frac{D_1}{D}$

⇒ $x = \frac{91}{91}$

⇒ $x = 1$

⇒ $y = \frac{D_2}{D}$

⇒ $y = \frac{182}{91}$

⇒ $y = 2$

⇒ $z = \frac{D_3}{D}$

⇒ $z = \frac{91}{91}$

⇒ $z = 1$

14. x + y = 5
y + z = 3
x + z = 4

Solution:

Given x + y = 5

y + z = 3

x + z = 4

Let there be a system of n simultaneous linear equations and with n unknown given by

$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$

⋮ ⋮ ⋮

$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \ldots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

x + y = 5

y + z = 3

x + z = 4

So by comparing with theorem, now we have to find D, D_1 and D_2

$$\Rightarrow \quad D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

⇒ D = 1[1] – 1[– 1] + 0[– 1]

⇒ D = 1 + 1 + 0

⇒ D = 2

Again, Solve D_1 formed by replacing 1^{st} column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 3 \\ 4 \end{vmatrix}$$

$$\Rightarrow \quad D_1 = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

⇒ D_1 = 5[1] – 1[(3) (1) – (4) (1)] + 0[0 – (4) (1)]

⇒ D_1 = 5 – 1[3 – 4] + 0[– 4]

⇒ D_1 = 5 – 1[– 1] + 0

⇒ D_1 = 5 + 1 + 0

⇒ D_1 = 6

Again, Solve D_2 formed by replacing 1^{st} column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 3 \\ 4 \end{vmatrix}$$

$$\Rightarrow \ D_2 = \begin{vmatrix} 1 & 5 & 0 \\ 0 & 3 & 1 \\ 1 & 4 & 1 \end{vmatrix}$$

Solving determinant

$\Rightarrow D_2 = 1[3 - 4] - 5[- 1] + 0[0 - 3]$

$\Rightarrow D_2 = 1[- 1] + 5 + 0$

$\Rightarrow D_2 = 4$

And, Solve D_3 formed by replacing 1st column by B matrices

Here

$$B = \begin{vmatrix} 5 \\ 3 \\ 4 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 0 & 1 & 3 \\ 1 & 0 & 4 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$\Rightarrow D_3 = 1[4 - 0] - 1[0 - 3] + 5[0 - 1]$

$\Rightarrow D_3 = 1[4] - 1(- 3) + 5(- 1)$

$\Rightarrow D_3 = 4 + 3 - 5$

$\Rightarrow D_3 = 2$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{6}{2}$

$\Rightarrow x = 3$

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{4}{2}$

$\Rightarrow y = 2$

$\Rightarrow z = \frac{D_3}{D}$

$\Rightarrow z = \frac{2}{2}$

$\Rightarrow z = 1$

15. 2y – 3z = 0
x + 3y = -4
3x + 4y = 3

Solution:
Given
2y – 3z = 0
x + 3y = -4
3x + 4y = 3
Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\vdots \; \vdots \; \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D} \text{ Provided that } D \neq 0$$

Now, here we have
2y – 3z = 0
x + 3y = – 4
3x + 4y = 3
So by comparing with theorem, now we have to find D, D_1 and D_2

$$\Rightarrow \; D = \begin{vmatrix} 0 & 2 & -3 \\ 1 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

Solving determinant, expanding along 1st Row
⇒ D = 0[0] – 2[(0) (1) – 0] – 3[1 (4) – 3 (3)]
⇒ D = 0 – 0 – 3[4 – 9]
⇒ D = 0 – 0 + 15
⇒ D = 15
Again, Solve D_1 formed by replacing 1st column by B matrices
Here

$$B = \begin{vmatrix} 0 \\ -4 \\ 3 \end{vmatrix}$$

$$\Rightarrow D_1 = \begin{vmatrix} 0 & 2 & -3 \\ -4 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

Solving determinant, expanding along 1st Row
⇒ D_1 = 0[0] – 2[(0) (– 4) – 0] – 3[4 (– 4) – 3(3)]
⇒ D_1 = 0 – 0 – 3[– 16 – 9]
⇒ D_1 = 0 – 0 – 3(– 25)
⇒ D_1 = 0 – 0 + 75
⇒ D_1 = 75
Again, Solve D_2 formed by replacing 2nd column by B matrices
Here

$$B = \begin{vmatrix} 0 \\ -4 \\ 3 \end{vmatrix}$$

$$\Rightarrow D_2 = \begin{vmatrix} 0 & 0 & -3 \\ 1 & -4 & 0 \\ 3 & 3 & 0 \end{vmatrix}$$

Solving determinant
⇒ D_2 = 0[0] – 0[(0) (1) – 0] – 3[1 (3) – 3(– 4)]
⇒ D_2 = 0 – 0 + (– 3) (3 + 12)
⇒ D_2 = – 45
And, Solve D_3 formed by replacing 3rd column by B matrices
Here

$$B = \begin{vmatrix} 0 \\ -4 \\ 3 \end{vmatrix}$$

$$\Rightarrow D_3 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & -4 \\ 3 & 4 & 3 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$\Rightarrow D_3 = 0[9 - (-4) 4] - 2[(3)(1) - (-4)(3)] + 0[1(4) - 3(3)]$

$\Rightarrow D_3 = 0[25] - 2(3 + 12) + 0(4 - 9)$

$\Rightarrow D_3 = 0 - 30 + 0$

$\Rightarrow D_3 = -30$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{75}{15}$

$\Rightarrow x = 5$

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-45}{15}$

$\Rightarrow y = -3$

$\Rightarrow z = \frac{D_3}{D}$

$\Rightarrow z = \frac{-30}{15}$

$\Rightarrow z = -2$

16. 5x – 7y + z = 11
6x – 8y – z = 15
3x + 2y – 6z = 7

Solution:

Given

$5x - 7y + z = 11$

$6x - 8y - z = 15$

$3x + 2y - 6z = 7$

Let there be a system of n simultaneous linear equations and with n unknown given by

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\vdots \; \vdots \; \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

$$\text{Let } D = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n1} & \dots & a_{nn} \end{vmatrix}$$

Let D_j be the determinant obtained from D after replacing the j^{th} column by

$$\begin{vmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix}$$

Then,

$x_1 = \frac{D_1}{D}, x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D}$ Provided that $D \neq 0$

Now, here we have

5x – 7y + z = 11

6x – 8y – z = 15

3x + 2y – 6z = 7

So by comparing with theorem, now we have to find D, D_1 and D_2

$$\Rightarrow \quad D = \begin{vmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{vmatrix}$$

Solving determinant, expanding along 1^{st} Row

⇒ D = 5[(– 8) (– 6) – (– 1) (2)] – 7[(– 6) (6) – 3(– 1)] + 1[2(6) – 3(– 8)]

⇒ D = 5[48 + 2] – 7[– 36 + 3] + 1[12 + 24]

⇒ D = 250 – 231 + 36

⇒ D = 55

Again, Solve D_1 formed by replacing 1^{st} column by B matrices

Here

$$B = \begin{vmatrix} 11 \\ 15 \\ 7 \end{vmatrix}$$

$$\Rightarrow\ D_1 = \begin{vmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$\Rightarrow D_1 = 11[(-8)(-6) - (2)(-1)] - (-7)[(15)(-6) - (-1)(7)] + 1[(15)2 - (7)(-8)]$

$\Rightarrow D_1 = 11[48 + 2] + 7[-90 + 7] + 1[30 + 56]$

$\Rightarrow D_1 = 11[50] + 7[-83] + 86$

$\Rightarrow D_1 = 550 - 581 + 86$

$\Rightarrow D_1 = 55$

Again, Solve D_2 formed by replacing 2nd column by B matrices

Here

$$B = \begin{vmatrix} 11 \\ 15 \\ 7 \end{vmatrix}$$

$$\Rightarrow\ D_2 = \begin{vmatrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$\Rightarrow D_2 = 5[(15)(-6) - (7)(-1)] - 11[(6)(-6) - (-1)(3)] + 1[(6)7 - (15)(3)]$

$\Rightarrow D_2 = 5[-90 + 7] - 11[-36 + 3] + 1[42 - 45]$

$\Rightarrow D_2 = 5[-83] - 11(-33) - 3$

$\Rightarrow D_2 = -415 + 363 - 3$

$\Rightarrow D_2 = -55$

And, Solve D_3 formed by replacing 3rd column by B matrices

Here

$$B = \begin{vmatrix} 11 \\ 15 \\ 7 \end{vmatrix}$$

$$\Rightarrow\ D_3 = \begin{vmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{vmatrix}$$

Solving determinant, expanding along 1st Row

$\Rightarrow D_3 = 5[(-8)(7) - (15)(2)] - (-7)[(6)(7) - (15)(3)] + 11[(6)2 - (-8)(3)]$

$\Rightarrow D_3 = 5[-56 - 30] - (-7)[42 - 45] + 11[12 + 24]$

$\Rightarrow D_3 = 5[-86] + 7[-3] + 11[36]$

$\Rightarrow D_3 = -430 - 21 + 396$

$\Rightarrow D_3 = -55$

Thus by Cramer's Rule, we have

$\Rightarrow x = \frac{D_1}{D}$

$\Rightarrow x = \frac{55}{55}$

$\Rightarrow x = 1$

$\Rightarrow y = \frac{D_2}{D}$

$\Rightarrow y = \frac{-55}{55}$

$\Rightarrow y = -1$

$\Rightarrow z = \frac{D_3}{D}$

$\Rightarrow z = \frac{-55}{55}$

$\Rightarrow z = -1$