## EXERCISE 23.1

1. Find the slopes of the lines which make the following angles with the positive direction of x - axis:
(i) $-\pi / 4$
(ii) $2 \pi / 3$

Solution:
(i) $-\pi / 4$

Let the slope of the line be ' $m$ '
Where, $m=\tan \theta$
So, the slope of Line is $m=\tan (-\pi / 4)$

$$
=-1
$$

$\therefore$ The slope of the line is -1 .
(ii) $2 \pi / 3$

Let the slope of the line be ' $m$ '
Where, $m=\tan \theta$
So, the slope of Line is $\mathrm{m}=\tan (2 \pi / 3)$
$\tan \left(\frac{2 \pi}{3}\right)=\tan \left(\pi-\frac{\pi}{3}\right)$
$\tan \left(\frac{2 \pi}{3}\right)=\tan \left(-\frac{\pi}{3}\right)$
$\tan \left(\frac{2 \pi}{3}\right)=-\sqrt{3}$
$\therefore$ The slope of the line is $-\sqrt{ } 3$
2. Find the slopes of a line passing through the following points :
(i) $(-3,2)$ and $(1,4)$
(ii) $\left(\mathrm{at}^{2}{ }_{\mathbf{1}}, \mathbf{2 a t} \mathbf{t}_{1}\right)$ and $\left(\mathrm{at}^{2}{ }_{2}, \mathbf{2 a t _ { 2 }}\right)$

Solution:
(i) $(-3,2)$ and $(1,4)$

By using the formula,
Slope of line, $\mathrm{m}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
So, the slope of the line, $m=\frac{4-2}{1-(-3)}$

$$
\begin{aligned}
& =2 / 4 \\
& =1 / 2
\end{aligned}
$$

$\therefore$ The slope of the line is $1 / 2$.
(ii) $\left(\mathrm{at}^{2}, 2 \mathrm{at}_{1}\right)$ and $\left(\mathrm{at}^{2}{ }_{2}, 2 \mathrm{at}_{2}\right)$

By using the formula,
Slope of line, $\mathrm{m}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
Now, substitute the values
The slope of the line, $\mathrm{m}=\frac{2 \mathrm{at}_{2}-2 \mathrm{at}}{\mathrm{at}_{2}^{2}-\mathrm{at}_{1}^{2}}$

$$
\begin{aligned}
& =\frac{2 a\left(t_{t_{2}}-t_{1}\right)}{a\left(t_{2}^{2}-t_{1}^{2}\right)} \\
& =\frac{2 a\left(t_{2}-t_{1}\right)}{\left.a\left(t_{2}-t_{1}\right) t_{2}+t_{1}\right)}\left[\text { Since, }\left(a^{2-} b^{2}=(a-b)(a+b)\right]\right. \\
& =\frac{2}{t_{2}+t_{1}}
\end{aligned}
$$

$\therefore$ The slope of the line is $\frac{2}{t_{2}+t_{1}}$
3. State whether the two lines in each of the following are parallel, perpendicular or neither:
(i) Through $(5,6)$ and $(2,3)$; through $(9,-2)$ and $(6,-5)$
(ii) Through $(9,5)$ and $(-1,1)$; through $(3,-5)$ and $(8,-3)$

Solution:
(i) Through $(5,6)$ and $(2,3)$; through $(9,-2)$ and $(6,-5)$

By using the formula,
Slope of line, $\mathrm{m}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
The slope of the line whose Coordinates are $(5,6)$ and $(2,3)$

$$
\begin{aligned}
\mathrm{m}_{1} & =\frac{3-6}{2-5} \\
& =\frac{-3}{-3} \\
& =1
\end{aligned}
$$

So, $\mathrm{m}_{1}=1$
The slope of the line whose Coordinates are $(9,-2)$ and $(6,-5)$

$$
\begin{aligned}
\mathrm{m}_{2} & =\frac{-5-(-2)}{6-9} \\
& =\frac{-3}{-3}
\end{aligned}
$$

So, $\mathrm{m}_{2}=1$

Here, $\mathrm{m}_{1}=\mathrm{m}_{2}=1$
$\therefore$ The lines are parallel to each other.
(ii) Through $(9,5)$ and $(-1,1)$; through $(3,-5)$ and $(8,-3)$

By using the formula,
Slope of line, $\mathrm{m}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
The slope of the line whose Coordinates are $(9,5)$ and ( $-1,1$ )

$$
\begin{aligned}
\mathrm{m}_{1} & =\frac{1-5}{-1-9} \\
& =\frac{-4}{-10} \\
& =2 / 5
\end{aligned}
$$

So, $\mathrm{m}_{1}=2 / 5$
The slope of the line whose Coordinates are $(3,-5)$ and $(8,-3)$

$$
\begin{aligned}
\mathrm{m}_{2} & =\frac{-3-(-5)}{8-3} \\
& =2 / 5
\end{aligned}
$$

So, $\mathrm{m}_{2}=2 / 5$
Here, $\mathrm{m}_{1}=\mathrm{m}_{2}=2 / 5$
$\therefore$ The lines are parallel to each other.

## 4. Find the slopes of a line

(i) which bisects the first quadrant angle
(ii) which makes an angle of $30^{\circ}$ with the positive direction of $\mathbf{y}$ - axis measured anticlockwise.

## Solution:

(i) Which bisects the first quadrant angle?

Given: Line bisects the first quadrant
We know that, if the line bisects in the first quadrant, then the angle must be between line and the positive direction of x - axis.
Since, angle $=90 / 2=45^{\circ}$
By using the formula,
The slope of the line, $m=\tan \theta$
The slope of the line for a given angle is $\mathrm{m}=\tan 45^{\circ}$
So, $\mathrm{m}=1$
$\therefore$ The slope of the line is 1 .
(ii) Which makes an angle of $30^{\circ}$ with the positive direction of $y$-axis measured
anticlockwise?
Given: The line makes an angle of $30^{\circ}$ with the positive direction of $y-$ axis.
We know that, angle between line and positive side of axis $=>90^{\circ}+30^{\circ}=120^{\circ}$
By using the formula,
The slope of the line, $m=\tan \theta$
The slope of the line for a given angle is $m=\tan 120^{\circ}$
So, $m=-\sqrt{ } 3$
$\therefore$ The slope of the line is $-\sqrt{ } 3$.
5. Using the method of slopes show that the following points are collinear:
(i) $\mathrm{A}(4,8), \mathrm{B}(5,12), \mathrm{C}(9,28)$
(ii) $\mathbf{A}(16,-18), \mathrm{B}(3,-6), \mathrm{C}(-10,6)$

## Solution:

(i) $\mathrm{A}(4,8), \mathrm{B}(5,12), \mathrm{C}(9,28)$

By using the formula,
The slope of the line $=\left[y_{2}-y_{1}\right] /\left[x_{2}-x_{1}\right]$
So,
The slope of line $\mathrm{AB}=[12-8] /[5-4]$

$$
=4 / 1
$$

The slope of line $\mathrm{BC}=[28-12] /[9-5]$

$$
\begin{aligned}
& =16 / 4 \\
& =4
\end{aligned}
$$

The slope of line $\mathrm{CA}=[8-28] /[4-9]$

$$
\begin{aligned}
& =-20 /-5 \\
& =4
\end{aligned}
$$

Here, $\mathrm{AB}=\mathrm{BC}=\mathrm{CA}$
$\therefore$ The Given points are collinear.
(ii) $\mathrm{A}(16,-18), \mathrm{B}(3,-6), \mathrm{C}(-10,6)$

By using the formula,
The slope of the line $=\left[y_{2}-y_{1}\right] /\left[x_{2}-x_{1}\right]$
So,
The slope of line $\mathrm{AB}=[-6-(-18)] /[3-16]$

$$
=12 /-13
$$

The slope of line $\mathrm{BC}=[6-(-6)] /[-10-3]$

$$
=12 /-13
$$

The slope of line CA $=[6-(-18)] /[-10-16]$
$=12 /-13$

$$
=4
$$

Here, $\mathrm{AB}=\mathrm{BC}=\mathrm{CA}$
$\therefore$ The Given points are collinear.

## EXERCISE 23.2

## 1. Find the equation of the line parallel to $x$-axis and passing through $(\mathbf{3},-5)$.

 Solution:Given: A line which is parallel to x -axis and passing through $(3,-5)$
By using the formula,
The equation of line: $\left[y-y_{1}=m\left(x-x_{1}\right)\right]$
We know that the parallel lines have equal slopes
And, the slope of x -axis is always 0
Then
The slope of line, $\mathrm{m}=0$
Coordinates of line are $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(3,-5)$
The equation of line $=y-y_{1}=m\left(x-x_{1}\right)$
Now, substitute the values, we get
$y-(-5)=0(x-3)$
$y+5=0$
$\therefore$ The equation of line is $\mathrm{y}+5=0$
2. Find the equation of the line perpendicular to $x$-axis and having intercept -2 on x -axis.

## Solution:

Given: A line which is perpendicular to x -axis and having intercept -2
By using the formula,
The equation of line: $\left[y-y_{1}=m\left(x-x_{1}\right)\right]$
We know that, the line is perpendicular to the x -axis, then x is 0 and y is -1 .
The slope of line is, $m=y / x$

$$
=-1 / 0
$$

It is given that x -intercept is -2 , so, y is 0 .
Coordinates of line are $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(-2,0)$
The equation of line $=y-y_{1}=m\left(x-x_{1}\right)$
Now, substitute the values, we get
$\mathrm{y}-0=(-1 / 0)(\mathrm{x}-(-2))$
$\mathrm{x}+2=0$
$\therefore$ The equation of line is $\mathrm{x}+2=0$
3. Find the equation of the line parallel to x -axis and having intercept - 2 on y axis.
Solution:

Given: A line which is parallel to x -axis and having intercept -2 on $\mathrm{y}-$ axis
By using the formula,
The equation of line: $\left[y-y_{1}=m\left(x-x_{1}\right)\right]$
The parallel lines have equal slopes,
And, the slope of x -axis is always 0
Then
The slope of line, $\mathrm{m}=0$
It is given that intercept is -2 , on $\mathrm{y}-$ axis then
Coordinates of line are $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(0,-2)$
The equation of line is $\mathrm{y}-\mathrm{y}_{1}=\mathrm{m}\left(\mathrm{x}-\mathrm{x}_{1}\right)$
Now, substitute the values, we get
$y-(-2)=0(x-0)$
$y+2=0$
$\therefore$ The equation of line is $\mathrm{y}+2=0$
4. Draw the lines $x=-3, x=2, y=-2, y=3$ and write the coordinates of the vertices of the square so formed.
Solution:
Given: $\mathrm{x}=-3, \mathrm{x}=2, \mathrm{y}=-2$ and $\mathrm{y}=3$

$\therefore$ The Coordinates of the square are: $\mathrm{A}(2,3), \mathrm{B}(2,-2), \mathrm{C}(-3,3)$, and $\mathrm{D}(-3,-2)$.
5. Find the equations of the straight lines which pass through $(4,3)$ and are respectively parallel and perpendicular to the $x$-axis.
Solution:

Given: A line which is perpendicular and parallel to x -axis respectively and passing through $(4,3)$
By using the formula,
The equation of line: $\left[y-y_{1}=m\left(x-x_{1}\right)\right]$
Let us consider,
Case 1: When Line is parallel to x -axis
The parallel lines have equal slopes,
And, the slope of x -axis is always 0 , then
The slope of line, $\mathrm{m}=0$
Coordinates of line are $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(4,3)$
The equation of line is $y-y_{1}=m\left(x-x_{1}\right)$
Now substitute the values, we get
$\mathrm{y}-(3)=0(\mathrm{x}-4)$
$y-3=0$
Case 2: When line is perpendicular to $x$-axis
The line is perpendicular to the x -axis, then x is 0 and y is -1 .
The slope of the line is, $m=y / x$

$$
=-1 / 0
$$

Coordinates of line are $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(4,3)$
The equation of line $=y-y_{1}=m\left(x-x_{1}\right)$
Now substitute the values, we get
$\mathrm{y}-3=(-1 / 0)(\mathrm{x}-4)$
$\mathrm{x}=4$
$\therefore$ The equation of line when it is parallel to $\mathrm{x}-$ axis is $\mathrm{y}=3$ and it is perpendicular is $\mathrm{x}=$ 4.

## EXERCISE 23.3

## 1. Find the equation of a line making an angle of $150^{\circ}$ with the $x$-axis and cutting off an intercept 2 from $y$-axis.

## Solution:

Given: A line which makes an angle of $150^{\circ}$ with the x -axis and cutting off an intercept at 2
By using the formula,
The equation of a line is $y=m x+c$
We know that angle, $\theta=150^{\circ}$
The slope of the line, $m=\tan \theta$
Where, $\mathrm{m}=\tan 150^{\circ}$

$$
=-1 / \sqrt{3}
$$

Coordinate of $y$-intercept is $(0,2)$
The required equation of the line is $y=m x+c$
Now substitute the values, we get
$y=-x / \sqrt{3}+2$
$\sqrt{3} y-2 \sqrt{3}+x=0$
$x+\sqrt{ } 3 y=2 \sqrt{ } 3$
$\therefore$ The equation of line is $x+\sqrt{ } 3 y=2 \sqrt{ } 3$
2. Find the equation of a straight line:
(i) with slope 2 and $y$ - intercept 3 ;
(ii) with slope $-1 / 3$ and $y-$ intercept -4 .
(iii) with slope -2 and intersecting the $x$-axis at a distance of 3 units to the left of origin.

## Solution:

(i) With slope 2 and $y$ - intercept 3

The slope is 2 and the coordinates are $(0,3)$
Now, the required equation of line is
$y=m x+c$
Substitute the values, we get
$y=2 x+3$
(ii) With slope $-1 / 3$ and $y$-intercept - 4

The slope is $-1 / 3$ and the coordinates are $(0,-4)$
Now, the required equation of line is
$y=m x+c$

Substitute the values, we get
$y=-1 / 3 x-4$
$3 y+x=-12$
(iii) With slope -2 and intersecting the x -axis at a distance of 3 units to the left of origin The slope is -2 and the coordinates are $(-3,0)$
Now, the required equation of line is $\mathrm{y}-\mathrm{y}_{1}=\mathrm{m}\left(\mathrm{x}-\mathrm{x}_{1}\right)$
Substitute the values, we get
$y-0=-2(x+3)$
$y=-2 x-6$
$2 x+y+6=0$

## 3. Find the equations of the bisectors of the angles between the coordinate axes. Solution:

There are two bisectors of the coordinate axes.
Their inclinations with the positive x -axis are $45^{\circ}$ and $135^{\circ}$
The slope of the bisector is $\mathrm{m}=\tan 45^{\circ}$ or $\mathrm{m}=\tan 135^{\circ}$
i.e., $m=1$ or $m=-1, c=0$

By using the formula, $\mathrm{y}=\mathrm{mx}+\mathrm{c}$
Now, substitute the values of $m$ and $c$, we get
$y=x+0$
$x-y=0$ or $y=-x+0$
$x+y=0$
$\therefore$ The equation of the bisector is $\mathrm{x} \pm \mathrm{y}=0$
4. Find the equation of a line which makes an angle of $\tan ^{-1}(3)$ with the $x$-axis and cuts off an intercept of 4 units on the negative direction of $y$-axis.

## Solution:

Given:
The equation which makes an angle of $\tan ^{-1}(3)$ with the $x$-axis and cuts off an intercept of 4 units on the negative direction of $y$-axis
By using the formula,
The equation of the line is $y=m x+c$
Here, angle $\theta=\tan ^{-1}(3)$
So, $\tan \theta=3$
The slope of the line is, $\mathrm{m}=3$
And, Intercept in the negative direction of $y$-axis is ( $0,-4$ )
The required equation of the line is $y=m x+c$
Now, substitute the values, we get

$$
y=3 x-4
$$

$\therefore$ The equation of the line is $\mathrm{y}=3 \mathrm{x}-4$.
5. Find the equation of a line that has $y$ - intercept -4 and is parallel to the line joining $(2,-5)$ and $(1,2)$.
Solution:
Given:
A line segment joining $(2,-5)$ and $(1,2)$ if it cuts off an intercept -4 from $y$-axis By using the formula,
The equation of line is $y=m x+C$
It is given that, $\mathrm{c}=-4$
Slope of line joining $\left(\mathrm{x}_{1}-\mathrm{x}_{2}\right)$ and $\left(\mathrm{y}_{1}-\mathrm{y}_{2}\right)$,

$$
\mathrm{m}=\frac{\mathrm{y}_{2}-\mathrm{y}_{1}}{\mathrm{x}_{2}-\mathrm{x}_{1}}
$$

So, Slope of line joining $(2,-5)$ and $(1,2)$,

$$
\begin{aligned}
& \mathrm{m}=\frac{2-(-5)}{1-2}=\frac{7}{-1} \\
& \mathrm{~m}=-7
\end{aligned}
$$

The equation of line is $y=m x+c$
Now, substitute the values, we get
$y=-7 x-4$
$y+7 x+4=0$
$\therefore$ The equation of line is $\mathrm{y}+7 \mathrm{x}+4=0$.

## EXERCISE 23.4

1. Find the equation of the straight line passing through the point $(6,2)$ and having slope - 3.

## Solution:

Given, A straight line passing through the point $(6,2)$ and the slope is -3
By using the formula,
The equation of line is $\left[y-y_{1}=m\left(x-x_{1}\right)\right.$ ]
Here, the line is passing through $(6,2)$
It is given that, the slope of line, $\mathrm{m}=-3$
Coordinates of line are $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(6,2)$
The equation of line $=y-y_{1}=m\left(x-x_{1}\right)$
Now, substitute the values, we get
$y-2=-3(x-6)$
$y-2=-3 x+18$
$y+3 x-20=0$
$\therefore$ The equation of line is $3 \mathrm{x}+\mathrm{y}-20=0$
2. Find the equation of the straight line passing through $(-2,3)$ and indicated at an angle of $45^{\circ}$ with the $x$-axis.

## Solution:

Given:
A line which is passing through $(-2,3)$, the angle is $45^{\circ}$.
By using the formula,
The equation of line is $\left[y-y_{1}=m\left(x-x_{1}\right)\right]$
Here, angle, $\theta=45^{\circ}$
The slope of the line, $\mathrm{m}=\tan \theta$

$$
\begin{aligned}
\mathrm{m} & =\tan 45^{\circ} \\
& =1
\end{aligned}
$$

The line passing through $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(-2,3)$
The required equation of line is $\mathrm{y}-\mathrm{y}_{1}=\mathrm{m}\left(\mathrm{x}-\mathrm{x}_{1}\right)$
Now, substitute the values, we get
$y-3=1(x-(-2))$
$y-3=x+2$
$x-y+5=0$
$\therefore$ The equation of line is $\mathrm{x}-\mathrm{y}+5=0$

## 3. Find the equation of the line passing through $(0,0)$ with slope $m$ Solution:

Given:
A straight line passing through the point $(0,0)$ and slope is $m$.
By using the formula,
The equation of line is $\left[y-y_{1}=m\left(x-x_{1}\right)\right]$
It is given that, the line is passing through $(0,0)$ and the slope of line, $m=m$
Coordinates of line are $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(0,0)$
The equation of line $=y-y_{1}=m\left(x-x_{1}\right)$
Now, substitute the values, we get
$\mathrm{y}-0=\mathrm{m}(\mathrm{x}-0)$
$y=m x$
$\therefore$ The equation of line is $\mathrm{y}=\mathrm{mx}$.
4. Find the equation of the line passing through $(2,2 \sqrt{ } 3)$ and inclined with $x-a x i s$ at an angle of $75^{\circ}$.
Solution:
Given:
A line which is passing through $(2,2 \sqrt{ } 3)$, the angle is $75^{\circ}$.
By using the formula,
The equation of line is $\left[y-y_{1}=m\left(x-x_{1}\right)\right.$ ]
Here, angle, $\theta=75^{\circ}$
The slope of the line, $m=\tan \theta$
$\mathrm{m}=\tan 75^{\circ}$

$$
=3.73=2+\sqrt{3}
$$

The line passing through $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(2,2 \sqrt{ } 3)$
The required equation of the line is $y-y_{1}=m\left(x-x_{1}\right)$
Now, substitute the values, we get
$y-2 \sqrt{ } 3=(2+\sqrt{ } 3)(x-2)$
$y-2 \sqrt{3}=(2+\sqrt{3}) x-7.46$
$(2+\sqrt{3}) x-y-4=0$
$\therefore$ The equation of the line is $(2+\sqrt{ } 3) x-y-4=0$
5. Find the equation of the straight line which passes through the point $(1,2)$ and makes such an angle with the positive direction of $x-$ axis whose sine is $3 / 5$. Solution:
A line which is passing through $(1,2)$
To Find: The equation of a straight line.
By using the formula,
The equation of line is $\left[y-y_{1}=m\left(x-x_{1}\right)\right]$
Here, $\sin \theta=3 / 5$

We know, $\sin \theta=$ perpendicular/hypotenuse

$$
=3 / 5
$$

So, according to Pythagoras theorem, $(\text { Hypotenuse })^{2}=(\text { Base })^{2}+(\text { Perpendicular })^{2}$
$(5)^{2}=(\text { Base })^{2}+(3)^{2}$
$($ Base $)=\sqrt{ }(25-9)$
$(\text { Base })^{2}=\sqrt{ } 16$
Base $=4$
Hence, $\tan \theta=$ perpendicular/base

$$
=3 / 4
$$

The slope of the line, $\mathrm{m}=\tan \theta$

$$
=3 / 4
$$

The line passing through $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(1,2)$
The required equation of line is $y-y_{1}=m\left(x-x_{1}\right)$
Now, substitute the values, we get
$\mathrm{y}-2=(3 / 4)(\mathrm{x}-1)$
$4 y-8=3 x-3$
$3 x-4 y+5=0$
$\therefore$ The equation of line is $3 x-4 y+5=0$

## EXERCISE 23.5

## 1. Find the equation of the straight lines passing through the following pair of points:

(i) $(0,0)$ and (2, -2)
(ii) $(a, b)$ and $(a+c \sin \alpha, b+c \cos \alpha)$

## Solution:

(i) $(0,0)$ and $(2,-2)$

Given:
$\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(0,0),\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)=(2,-2)$
The equation of the line passing through the two points $(0,0)$ and $(2,-2)$ is
By using the formula,

$$
y-y_{1}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\left(x-x_{1}\right)
$$

Now, substitute the values, we get

$$
y-0=\frac{-2-0}{2-0}(x-0)
$$

$$
y=-x
$$

$\therefore$ The equation of line is $\mathrm{y}=-\mathrm{x}$
(ii) $(\mathrm{a}, \mathrm{b})$ and $(\mathrm{a}+\mathrm{c} \sin \alpha, \mathrm{b}+\mathrm{c} \cos \alpha)$

Given:
$\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=(\mathrm{a}, \mathrm{b}),\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)=(\mathrm{a}+\mathrm{c} \sin \alpha, \mathrm{b}+\mathrm{c} \cos \alpha)$
So, the equation of the line passing through the two points $(0,0)$ and $(2,-2)$ is
By using the formula,

$$
\mathrm{y}-\mathrm{y}_{1}=\frac{\mathrm{y}_{2}-\mathrm{y}_{1}}{\mathrm{x}_{2}-\mathrm{x}_{1}}\left(\mathrm{x}-\mathrm{x}_{1}\right)
$$

Now, substitute the values, we get

$$
y-b=\frac{b+c \cos \alpha-b}{a+c \sin \alpha-a}(x-a)
$$

$\mathrm{y}-\mathrm{b}=\cot \alpha(\mathrm{x}-\mathrm{a})$
$\therefore$ The equation of line is $\mathrm{y}-\mathrm{b}=\cot \alpha(\mathrm{x}-\mathrm{a})$
2. Find the equations to the sides of the triangles the coordinates of whose angular points are respectively:
(i) $(1,4),(2,-3)$ and $(-1,-2)$
(ii) $(0,1),(2,0)$ and $(-1,-2)$

## Solution:

(i) $(1,4),(2,-3)$ and $(-1,-2)$

Given:

Points A $(1,4)$, B (2, -3 ) and C ( $-1,-2$ ).
Let us assume,
$\mathrm{m}_{1}, \mathrm{~m}_{2}$, and $\mathrm{m}_{3}$ be the slope of the sides $\mathrm{AB}, \mathrm{BC}$ and CA , respectively.
So,
The equation of the line passing through the two points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$.
Then,

$$
\begin{aligned}
& \mathrm{m}_{1}=\frac{-3-4}{2-1}, \\
& \mathrm{~m}_{2}=\frac{-2+3}{-1-2}, \\
& \mathrm{~m}_{3}=\frac{4+2}{1+1}
\end{aligned}
$$

$m_{1}=-7, m_{2}=-1 / 3$ and $m_{3}=3$
So, the equation of the sides $\mathrm{AB}, \mathrm{BC}$ and CA are
By using the formula,

$$
\begin{aligned}
& y-y_{1}=m\left(x-x_{1}\right) \\
& \Rightarrow>-4=-7(x-1) \\
& y-4=-7 x+7 \\
& y+7 x=11, \\
& \Rightarrow y+3=(-1 / 3)(x-2) \\
& \Rightarrow y+9=-x+2 \\
& 3 y+x=-7 \\
& x+3 y+7=0 \text { and } \\
& \Rightarrow y+2=3(x+1) \\
& \Rightarrow>+2=3 x+3 \\
& y+3 x=1
\end{aligned}
$$

So, we get
$y+7 x=11, x+3 y+7=0$ and $y-3 x=1$
$\therefore$ The equation of sides are $\mathrm{y}+7 \mathrm{x}=11, \mathrm{x}+3 \mathrm{y}+7=0$ and $\mathrm{y}-3 \mathrm{x}=1$
(ii) $(0,1),(2,0)$ and $(-1,-2)$

Given:
Points A $(0,1), \mathrm{B}(2,0)$ and $\mathrm{C}(-1,-2)$.
Let us assume,
$\mathrm{m}_{1}, \mathrm{~m}_{2}$, and $\mathrm{m}_{3}$ be the slope of the sides $\mathrm{AB}, \mathrm{BC}$ and $C A$, respectively.
So,
The equation of the line passing through the two points $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$.
Then,

$$
\begin{aligned}
\mathrm{m}_{1} & =\frac{0-1}{2-0} \\
\mathrm{~m}_{2} & =\frac{-2-0}{-1-2} \\
\mathrm{~m}_{3} & =\frac{1+2}{1+0} \\
\mathrm{~m}_{1} & =-1 / 2, \mathrm{~m}_{2}=2 / 3 \text { and } \mathrm{m}_{3}=3
\end{aligned}
$$

So, the equation of the sides $\mathrm{AB}, \mathrm{BC}$ and CA are
By using the formula,

$$
\begin{gathered}
y-y_{1}=m\left(x-x_{1}\right) \\
\Rightarrow y_{1}-1=(-1 / 2)(x-0) \\
2 y-2=-x \\
x+2 y=2 \\
\Rightarrow y-0=(2 / 3)(x-2) \\
3 y=2 x-4 \\
2 x-3 y=4 \\
\Rightarrow y+2=3(x+1) \\
\Rightarrow y+2=3 x+3 \\
y-3 x=1
\end{gathered}
$$

So, we get
$x+2 y=2,2 x-3 y=4$ and $y-3 x=1$
$\therefore$ The equation of sides are $\mathrm{x}+2 \mathrm{y}=2,2 \mathrm{x}-3 \mathrm{y}=4$ and $\mathrm{y}-3 \mathrm{x}=1$

## 3. Find the equations of the medians of a triangle, the coordinates of whose vertices

 are $(-1,6),(-3,-9)$ and $(5,-8)$.
## Solution:

## Given:

A $(-1,6), \mathrm{B}(-3,-9)$ and $\mathrm{C}(5,-8)$ be the coordinates of the given triangle.
Let us assume: D, E, and F be midpoints of BC, CA and AB, respectively. So, the coordinates of $\mathrm{D}, \mathrm{E}$ and F are


Median AD passes through A $(-1,6)$ and $\mathrm{D}(1,-17 / 2)$
So, by using the formula,

$$
\begin{aligned}
& y-y_{1}=\left(\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\right)\left(x-x_{1}\right) \\
& y-6=\frac{-\frac{17}{2}-6}{1+1}(x+1)
\end{aligned}
$$

$$
4 y-24=-29 x-29
$$

$$
29 x+4 y+5=0
$$

Similarly, Median BE passes through B $(-3,-9)$ and E $(2,-1)$
So, by using the formula,

$$
\begin{aligned}
& y-y_{1}=\left(\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\right)\left(x-x_{1}\right) \\
& y+9=\frac{-1+9}{2+3}(x+3) \\
& 5 y+45=8 x+24 \\
& 8 x-5 y-21=0
\end{aligned}
$$

Similarly, Median CF passes through C (5,-8) and F(-2,-3/2)
So, by using the formula,

$$
\begin{aligned}
& y-y_{1}=\left(\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\right)\left(x-x_{1}\right) \\
& y+8=\frac{-\frac{3}{2}+8}{-2-5}(x-5) \\
& y+8=[(-3+16) /(2(-7))](x-5) \\
& y+8=(-13 / 14)(x-5) \\
& -14 y-112=13 x-65 \\
& 13 x+14 y+47=0
\end{aligned}
$$

$\therefore$ The equation of lines are: $29 x+4 y+5=0,8 x-5 y-21=0$ and $13 x+14 y+47=0$

## 4. Find the equations to the diagonals of the rectangle the equations of whose sides

 are $x=a, x=a^{\prime}, y=b$ and $y=b^{\prime}$.
## Solution:

## Given:

The rectangle formed by the lines $\mathrm{x}=\mathrm{a}, \mathrm{x}=\mathrm{a}^{\prime}, \mathrm{y}=\mathrm{b}$ and $\mathrm{y}=\mathrm{b}^{\prime}$
It is clear that, the vertices of the rectangle are $A(a, b), B\left(a^{\prime}, b\right), C\left(a^{\prime}, b^{\prime}\right)$ and $D\left(a, b^{\prime}\right)$.
The diagonal passing through $\mathrm{A}(\mathrm{a}, \mathrm{b})$ and $\mathrm{C}\left(\mathrm{a}^{\prime}, \mathrm{b}^{\prime}\right)$ is
By using the formula,

$$
\begin{aligned}
& y-y_{1}=\left(\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\right)\left(x-x_{1}\right) \\
& y-b=\frac{b^{\prime}-b}{a^{\prime}-a}(x-a) \\
& \left(a^{\prime}-a\right) y-b\left(a^{\prime}-a\right)=\left(b^{\prime}-b\right) x-a\left(b^{\prime}-b\right) \\
& \left(a^{\prime}-a\right)-\left(b^{\prime}-b\right) x=b a^{\prime}-a b^{\prime}
\end{aligned}
$$

Similarly, the diagonal passing through $B\left(a^{\prime}, b\right)$ and $D\left(a, b^{\prime}\right)$ is
By using the formula,

$$
\begin{aligned}
& y-y_{1}=\left(\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\right)\left(x-x_{1}\right) \\
& y-b=\frac{b^{\prime}-b}{a-a^{\prime}}\left(x-a^{\prime}\right) \\
& \left(a^{\prime}-a\right) y-b\left(a-a^{\prime}\right)=\left(b^{\prime}-b\right) x-a^{\prime}\left(b^{\prime}-b\right) \\
& \left(a^{\prime}-a\right)+\left(b^{\prime}-b\right) x=a^{\prime} b^{\prime}-a b
\end{aligned}
$$

$\therefore$ The equation of diagonals are $\mathrm{y}\left(\mathrm{a}^{\prime}-\mathrm{a}\right)-\mathrm{x}\left(\mathrm{b}^{\prime}-\mathrm{b}\right)=\mathrm{a}^{\prime} \mathrm{b}-\mathrm{ab}$ ' and $y\left(a^{\prime}-a\right)+x\left(b^{\prime}-b\right)=a^{\prime} b^{\prime}-a b$
5. Find the equation of the side $B C$ of the triangle $A B C$ whose vertices are $A(-1,-2)$, $B(0,1)$ and $C(2,0)$ respectively. Also, find the equation of the median through $A(-$ 1, -2).

## Solution:

Given:
The vertices of triangle ABC are $\mathrm{A}(-1,-2), \mathrm{B}(0,1)$ and $\mathrm{C}(2,0)$.
Let us find the equation of median through $A$.
So, the equation of BC is
By using the formula,

$$
\begin{aligned}
& y-y_{1}=\left(\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\right)\left(x-x_{1}\right) \\
& y-1=\frac{0-1}{2-0}(x-0) \\
& y-1=\frac{-1}{2}(x-0) \\
& x+2 y-2=0
\end{aligned}
$$

Let D be the midpoint of median AD , So, $D\left(\frac{0+2}{2}, \frac{1+0}{2}\right)=\left(1, \frac{1}{2}\right)$
The equation of the median AD is
By using the formula,
$y-y_{1}=\left(\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\right)\left(x-x_{1}\right)$
$y+2=\frac{\frac{1}{2}+2}{1+1}(x+1)$
$4 y+8=5 x+5$
$5 x-4 y-3=0$
$\therefore$ The equation of line BC is $\mathrm{x}+2 \mathrm{y}-2=0$
The equation of median is $5 x-4 y-3=0$

## EXERCISE 23.6

## 1. Find the equation to the straight line

(i) cutting off intercepts 3 and 2 from the axes.
(ii) cutting off intercepts $\mathbf{- 5}$ and 6 from the axes.

## Solution:

(i) Cutting off intercepts 3 and 2 from the axes.

Given:
$\mathrm{a}=3, \mathrm{~b}=2$
Let us find the equation of line cutoff intercepts from the axes.
By using the formula,
The equation of the line is $x / a+y / b=1$

$$
x / 3+y / 2=1
$$

By taking LCM,

$$
2 x+3 y=6
$$

$\therefore$ The equation of line cut off intercepts 3 and 2 from the axes is $2 x+3 y=6$
(ii) Cutting off intercepts -5 and 6 from the axes.

Given:
$a=-5, b=6$
Let us find the equation of line cutoff intercepts from the axes.
By using the formula,
The equation of the line is $x / a+y / b=1$

$$
x /-5+y / 6=1
$$

By taking LCM,

$$
6 x-5 y=-30
$$

$\therefore$ The equation of line cut off intercepts 3 and 2 from the axes is $6 x-5 y=-30$
2. Find the equation of the straight line which passes through (1,-2) and cuts off equal intercepts on the axes.

## Solution:

## Given:

A line passing through ( $1,-2$ )
Let us assume, the equation of the line cutting equal intercepts at coordinates of length ' $a$ ' is
By using the formula,
The equation of the line is $x / a+y / b=1$

$$
\begin{aligned}
& x / a+y / a=1 \\
& x+y=a
\end{aligned}
$$

The line $\mathrm{x}+\mathrm{y}=\mathrm{a}$ passes through $(1,-2)$
Hence, the point satisfies the equation.
$1-2=\mathrm{a}$
$\mathrm{a}=-1$
$\therefore$ The equation of the line is $\mathrm{x}+\mathrm{y}=-1$
3. Find the equation to the straight line which passes through the point $(5,6)$ and has intercepts on the axes
(i) Equal in magnitude and both positive
(ii) Equal in magnitude but opposite in sign

Solution:
(i) Equal in magnitude and both positive

Given:
$\mathrm{a}=\mathrm{b}$
Let us find the equation of line cutoff intercepts from the axes.
By using the formula,
The equation of the line is $x / a+y / b=1$

$$
\begin{aligned}
& x / a+y / a=1 \\
& x+y=a
\end{aligned}
$$

The line passes through the point $(5,6)$
Hence, the equation satisfies the points.
$5+6=\mathrm{a}$
$\mathrm{a}=11$
$\therefore$ The equation of the line is $\mathrm{x}+\mathrm{y}=11$
(ii) Equal in magnitude but opposite in sign

Given:
$\mathrm{b}=-\mathrm{a}$
Let us find the equation of line cutoff intercepts from the axes.
By using the formula,
The equation of the line is $x / a+y / b=1$

$$
\begin{aligned}
& x / a+y /-a=1 \\
& x-y=a
\end{aligned}
$$

The line passes through the point $(5,6)$
Hence, the equation satisfies the points.
5-6 = a
$\mathrm{a}=-1$
$\therefore$ The equation of the line is $\mathrm{x}-\mathrm{y}=-1$
4. For what values of $a$ and $b$ the intercepts cut off on the coordinate axes by the line $\mathbf{a x}+\mathrm{by}+\mathbf{8}=\mathbf{0}$ are equal in length but opposite in signs to those cut off by the line 2 x $-3 y+6=0$ on the axes.

## Solution:

Given:
Intercepts cut off on the coordinate axes by the line ax + by $+8=0$
And are equal in length but opposite in sign to those cut off by the line
$2 x-3 y+6=0$ $\qquad$
We know that, the slope of two lines is equal
The slope of the line (i) is $-\mathrm{a} / \mathrm{b}$
The slope of the line (ii) is $2 / 3$
So let us equate,
$-\mathrm{a} / \mathrm{b}=2 / 3$
$\mathrm{a}=-2 \mathrm{~b} / 3$
The length of the perpendicular from the origin to the line (i) is
By using the formula,

$$
\begin{aligned}
\mathrm{d} & =\left|\frac{a x+b y+d}{\sqrt{\mathrm{a}^{2}+\mathrm{b}^{2}}}\right| \\
\mathrm{d}_{1} & =\left|\frac{\mathrm{a}(0)+\mathrm{b}(0)+8}{\sqrt{\mathrm{a}^{2}+\mathrm{b}^{2}}}\right| \\
& =\frac{8 \times 3}{\sqrt{13 b^{2}}}
\end{aligned}
$$

The length of the perpendicular from the origin to the line (ii) is
By using the formula,

$$
\begin{aligned}
& \mathrm{d}=\left|\frac{\mathrm{ax}+\mathrm{by}+\mathrm{d}}{\sqrt{\mathrm{a}^{2}+\mathrm{b}^{2}}}\right| \\
& \mathrm{d}_{2}=\left|\frac{2(0)-3(0)+6}{\sqrt{2^{2}+3^{2}}}\right|
\end{aligned}
$$

It is given that, $\mathrm{d}_{1}=\mathrm{d}_{2}$

$$
\frac{8 \times 3}{\sqrt{13 b^{2}}}=\frac{6}{\sqrt{13}}
$$

b $=4$
So, $a=-2 b / 3$

$$
=-8 / 3
$$

$\therefore$ The value of a is $-8 / 3$ and b is 4 .
5. Find the equation to the straight line which cuts off equal positive intercepts on the axes and their product is 25 .
Solution:
Given:
$\mathrm{a}=\mathrm{b}$ and $\mathrm{ab}=25$
Let us find the equation of the line which cutoff intercepts on the axes.
$\therefore \mathrm{a}^{2}=25$
$\mathrm{a}=5$ [considering only positive value of intercepts]
By using the formula,
The equation of the line with intercepts $a$ and $b$ is $x / a+y / b=1$

$$
x / 5+y / 5=1
$$

By taking LCM

$$
x+y=5
$$

$\therefore$ The equation of line is $\mathrm{x}+\mathrm{y}=5$

## EXERCISE 23.7

## 1. Find the equation of a line for which

(i) $p=5, \alpha=60^{\circ}$
(ii) $p=4, \alpha=150^{\circ}$

## Solution:

(i) $p=5, \alpha=60^{\circ}$

Given:
$\mathrm{p}=5, \alpha=60^{\circ}$
The equation of the line in normal form is given by
Using the formula,
$\mathrm{x} \cos \alpha+\mathrm{y} \sin \alpha=\mathrm{p}$
Now, substitute the values, we get
$x \cos 60^{\circ}+y \sin 60^{\circ}=5$
$x / 2+\sqrt{ } 3 y / 2=5$
$x+\sqrt{ } 3 y=10$
$\therefore$ The equation of line in normal form is $\mathrm{x}+\sqrt{ } 3 \mathrm{y}=10$.
(ii) $p=4, \alpha=150^{\circ}$

Given:
$\mathrm{p}=4, \alpha=150^{\circ}$
The equation of the line in normal form is given by
Using the formula,
$\mathrm{x} \cos \alpha+\mathrm{y} \sin \alpha=\mathrm{p}$
Now, substitute the values, we get
$x \cos 150^{\circ}+y \sin 150^{\circ}=4$
$\cos \left(180^{\circ}-\theta\right)=-\cos \theta, \sin \left(180^{\circ}-\theta\right)=\sin \theta$
$\mathrm{x} \cos \left(180^{\circ}-30^{\circ}\right)+\mathrm{y} \sin \left(180^{\circ}-30^{\circ}\right)=4$
$-x \cos 30^{\circ}+y \sin 30^{\circ}=4$
$-\sqrt{ } 3 x / 2+y / 2=4$
$-\sqrt{ } 3 x+y=8$
$\therefore$ The equation of line in normal form is $-\sqrt{ } 3 x+y=8$.
2. Find the equation of the line on which the length of the perpendicular segment from the origin to the line is 4 and the inclination of the perpendicular segment with the positive direction of $x$-axis is $30^{\circ}$.
Solution:
Given:
$\mathrm{p}=4, \alpha=30^{\circ}$

The equation of the line in normal form is given by
Using the formula,
$\mathrm{x} \cos \alpha+\mathrm{y} \sin \alpha=\mathrm{p}$
Now, substitute the values, we get
$\mathrm{x} \cos 30^{\circ}+\mathrm{y} \sin 30^{\circ}=4$
$\mathrm{x} \sqrt{3} / 2+\mathrm{y} 1 / 2=4$
$\sqrt{ } 3 x+y=8$
$\therefore$ The equation of line in normal form is $\sqrt{ } 3 x+y=8$.
3. Find the equation of the line whose perpendicular distance from the origin is 4 units and the angle which the normal makes with the positive direction of $x$-axis is $15^{\circ}$.
Solution:
Given:
$\mathrm{p}=4, \alpha=15^{\circ}$
The equation of the line in normal form is given by
We know that, $\cos 15^{\circ}=\cos \left(45^{\circ}-30^{\circ}\right)=\cos 45^{\circ} \cos 30^{\circ}+\sin 45^{\circ} \sin 30^{\circ}$
$\operatorname{Cos}(\mathrm{A}-\mathrm{B})=\cos \mathrm{A} \cos \mathrm{B}+\sin \mathrm{A} \sin \mathrm{B}$
So,
$\cos 15=\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}} \times \frac{1}{2}=\frac{\sqrt{3}+1}{2 \sqrt{2}}$
And $\sin 15=\sin \left(45^{\circ}-30^{\circ}\right)=\sin 45^{\circ} \cos 30^{\circ}-\cos 45^{\circ} \sin 30^{\circ}$
$\operatorname{Sin}(A-B)=\sin A \cos B-\cos A \sin B$
So,

$$
\sin 15=\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}} \times \frac{1}{2}=\frac{\sqrt{3}-1}{2 \sqrt{2}}
$$

Now, by using the formula,
$\mathrm{x} \cos \alpha+\mathrm{y} \sin \alpha=\mathrm{p}$
Now, substitute the values, we get

$$
\begin{aligned}
& \frac{\sqrt{3}+1}{2 \sqrt{2}} x+\frac{\sqrt{3}-1}{2 \sqrt{2}} y=4 \\
& (\sqrt{3}+1) x+(\sqrt{3}-1) y=8 \sqrt{ } 2
\end{aligned}
$$

$\therefore$ The equation of line in normal form is $(\sqrt{ } 3+1) x+(\sqrt{ } 3-1) y=8 \sqrt{ } 2$.
4. Find the equation of the straight line at a distance of 3 units from the origin such that the perpendicular from the origin to the line makes an angle $\alpha$ given by $\tan \alpha=$
$5 / 12$ with the positive direction of $x$-axis.

## Solution:

Given:
$\mathrm{p}=3, \alpha=\tan ^{-1}(5 / 12)$
So, $\tan \alpha=5 / 12$
$\sin \alpha=5 / 13$
$\cos \alpha=12 / 13$
The equation of the line in normal form is given by
By using the formula,
$\mathrm{x} \cos \alpha+\mathrm{y} \sin \alpha=\mathrm{p}$
Now, substitute the values, we get
$12 \mathrm{x} / 13+5 \mathrm{y} / 13=3$
$12 x+5 y=39$
$\therefore$ The equation of line in normal form is $12 \mathrm{x}+5 \mathrm{y}=39$.
5. Find the equation of the straight line on which the length of the perpendicular from the origin is 2 and the perpendicular makes an angle $\alpha$ with $x$-axis such that $\sin \alpha=1 / 3$.

## Solution:

Given:
$\mathrm{p}=2, \sin \alpha=1 / 3$
We know that $\cos \alpha=\sqrt{ }\left(1-\sin ^{2} \alpha\right)$

$$
\begin{aligned}
& =\sqrt{ }(1-1 / 9) \\
& =2 \sqrt{ } 2 / 3
\end{aligned}
$$

The equation of the line in normal form is given by
By using the formula,
$\mathrm{x} \cos \alpha+\mathrm{y} \sin \alpha=\mathrm{p}$
Now, substitute the values, we get
$\mathrm{x} 2 \sqrt{ } 2 / 3+\mathrm{y} / 3=2$
$2 \sqrt{ } 2 x+y=6$
$\therefore$ The equation of line in normal form is $2 \sqrt{ } 2 \mathrm{x}+\mathrm{y}=6$.

1. A line passes through a point $A(1,2)$ and makes an angle of $60^{\circ}$ with the $x$-axis and intercepts the line $x+y=6$ at the point P. Find AP.

## Solution:

Given:
$\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=\mathrm{A}(1,2), \theta=60^{\circ}$
Let us find the distance AP.
By using the formula,
The equation of the line is given by:

$$
\frac{x-x_{1}}{\cos \theta}=\frac{y-y_{1}}{\sin \theta}=r
$$

Now, substitute the values, we get

$$
\begin{aligned}
& \frac{x-1}{\cos 60 \circ}=\frac{y-2}{\sin 60 \circ}=r \\
& \frac{x-1}{\frac{1}{2}}=\frac{y-2}{\frac{\sqrt{3}}{2}}=r
\end{aligned}
$$

Here, $r$ represents the distance of any point on the line from point $\mathrm{A}(1,2)$.
The coordinate of any point $P$ on this line are $(1+r / 2,2+\sqrt{ } 3 r / 2)$
It is clear that, $P$ lies on the line $x+y=6$
So,

$$
\begin{aligned}
& 1+\frac{r}{2}+2+\frac{\sqrt{3}}{2} r=6 \\
& \frac{\sqrt{3}}{2} r+\frac{r}{2}=3 \\
& r(\sqrt{3}+1)=6 \\
& r=\frac{6}{\sqrt{3}+1}=3(\sqrt{3}-1)
\end{aligned}
$$

$\therefore$ The value of AP is $3(\sqrt{ } 3-1)$
2. If the straight line through the point $P(3,4)$ makes an angle $\pi / 6$ with the $x$-axis and meets the line $12 x+5 y+10=0$ at $Q$, find the length $P Q$.

## Solution:

Given:
$\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=\mathrm{A}(3,4), \theta=\pi / 6=30^{\circ}$
Let us find the length PQ.
By using the formula,
The equation of the line is given by:

$$
\frac{\mathrm{x}-\mathrm{x}_{1}}{\cos \theta}=\frac{\mathrm{y}-\mathrm{y}_{1}}{\sin \theta}=\mathrm{r}
$$

Now, substitute the values, we get

$$
\begin{aligned}
& \frac{x-3}{\cos 300}=\frac{y-4}{\sin 30 \circ}=r \\
& \frac{x-3}{\frac{\sqrt{3}}{2}}-\frac{y-4}{\frac{1}{2}}=r \\
& x-\sqrt{ } 3 y+4 \sqrt{ } 3-3=0
\end{aligned}
$$

Let $\mathrm{PQ}=\mathrm{r}$
Then, the coordinate of Q are given by

$$
\frac{x-3}{\cos 30^{\circ}}=\frac{y-4}{\sin 30^{\circ}}=r
$$

$$
x=3+\frac{\sqrt{3}}{2} r, y=4+\frac{r}{2}
$$

The coordinate of point $Q$ is $\left(3+\frac{\sqrt{3}}{2} r, 4+\frac{r}{2}\right)$
It is clear that, Q lies on the line $12 \mathrm{x}+5 \mathrm{y}+10=0$
So,
$12\left(3+\frac{\sqrt{3}}{2} r\right)+5\left(4+\frac{r}{2}\right)+10=0$
$66+\frac{12 \sqrt{3}+5}{2} \mathrm{r}=0$
$r=-\frac{132}{5+12 \sqrt{3}}$
$\mathrm{PQ}=|\mathrm{r}|=\frac{132}{5+12 \sqrt{3}}$
$\therefore$ The value of $P Q$ is $\frac{132}{5+12 \sqrt{3}}$
3. A straight line drawn through the point $A(2,1)$ making an angle $\pi / 4$ with positive $x-$ axis intersects another line $x+2 y+1=0$ in the point B. Find length AB.

## Solution:

Given:
$\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=\mathrm{A}(2,1), \theta=\pi / 4=45^{\circ}$
Let us find the length AB .
By using the formula,
The equation of the line is given by:

$$
\frac{x-x_{1}}{\cos \theta}=\frac{y-y_{1}}{\sin \theta}=r
$$

Now, substitute the values, we get

$$
\frac{x-2}{\cos 45^{\circ}}=\frac{y-1}{\sin 45^{\circ}}=r
$$

$$
\frac{\mathrm{x}-2}{\frac{1}{\sqrt{2}}}=\frac{\mathrm{y}-1}{\frac{1}{\sqrt{2}}}=\mathrm{r}
$$

$\mathrm{x}-\mathrm{y}-1=0$
Let $A B=r$
Then, the coordinate of $B$ is given by

$$
\begin{aligned}
& \frac{x-2}{\cos 45 \circ}=\frac{y-1}{\sin 45 \circ}=r \\
& x=2+\frac{1}{\sqrt{2}} r, y=1+\frac{r}{\sqrt{2}}
\end{aligned}
$$

The coordinate of point $B$ is $\left(2+\frac{1}{\sqrt{2}} r, 1+\frac{r}{\sqrt{2}}\right)$
It is clear that, $B$ lies on the line $x+2 y+1=0$

$$
\begin{aligned}
& 2+\frac{1}{\sqrt{2}} r+2\left(1+\frac{r}{\sqrt{2}}\right)+1=0 \\
& 5+\frac{3 r}{\sqrt{2}} r=0 \\
& r=\frac{5 \sqrt{2}}{3}
\end{aligned}
$$

$\therefore$ The value of $A B$ is $\frac{5 \sqrt{2}}{3}$
4. A line a drawn through $A(4,-1)$ parallel to the line $3 x-4 y+1=0$. Find the coordinates of the two points on this line which are at a distance of 5 units from $A$. Solution:
Given:
$\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)=\mathrm{A}(4,-1)$
Let us find Coordinates of the two points on this line which are at a distance of 5 units from A.
Given: Line $3 \mathrm{x}-4 \mathrm{y}+1=0$
$4 y=3 x+1$
$y=3 x / 4+1 / 4$
Slope $\tan \theta=3 / 4$
So,
Sin $\theta=3 / 5$
$\operatorname{Cos} \theta=4 / 5$

The equation of the line passing through $\mathrm{A}(4,-1)$ and having slope $3 / 4$ is
By using the formula,
The equation of the line is given by:
$\frac{x-x_{1}}{\cos \theta}=\frac{\mathrm{y}-\mathrm{y}_{1}}{\sin \theta}$
Now, substitute the values, we get
$\frac{x-4}{\frac{4}{5}}=\frac{y+1}{\frac{3}{5}}$
$3 x-4 y=16$
Here, $\mathrm{AP}=\mathrm{r}= \pm 5$
Thus, the coordinates of P are given by
$\frac{\mathrm{x}-\mathrm{x}_{1}}{\cos \theta}=\frac{\mathrm{y}-\mathrm{y}_{1}}{\sin \theta}=\mathrm{r}$
$\frac{\mathrm{x}-4}{\frac{4}{5}}=\frac{\mathrm{y}+1}{\frac{3}{5}}=\mathrm{r}$
$x=\frac{4 r}{5}+4$ and $y=\frac{3 r}{5}-1$
$x=\frac{4( \pm 5)}{5}+4$ and $y=\frac{3( \pm 5)}{5}-1$
$\mathrm{x}= \pm 4+4$ and $\mathrm{y}= \pm 3-1$
$x=8,0$ and $y=2,-4$
$\therefore$ The coordinates of the two points at a distance of 5 units from A are $(8,2)$ and $(0,-4)$.
5. The straight line through $P\left(x_{1}, y_{1}\right)$ inclined at an angle $\theta$ with the $x$-axis meets the line $a x+b y+c=0$ in $Q$. Find the length of PQ.

## Solution:

## Given:

The equation of the line that passes through $P\left(x_{1}, y_{1}\right)$ and makes an angle of $\theta$ with the $x-$ axis.
Let us find the length of PQ.
We know that,

$$
\frac{x-x_{1}}{\cos \theta}=\frac{y-y_{1}}{\sin \theta}
$$

Let $P Q=r$
Then, the coordinates of $Q$ are given by
By using the formula,
The equation of the line is given by:

$$
\begin{aligned}
& \frac{x-x_{1}}{\cos \theta}=\frac{y-y_{1}}{\sin \theta}=r \\
& x=x_{1}+r \cos \theta, y=y_{1}+r \sin \theta
\end{aligned}
$$

Thus, the coordinates of $Q$ are $\left(x_{1}+r \cos \theta, y_{1}+r \sin \theta\right)$ It is clear that, Q lies on the line $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$.
So,

$$
\begin{aligned}
& a\left(x_{1}+r \cos \theta\right)+b\left(y_{1}+r \sin \theta\right)+c=0 \\
& r=P Q=\left|\frac{a x_{1}+b y_{1}+c}{a \cos \theta+b \sin \theta}\right|
\end{aligned}
$$

$\therefore$ The value of PQ is $\left|\frac{a x_{1}+\mathrm{by}_{1}+\mathrm{c}}{\mathrm{a} \cos \theta+\mathrm{b} \sin \theta}\right|$

## EXERCISE 23.9

1. Reduce the equation $\sqrt{ } 3 x+y+2=0$ to:
(i) slope - intercept form and find slope and $y$ - intercept;
(ii) Intercept form and find intercept on the axes
(iii) The normal form and find $p$ and $\alpha$.

Solution:
(i) Given:
$\sqrt{3} x+y+2=0$
$y=-\sqrt{3 x}-2$
This is the slope intercept form of the given line.
$\therefore$ The slope $=-\sqrt{3}$ and y - intercept $=-2$
(ii) Given:

$$
\begin{aligned}
& \sqrt{ } 3 x+y+2=0 \\
& \sqrt{3} x+y=-2
\end{aligned}
$$

Divide both sides by -2 , we get
$\sqrt{3} \mathrm{x} /-2+\mathrm{y} /-2=1$
$\therefore$ The intercept form of the given line. Here, $\mathrm{x}-$ intercept $=-2 / \sqrt{3}$ and y - intercept $=-2$
(iii) Given:
$\sqrt{3} \mathrm{x}+\mathrm{y}+2=0$
$-\sqrt{3} x-y=2$

$$
-\frac{\sqrt{3} x}{\sqrt{(-\sqrt{3})^{2}+(-1)^{2}}}-\frac{y}{\sqrt{(-\sqrt{3})^{2}+(-1)^{2}}}=\frac{2}{\sqrt{(-\sqrt{3})^{2}+(-1)^{2}}}
$$

Divide both sides by $\sqrt{(\text { coefficient of } \mathrm{x})^{2}+(\text { coefficient of } \mathrm{y})^{2}}$
$-\frac{\sqrt{3} \mathrm{x}}{2}-\frac{\mathrm{y}}{2}=1$
This is the normal form of the given line.
So, $p=1 \cos \alpha=-\sqrt{3} / 2$ and $\sin \alpha=-1 / 2$
$\therefore \mathrm{p}=1$ and $\alpha=210$
2. Reduce the following equations to the normal form and find $p$ and $\alpha$ in each case:
(i) $x+\sqrt{ } 3 y-4=0$
(ii) $\mathbf{x}+\mathbf{y}+\sqrt{ } \mathbf{2}=\mathbf{0}$

Solution:
(i) $x+\sqrt{3} y-4=0$
$x+\sqrt{ } 3 y=4$

$$
\frac{x}{\sqrt{1^{2}+(\sqrt{3})^{2}}}+\frac{\sqrt{3} y}{\sqrt{1^{2}+(\sqrt{3})^{2}}}=\frac{4}{\sqrt{1^{2}+(\sqrt{3})^{2}}}
$$

Divide both sides by $\sqrt{(\text { coefficient of } \mathrm{x})^{2}+(\text { coefficient of } \mathrm{y})^{2}}$

$$
\frac{x}{2}+\frac{\sqrt{3} y}{2}=2
$$

The normal form of the given line, where $p=2, \cos \alpha=1 / 2$ and $\sin \alpha=\sqrt{3} / 2$
$\therefore \mathrm{p}=2$ and $\alpha=\pi / 3$
(ii) $x+y+\sqrt{ } 2=0$
$-x-y=\sqrt{ } 2$

$$
\frac{-\mathrm{x}}{\sqrt{(-1)^{2}+(-1)^{2}}}+\frac{\mathrm{y}}{\sqrt{(-1)^{2}+(-1)^{2}}}=\frac{\sqrt{2}}{\sqrt{(-1)^{2}+(-1)^{2}}}
$$

Divide both sides by $\sqrt{(\text { coefficient of } \mathrm{x})^{2}+(\text { coefficient of } \mathrm{y})^{2}}$

$$
-\frac{x}{\sqrt{2}}-\frac{y}{\sqrt{2}}=1
$$

The normal form of the given line, where $p=1, \cos \alpha=-1 / \sqrt{ } 2$ and $\sin \alpha=-1 / \sqrt{ } 2$
$\therefore \mathrm{p}=1$ and $\alpha=225^{\circ}$
3. Put the equation $x / a+y / b=1$ the slope intercept form and find its slope and $\mathbf{y}$ intercept.

## Solution:

Given: the equation is $x / a+y / b=1$
We know that,
General equation of line $\mathrm{y}=\mathrm{mx}+\mathrm{c}$.
$b x+a y=a b$
$a y=-b x+a b$
$y=-b x / a+b$
The slope intercept form of the given line.
$\therefore$ Slope $\mathrm{=}-\mathrm{b} / \mathrm{a}$ and y - intercept b
4. Reduce the lines $3 x-4 y+4=0$ and $2 x+4 y-5=0$ to the normal form and hence find which line is nearer to the origin.

## Solution:

Given:

The normal forms of the lines $3 x-4 y+4=0$ and $2 x+4 y-5=0$.
Let us find, in given normal form of a line, which is nearer to the origin.
$-3 x+4 y=4$

$$
-\frac{3 x}{\sqrt{(-3)^{2}+(4)^{2}}}+4 \frac{y}{\sqrt{(-3)^{2}+(4)^{2}}}=\frac{4}{\sqrt{(-3)^{2}+(4)^{2}}}
$$

Divide both sides by $\sqrt{(\text { coefficient of } \mathrm{x})^{2}+(\text { coefficient of } \mathrm{y})^{2}}$

$$
\begin{equation*}
-\frac{3}{5} x+\frac{4}{5} y=\frac{4}{5} \tag{1}
\end{equation*}
$$

Now $2 \mathrm{x}+4 \mathrm{y}=-5$
$-2 x-4 y=5$

$$
-\frac{2 \mathrm{x}}{\sqrt{(-2)^{2}+(-4)^{2}}}-4 \frac{\mathrm{y}}{\sqrt{(-2)^{2}+(-4)^{2}}}=\frac{5}{\sqrt{(-2)^{2}+(-4)^{2}}}
$$

Divide both sides by $\sqrt{(\text { coefficient of } \mathrm{x})^{2}+(\text { coefficient of } \mathrm{y})^{2}}$

$$
\begin{equation*}
-\frac{2}{2 \sqrt{5}} x-\frac{4}{2 \sqrt{5}} y=\frac{5}{2 \sqrt{5}} \tag{2}
\end{equation*}
$$

From equations (1) and (2):
$45<525$
$\therefore$ The line $3 x-4 y+4=0$ is nearer to the origin.
5. Show that the origin is equidistant from the lines $4 x+3 y+10=0 ; 5 x-12 y+26=$ 0 and $7 x+24 y=50$.

## Solution:

Given:
The lines $4 x+3 y+10=0 ; 5 x-12 y+26=0$ and $7 x+24 y=50$.
We need to prove that, the origin is equidistant from the lines $4 x+3 y+10=0 ; 5 x-12 y$ $+26=0$ and $7 x+24 y=50$.
Let us write down the normal forms of the given lines.
First line: $4 \mathrm{x}+3 \mathrm{y}+10=0$

$$
\begin{aligned}
& -4 x-3 y=10 \\
& -\frac{4 x}{\sqrt{(-4)^{2}+(-3)^{2}}}-3 \frac{y}{\sqrt{(-4)^{2}+(-3)^{2}}}=\frac{10}{\sqrt{(-4)^{2}+(-3)^{2}}}
\end{aligned}
$$

Divide both sides by $\sqrt{(\text { coefficient of } \mathrm{x})^{2}+(\text { coefficient of } \mathrm{y})^{2}}$

$$
-\frac{4}{5} x-\frac{3}{5} y=2
$$

So, $\mathrm{p}=2$

Second line: $5 x-12 y+26=0$
$-5 x+12 y=26$

$$
-\frac{5 x}{\sqrt{(-5)^{2}+(12)^{2}}}+12 \frac{y}{\sqrt{(-5)^{2}+(12)^{2}}}=\frac{26}{\sqrt{(-5)^{2}+(12)^{2}}}
$$

Divide both sides by $\sqrt{(\text { coefficient of } \mathrm{x})^{2}+(\text { coefficient of } \mathrm{y})^{2}}$
$-\frac{5}{13} x+\frac{12}{13} y=2$
So, $\mathrm{p}=2$
Third line: $7 x+24 y=50$

$$
\frac{7 x}{\sqrt{(7)^{2}+(24)^{2}}}+24 \frac{\mathrm{y}}{\sqrt{(7)^{2}+(24)^{2}}}=\frac{50}{\sqrt{(7)^{2}+(24)^{2}}}
$$

Divide both sides by $\sqrt{(\text { coefficient of } \mathrm{x})^{2}+(\text { coefficient of } \mathrm{y})^{2}}$
$\frac{7}{25} x+\frac{24}{25} y=2$
So, $\mathrm{p}=2$
$\therefore$ The origin is equidistant from the given lines.

## EXERCISE 23.10

## 1. Find the point of intersection of the following pairs of lines:

(i) $2 x-y+3=0$ and $x+y-5=0$
(ii) $\mathbf{b x}+\mathbf{a y}=\mathbf{a b}$ and $a x+b y=a b$

## Solution:

(i) $2 x-y+3=0$ and $x+y-5=0$

Given:
The equations of the lines are as follows:

$$
\begin{align*}
& 2 x-y+3=0 \ldots(1)  \tag{1}\\
& x+y-5=0 \ldots
\end{align*}
$$

Let us find the point of intersection of pair of lines.
By solving (1) and (2) using cross - multiplication method, we get

$$
\begin{aligned}
& \frac{x}{5-3}=\frac{y}{3+10}=\frac{1}{2+1} \\
& \frac{x}{2}=\frac{y}{13}=\frac{1}{3}
\end{aligned}
$$

$x=2 / 3$ and $y=13 / 3$
$\therefore$ The point of intersection is $(2 / 3,13 / 3)$
(ii) $b x+a y=a b$ and $a x+b y=a b$

Given:
The equations of the lines are as follows:
$b x+a y-a b=0 \ldots$ (1)
$a x+b y=a b \Rightarrow a x+b y-a b=0$
Let us find the point of intersection of pair of lines.
By solving (1) and (2) using cross - multiplication method, we get

$$
\begin{aligned}
& \frac{x}{-a^{2} b+a b^{2}}=\frac{y}{-a^{2} b+a b^{2}}=\frac{1}{b^{2}-a^{2}} \\
& \frac{x}{a b(b-a)}=\frac{y}{a b(b-a)}=\frac{1}{(a+b)(b-a)} \\
& x=\frac{a b}{a+b} \text { and } y=\frac{a b}{a+b}
\end{aligned}
$$

$\therefore$ The point of intersection is $(a b / a+b, a b / a+b)$
2. Find the coordinates of the vertices of a triangle, the equations of whose sides are:
(i) $x+y-4=0,2 x-y+3=0$ and $x-3 y+2=0$
(ii) $y\left(t_{1}+t_{2}\right)=2 x+2 a t_{1} t_{2}, y\left(t_{2}+t_{3}\right)=2 x+2 a_{2} t_{3}$ and, $y\left(t_{3}+t_{1}\right)=2 x+$

2at $\mathbf{t}_{1}$. Solution:
(i) $x+y-4=0,2 x-y+3=0$ and $x-3 y+2=0$

Given:
$x+y-4=0,2 x-y+3=0$ and $x-3 y+2=0$
Let us find the point of intersection of pair of lines.
$\mathrm{x}+\mathrm{y}-4=0 \ldots$ (1)
$2 x-y+3=0 \ldots$ (2)
$x-3 y+2=0 \ldots$ (3)
By solving (1) and (2) using cross - multiplication method, we get

$$
\begin{aligned}
& \frac{x}{3-4}=\frac{y}{-8-3}=\frac{1}{-1-2} \\
& x=1 / 3, y=11 / 3
\end{aligned}
$$

Solving (1) and (3) using cross - multiplication method, we get

$$
\frac{x}{2-12}=\frac{y}{-4-2}=\frac{1}{-3-1}
$$

$\mathrm{x}=5 / 2, \mathrm{y}=3 / 2$
Similarly, solving (2) and (3) using cross - multiplication method, we get
$\frac{x}{-2+9}=\frac{y}{3-4}=\frac{1}{-6+1}$
$\mathrm{x}=-7 / 5, \mathrm{y}=1 / 5$
$\therefore$ The coordinates of the vertices of the triangle are $(1 / 3,11 / 3),(5 / 2,3 / 2)$ and $(-7 / 5,1 / 5)$
(ii) $y\left(t_{1}+t_{2}\right)=2 x+2 a t_{1} t_{2}, y\left(t_{2}+t_{3}\right)=2 x+2 a t_{2} t_{3}$ and, $y\left(t_{3}+t_{1}\right)=2 x+2 a t_{1} t_{3}$.

Given:
$y\left(t_{1}+t_{2}\right)=2 x+2 a t_{1} t_{2}, y\left(t_{2}+t_{3}\right)=2 x+2 a t_{2} t_{3}$ and $y\left(t_{3}+t_{1}\right)=2 x+2 a t_{1} t_{3}$
Let us find the point of intersection of pair of lines.
$2 \mathrm{x}-\mathrm{y}\left(\mathrm{t}_{1}+\mathrm{t}_{2}\right)+2 \mathrm{a} \mathrm{t}_{1} \mathrm{t}_{2}=0 \ldots$ (1)
$2 \mathrm{x}-\mathrm{y}\left(\mathrm{t}_{2}+\mathrm{t}_{3}\right)+2 \mathrm{a} \mathrm{t}_{2} \mathrm{t}_{3}=0 \ldots$ (2)
$2 \mathrm{x}-\mathrm{y}\left(\mathrm{t}_{3}+\mathrm{t}_{1}\right)+2 \mathrm{a} \mathrm{t}_{1} \mathrm{t}_{3}=0 \ldots$ (3)
By solving (1) and (2) using cross - multiplication method, we get

$$
\begin{gathered}
\frac{x}{-\left(t_{1}+t_{2}\right) \times 2 a t_{2} t_{3}+\left(t_{2}+t_{3}\right) 2 \mathrm{at}_{1} t_{2}}=\frac{1}{4 a t_{2} t_{3}-4 a t_{1} t_{2}} \\
=\frac{1}{-2\left(t_{2}+t_{3}\right)+2\left(t_{1}+t_{2}\right)} \\
x=\frac{-\left(t_{1}+t_{2}\right) \times 2 a t_{2} t_{3}+\left(t_{2}+t_{3}\right) 2 \mathrm{at}_{1} t_{2}}{-2\left(t_{2}+t_{3}\right)+2\left(t_{1}+t_{2}\right)}=a t_{2}^{2} \\
y=-\frac{4 a t_{2} t_{3}-4 a t_{1} t_{2}}{-2\left(t_{2}+t_{3}\right)+2\left(t_{1}+t_{2}\right)}=2 a t_{2}
\end{gathered}
$$

Solving (1) and (3) using cross - multiplication method, we get

$$
\begin{gathered}
\frac{x}{-\left(t_{1}+t_{2}\right) \times 2 a t_{1} t_{3}+\left(t_{3}+t_{1}\right) 2 a t_{1} t_{2}}=\frac{-y}{4 a t_{1} t_{3}-4 a_{1} t_{2}} \\
=\frac{1}{-2\left(t_{3}+t_{1}\right)+2\left(t_{1}+t_{2}\right)} \\
x=\frac{-\left(t_{1}+t_{2}\right) \times 2 a_{1} t_{3}+\left(t_{3}+t_{1}\right) 2 a t_{1} t_{2}}{-2\left(t_{3}+t_{1}\right)+2\left(t_{1}+t_{2}\right)}=a t_{1}^{2} \\
y=-\frac{4 a t_{1} t_{3}-4 a t_{1} t_{2}}{-2\left(t_{3}+t_{1}\right)+2\left(t_{1}+t_{2}\right)}=2 a t_{1}
\end{gathered}
$$

Solving (2) and (3) using cross - multiplication method, we get

$$
\begin{gathered}
\frac{x}{-\left(t_{2}+t_{3}\right) \times 2 a t_{1} t_{3}+\left(t_{3}+t_{1}\right) 2 a t_{2} t_{3}}=\frac{-y}{4 a t_{1} t_{3}-4 a_{2} t_{3}} \\
=\frac{1}{-2\left(t_{3}+t_{1}\right)+2\left(t_{2}+t_{3}\right)} \\
x=\frac{-\left(t_{2}+t_{3}\right) \times 2 a t_{1} t_{3}+\left(t_{3}+t_{1}\right) 2 a_{2} t_{3}}{-2\left(t_{3}+t_{1}\right)+2\left(t_{2}+t_{3}\right)}=a t_{3}^{2} \\
y=-\frac{4 a t_{1} t_{3}-4 a t_{2} t_{3}}{-2\left(t_{3}+t_{1}\right)+2\left(t_{2}+t_{3}\right)}=2 a t_{3}
\end{gathered}
$$

$\therefore$ The coordinates of the vertices of the triangle are $\left(\mathrm{at}^{2}, 2 \mathrm{at}_{1}\right),\left(\mathrm{at}^{2}, 2 \mathrm{at}\right)$ and $\left(\mathrm{at}^{2}, 2 \mathrm{at}_{3}\right)$.

## 3. Find the area of the triangle formed by the lines

$\mathrm{y}=\mathrm{m}_{1} \mathrm{x}+\mathrm{c}_{1}, \mathrm{y}=\mathrm{m}_{2} \mathrm{x}+\mathrm{c}_{2}$ and $\mathrm{x}=0$

## Solution:

Given:
$\mathrm{y}=\mathrm{m}_{1} \mathrm{x}+\mathrm{c}_{1} \ldots$
$\mathrm{y}=\mathrm{m}_{2} \mathrm{x}+\mathrm{c}_{2} \ldots$
$\mathrm{x}=0$
In triangle ABC , let equations (1), (2) and (3) represent the sides $\mathrm{AB}, \mathrm{BC}$ and CA , respectively.
Solving (1) and (2), we get

$$
\mathrm{x}=\frac{\mathrm{c}_{2}-\mathrm{c}_{1}}{\mathrm{~m}_{1}-\mathrm{m}_{2}}, \mathrm{y}=\frac{\mathrm{m}_{1} \mathrm{c}_{2}-\mathrm{m}_{2} \mathrm{c}_{1}}{\mathrm{~m}_{1}-\mathrm{m}_{2}}
$$

Thus, $A B$ and $B C$ intersect at $B\left(\frac{c_{2}-c_{1}}{m_{1}-m_{2}}, \frac{m_{1} c_{2}-m_{2} c_{1}}{m_{1}-m_{2}}\right)$
Solving (1) and (3):
$\mathrm{x}=0, \mathrm{y}=\mathrm{c}_{1}$
Thus, AB and CA intersect at $\mathrm{A} 0, \mathrm{c}_{1}$.
Similarly, solving (2) and (3):
$\mathrm{x}=0, \mathrm{y}=\mathrm{c}_{2}$
Thus, BC and CA intersect at $\mathrm{C} 0, \mathrm{c}_{2}$.
$\therefore$ Area of triangle $A B C=\frac{1}{2}\left|\begin{array}{ccc}0 & c_{1} & 1 \\ 0 & c_{2} & 1 \\ \frac{c_{2}-c_{1}}{m_{1}-m_{2}} & \frac{m_{1} c_{2}-m_{2} c_{1}}{m_{1}-m_{2}} & 1\end{array}\right|$

$$
\begin{aligned}
& =\frac{1}{2}\left(\frac{c_{2}-c_{1}}{m_{1}-m_{2}}\right)\left(c_{1}-c_{2}\right) \\
& =\frac{\frac{1}{2}\left(c_{1}-c_{2}\right)^{2}}{m_{2}-m_{1}}
\end{aligned}
$$

4. Find the equations of the medians of a triangle, the equations of whose sides are: $3 x+2 y+6=0,2 x-5 y+4=0$ and $x-3 y-6=0$
Solution:
Given:
$3 x+2 y+6=0 \ldots$ (1)
$2 \mathrm{x}-5 \mathrm{y}+4=0 \ldots$ (2)
$x-3 y-6=0 \ldots$ (3)
Let us assume, in triangle ABC , let equations (1), (2) and (3) represent the sides $\mathrm{AB}, \mathrm{BC}$ and CA, respectively.
Solving equations (1) and (2), we get
$\mathrm{x}=-2, \mathrm{y}=0$
Thus, AB and BC intersect at $\mathrm{B}(-2,0)$.

Now, solving (1) and (3), we get
$x=-6 / 11, y=-24 / 11$
Thus, AB and CA intersect at $\mathrm{A}(-6 / 11,-24 / 11)$
Similarly, solving (2) and (3), we get
$\mathrm{x}=-42, \mathrm{y}=-16$
Thus, BC and CA intersect at C $(-42,-16)$.
Now, let $\mathrm{D}, \mathrm{E}$ and F be the midpoints the sides $\mathrm{BC}, \mathrm{CA}$ and AB , respectively. Then, we have:

$$
\begin{aligned}
& D=\left(\frac{-2-42}{2}, \frac{0-16}{2}\right)=(-22,-8) \\
& E=\left(\frac{-\frac{6}{11}-42}{2}, \frac{-\frac{24}{11}-16}{2}\right)=\left(-\frac{234}{11},-\frac{100}{11}\right) \\
& F=\left(\frac{-\frac{6}{11}-2}{2}, \frac{-\frac{24}{11}+0}{2}\right)=\left(-\frac{14}{11},-\frac{12}{11}\right)
\end{aligned}
$$

Now, the equation of the median AD is

$$
\begin{aligned}
& y+\frac{24}{11}=\frac{-8+\frac{24}{11}}{-22+\frac{6}{11}}\left(x+\frac{6}{11}\right) \\
& 16 x-59 y-120=0
\end{aligned}
$$

The equation of median CF is
$y+16=\frac{-\frac{12}{11}+16}{-\frac{14}{11}+42}(x+42)$
$41 \mathrm{x}-112 \mathrm{y}-70=0$
And, the equation of the median BE is
$y-0=\frac{-\frac{100}{11}-0}{-\frac{234}{11}+2}(x+2)$
$25 \mathrm{x}-53 \mathrm{y}+50=0$
$\therefore$ The equations of the medians of a triangle are: $41 \mathrm{x}-112 \mathrm{y}-70=0$,
$16 x-59 y-120=0,25 x-53 y+50=0$
5. Prove that the lines $y=\sqrt{ } 3 x+1, y=4$ and $y=-\sqrt{ } 3 x+2$ form an equilateral triangle.

## Solution:

Given:
$y=\sqrt{ } 3 x+1$
$y=4$
$y=-\sqrt{3} x+2$.
Let us assume in triangle ABC , let equations (1), (2) and (3) represent the sides $\mathrm{AB}, \mathrm{BC}$ and CA, respectively.
By solving equations (1) and (2), we get
$x=\sqrt{ } 3, y=4$
Thus, AB and BC intersect at $\mathrm{B}(\sqrt{3}, 4)$
Now, solving equations (1) and (3), we get
$x=1 / 2 \sqrt{ } 3, y=3 / 2$
Thus, $A B$ and $C A$ intersect at $A(1 / 2 \sqrt{ } 3,3 / 2)$
Similarly, solving equations (2) and (3), we get
$x=-2 / \sqrt{3}, y=4$
Thus, BC and AC intersect at $C(-2 / \sqrt{3}, 4)$
Now, we have:

$$
\begin{aligned}
& A B=\sqrt{\left(\frac{1}{2 \sqrt{3}}-\sqrt{3}\right)^{2}+\left(\frac{3}{2}-4\right)^{2}}=\frac{5}{\sqrt{3}} \\
& \mathrm{BC}=\sqrt{\left(\frac{1}{2 \sqrt{3}}+\frac{2}{\sqrt{3}}\right)^{2}+\left(\frac{3}{2}-4\right)^{2}}=\frac{5}{\sqrt{3}} \\
& \mathrm{AC}=\sqrt{\left(\frac{1}{2 \sqrt{3}}+\frac{2}{\sqrt{3}}\right)^{2}+\left(\frac{3}{2}-4\right)^{2}}=\frac{5}{\sqrt{3}}
\end{aligned}
$$

Hence proved, the given lines form an equilateral triangle.

## EXERCISE 23.11

## 1. Prove that the following sets of three lines are concurrent:

(i) $15 x-18 y+1=0,12 x+10 y-3=0$ and $6 x+66 y-11=0$
(ii) $3 x-5 y-11=0,5 x+3 y-7=0$ and $x+2 y=0$

## Solution:

(i) $15 x-18 y+1=0,12 x+10 y-3=0$ and $6 x+66 y-11=0$

Given:

$$
\begin{align*}
& 15 x-18 y+1=0 \\
& 12 x+10 y-3=0  \tag{ii}\\
& 6 x+66 y-11=0 \tag{iii}
\end{align*}
$$

Now, consider the following determinant:

$$
\left|\begin{array}{ccc}
15 & -18 & 1 \\
12 & 10 & -3 \\
6 & 66 & -11
\end{array}\right|=15(-110+198)+18(-132+18)+1(792-60)
$$

$\Rightarrow 1320-2052+732=0$
Hence proved, the given lines are concurrent.
(ii) $3 x-5 y-11=0,5 x+3 y-7=0$ and $x+2 y=0$

Given:
$3 x-5 y-11=0$
$5 x+3 y-7=0$
$x+2 y=0$ $\qquad$
Now, consider the following determinant:
$\left|\begin{array}{ccc}3 & -5 & -11 \\ 5 & 3 & -7 \\ 1 & 2 & 0\end{array}\right|=3 \times 14+5 \times 7-11 \times 7=0$
Hence, the given lines are concurrent.
2. For what value of $\lambda$ are the three lines $2 x-5 y+3=0,5 x-9 y+\lambda=0$ and $x-2 y+$ 1 = 0 concurrent?

## Solution:

Given:
$2 \mathrm{x}-5 \mathrm{y}+3=0$
$5 \mathrm{x}-9 \mathrm{y}+\lambda=0$
$\mathrm{x}-2 \mathrm{y}+1=0$
It is given that the three lines are concurrent.
Now, consider the following determinant:

$$
\therefore\left|\begin{array}{lll}
2 & -5 & 3 \\
5 & -9 & \lambda \\
1 & -2 & 1
\end{array}\right|=0
$$

$2(-9+2 \lambda)+5(5-\lambda)+3(-10+9)=0$
$-18+4 \lambda+25-5 \lambda-3=0$
$\lambda=4$
$\therefore$ The value of $\lambda$ is 4 .
3. Find the conditions that the straight lines $y=m_{1} x+c_{1}, y=m_{2} x+c_{2}$ and $y=m_{3} x+$ $\mathbf{c}_{3}$ may meet in a point.
Solution:
Given:

$$
\begin{align*}
& \mathrm{m}_{1} \mathrm{x}-\mathrm{y}+\mathrm{c}_{1}=0 \ldots \text { (1) } \\
& \mathrm{m}_{2} \mathrm{x}-\mathrm{y}+\mathrm{c}_{2}=0 \ldots \text { (2) } \\
& \mathrm{m}_{3} \mathrm{x}-\mathrm{y}+\mathrm{c}_{3}=0 \ldots \text { (3) } \tag{3}
\end{align*}
$$

It is given that the three lines are concurrent.
Now, consider the following determinant:
$\therefore\left|\begin{array}{lll}\mathrm{m}_{1} & -1 & \mathrm{c}_{1} \\ \mathrm{~m}_{2} & -1 & \mathrm{c}_{2} \\ \mathrm{~m}_{3} & -1 & \mathrm{c}_{3}\end{array}\right|=0$
$m_{1}\left(-c_{3}+c_{2}\right)+1\left(m_{2} c_{3}-m_{3} c_{2}\right)+c_{1}\left(-m_{2}+m_{3}\right)=0$
$\mathrm{m}_{1}\left(\mathrm{c}_{2}-\mathrm{c}_{3}\right)+\mathrm{m}_{2}\left(\mathrm{c}_{3}-\mathrm{c}_{1}\right)+\mathrm{m}_{3}\left(\mathrm{c}_{1}-\mathrm{c}_{2}\right)=0$
$\therefore$ The required condition is $\mathrm{m}_{1}\left(\mathrm{c}_{2}-\mathrm{c}_{3}\right)+\mathrm{m}_{2}\left(\mathrm{c}_{3}-\mathrm{c}_{1}\right)+\mathrm{m}_{3}\left(\mathrm{c}_{1}-\mathrm{c}_{2}\right)=0$
4. If the lines $p_{1} x+q_{1} y=1, p_{2} x+q_{2} y=1$ and $p_{3} x+q_{3} y=1$ be concurrent, show that the points $\left(p_{1}, q_{1}\right),\left(p_{2}, q_{2}\right)$ and $\left(p_{3}, q_{3}\right)$ are collinear.
Solution:
Given:
$\mathrm{p}_{1} \mathrm{x}+\mathrm{q}_{1} \mathrm{y}=1$
$\mathrm{p}_{2} \mathrm{x}+\mathrm{q}_{2} \mathrm{y}=1$
$p_{3} \mathrm{x}+\mathrm{q}_{3} \mathrm{y}=1$
The given lines can be written as follows:
$\mathrm{p}_{1} \mathrm{x}+\mathrm{q}_{1} \mathrm{y}-1=0 \ldots$ (1)
$\mathrm{p}_{2} \mathrm{x}+\mathrm{q}_{2} \mathrm{y}-1=0 \ldots$ (2)
$\mathrm{p}_{3} \mathrm{x}+\mathrm{q}_{3} \mathrm{y}-1=0 \ldots$ (3)
It is given that the three lines are concurrent.
Now, consider the following determinant:

$$
\left|\begin{array}{lll}
\mathrm{p}_{1} & \mathrm{q}_{1} & -1 \\
\mathrm{p}_{2} & \mathrm{q}_{2} & -1 \\
\mathrm{p}_{3} & \mathrm{q}_{3} & -1
\end{array}\right|=0
$$

$-\left|\begin{array}{lll}\mathrm{p}_{1} & \mathrm{q}_{1} & 1 \\ \mathrm{p}_{2} & \mathrm{q}_{2} & 1 \\ \mathrm{p}_{3} & \mathrm{q}_{3} & 1\end{array}\right|=0$
$\left|\begin{array}{lll}\mathrm{p}_{1} & \mathrm{q}_{1} & 1 \\ \mathrm{p}_{2} & \mathrm{q}_{2} & 1 \\ \mathrm{p}_{3} & \mathrm{q}_{3} & 1\end{array}\right|=0$
Hence proved, the given three points, $\left(p_{1}, q_{1}\right),\left(p_{2}, q_{2}\right)$ and $\left(p_{3}, q_{3}\right)$ are collinear.
5. Show that the straight lines $L_{1}=(b+c) x+a y+1=0, L_{2}=(c+a) x+b y+1=0$ and $L_{3}=(a+b) x+c y+1=0$ are concurrent.
Solution:
Given:
$\mathrm{L}_{1}=(\mathrm{b}+\mathrm{c}) \mathrm{x}+\mathrm{ay}+1=0$
$L_{2}=(c+a) x+b y+1=0$
$\mathrm{L}_{3}=(\mathrm{a}+\mathrm{b}) \mathrm{x}+\mathrm{cy}+1=0$
The given lines can be written as follows:
$(b+c) x+a y+1=0 \ldots$ (1)
$(c+a) x+b y+1=0 \ldots$ (2)
$(a+b) x+c y+1=0 \ldots$ (3)
Consider the following determinant.

$$
\left|\begin{array}{lll}
\mathrm{b}+\mathrm{c} & \mathrm{a} & 1 \\
\mathrm{c}+\mathrm{a} & \mathrm{~b} & 1 \\
\mathrm{a}+\mathrm{b} & \mathrm{c} & 1
\end{array}\right|
$$

Let us apply the transformation $\mathrm{C}_{1} \rightarrow \mathrm{C}_{1}+\mathrm{C}_{2}$, we get

$$
\begin{aligned}
& \left|\begin{array}{lll}
\mathrm{b}+\mathrm{c} & \mathrm{a} & 1 \\
\mathrm{c}+\mathrm{a} & \mathrm{~b} & 1 \\
\mathrm{a}+\mathrm{b} & \mathrm{c} & 1
\end{array}\right|=\left|\begin{array}{lll}
\mathrm{a}+\mathrm{b}+\mathrm{c} & \mathrm{a} & 1 \\
\mathrm{c}+\mathrm{a}+\mathrm{b} & \mathrm{~b} & 1 \\
\mathrm{a}+\mathrm{b}+\mathrm{c} & \mathrm{c} & 1
\end{array}\right| \\
& \left|\begin{array}{lll}
\mathrm{b}+\mathrm{c} & \mathrm{a} & 1 \\
\mathrm{c}+\mathrm{a} & \mathrm{~b} & 1 \\
\mathrm{a}+\mathrm{b} & \mathrm{c} & 1
\end{array}\right|
\end{aligned}=(\mathrm{a}+\mathrm{b}+\mathrm{c})\left|\begin{array}{lll}
1 & \mathrm{a} & 1 \\
1 & \mathrm{~b} & 1 \\
1 & \mathrm{c} & 1
\end{array}\right|,
$$

Hence proved, the given lines are concurrent.

## EXERCISE 23.12

1. Find the equation of a line passing through the point $(2,3)$ and parallel to the line $3 x-4 y+5=0$.

## Solution:

Given:
The equation is parallel to $3 x-4 y+5=0$ and pass through $(2,3)$
The equation of the line parallel to $3 x-4 y+5=0$ is
$3 x-4 y+\lambda=0$,
Where, $\lambda$ is a constant.
It passes through $(2,3)$.
Substitute the values in above equation, we get
$3(2)-4(3)+\lambda=0$
$6-12+\lambda=0$
$\lambda=6$
Now, substitute the value of $\lambda=6$ in $3 x-4 y+\lambda=0$, we get
$3 x-4 y+6$
$\therefore$ The required line is $3 x-4 y+6=0$.
2. Find the equation of a line passing through ( $3,-2$ ) and perpendicular to the line $x$ $-3 y+5=0$.
Solution:
Given:
The equation is perpendicular to $x-3 y+5=0$ and passes through $(3,-2)$
The equation of the line perpendicular to $x-3 y+5=0$ is
$3 \mathrm{x}+\mathrm{y}+\lambda=0$,
Where, $\lambda$ is a constant.
It passes through $(3,-2)$.
Substitute the values in above equation, we get
$3(3)+(-2)+\lambda=0$
$9-2+\lambda=0$
$\lambda=-7$
Now, substitute the value of $\lambda=-7$ in $3 x+y+\lambda=0$, we get
$3 \mathrm{x}+\mathrm{y}-7=0$
$\therefore$ The required line is $3 \mathrm{x}+\mathrm{y}-7=0$.
3. Find the equation of the perpendicular bisector of the line joining the points (1, 3) and ( 3,1 ).
Solution:

Given:
A $(1,3)$ and $B(3,1)$ be the points joining the perpendicular bisector
Let C be the midpoint of AB .
So, coordinates of $\mathrm{C}=[(1+3) / 2,(3+1) / 2]$

$$
=(2,2)
$$

Slope of $\mathrm{AB}=[(1-3) /(3-1)]$

$$
=-1
$$

Slope of the perpendicular bisector of $\mathrm{AB}=1$
Thus, the equation of the perpendicular bisector of $A B$ is given as,
$y-2=1(x-2)$
$y=x$
$x-y=0$
$\therefore$ The required equation is $\mathrm{y}=\mathrm{x}$.
4. Find the equations of the altitudes of a $\triangle \mathrm{ABC}$ whose vertices are $\mathrm{A}(1,4), B(-3,2)$ and $C(-5,-3)$.

## Solution:

Given:
The vertices of $\triangle \mathrm{ABC}$ are $\mathrm{A}(1,4), \mathrm{B}(-3,2)$ and $\mathrm{C}(-5,-3)$.
Now let us find the slopes of $\triangle A B C$.


Slope of $A B=[(2-4) /(-3-1)]$

$$
=1 / 2
$$

Slope of BC $=[(-3-2) /(-5+3)]$

$$
=5 / 2
$$

Slope of CA $=[(4+3) /(1+5)]$

$$
=7 / 6
$$

Thus, we have:

Slope of CF $=-2$
Slope of AD $=-2 / 5$
Slope of BE $=-6 / 7$
Hence,
Equation of CF is:
$y+3=-2(x+5)$
$y+3=-2 x-10$
$2 x+y+13=0$
Equation of AD is:
$y-4=(-2 / 5)(x-1)$
$5 \mathrm{y}-20=-2 \mathrm{x}+2$
$2 x+5 y-22=0$
Equation of BE is:
$y-2=(-6 / 7)(x+3)$
$7 \mathrm{y}-14=-6 \mathrm{x}-18$
$6 x+7 y+4=0$
$\therefore$ The required equations are $2 x+y+13=0,2 x+5 y-22=0,6 x+7 y+4=0$.
5. Find the equation of a line which is perpendicular to the line $\sqrt{ } 3 x-y+5=0$ and which cuts off an intercept of 4 units with the negative direction of $y$-axis. Solution:

## Given:

The equation is perpendicular to $\sqrt{ } 3 x-y+5=0$ equation and cuts off an intercept of 4 units with the negative direction of $y$-axis.
The line perpendicular to $\sqrt{ } 3 x-y+5=0$ is $x+\sqrt{ } 3 y+\lambda=0$
It is given that the line $x+\sqrt{ } 3 y+\lambda=0$ cuts off an intercept of 4 units with the negative direction of the $y$-axis.
This means that the line passes through $(0,-4)$.
So,
Let us substitute the values in the equation $x+\sqrt{ } 3 y+\lambda=0$, we get
$0-\sqrt{ } 3(4)+\lambda=0$
$\lambda=4 \sqrt{ } 3$
Now, substitute the value of $\lambda$ back, we get
$x+\sqrt{ } 3 y+4 \sqrt{ } 3=0$
$\therefore$ The required equation of line is $x+\sqrt{ } 3 y+4 \sqrt{ } 3=0$.

## EXERCISE 23.13

## 1. Find the angles between each of the following pairs of straight lines:

(i) $3 x+y+12=0$ and $x+2 y-1=0$
(ii) $3 x-y+5=0$ and $x-3 y+1=0$

## Solution:

(i) $3 x+y+12=0$ and $x+2 y-1=0$

Given:
The equations of the lines are

$$
\begin{aligned}
& 3 x+y+12=0 \ldots(1) \\
& x+2 y-1=0 \ldots(2)
\end{aligned}
$$

Let $\mathrm{m}_{1}$ and $\mathrm{m}_{2}$ be the slopes of these lines.
$m_{1}=-3, m_{2}=-1 / 2$
Let $\theta$ be the angle between the lines.
Then, by using the formula

$$
\begin{aligned}
\tan \theta & =\left[\left(m_{1}-m_{2}\right) /\left(1+m_{1} m_{2}\right)\right] \\
& =[(-3+1 / 2) /(1+3 / 2)] \\
& =1
\end{aligned}
$$

So,
$\theta=\pi / 4$ or $45^{\circ}$
$\therefore$ The acute angle between the lines is $45^{\circ}$
(ii) $3 x-y+5=0$ and $x-3 y+1=0$

Given:
The equations of the lines are

$$
\begin{aligned}
& 3 x-y+5=0 \ldots \text { (1) } \\
& x-3 y+1=0 \ldots \text { (2) }
\end{aligned}
$$

Let $\mathrm{m}_{1}$ and $\mathrm{m}_{2}$ be the slopes of these lines.
$\mathrm{m}_{1}=3, \mathrm{~m}_{2}=1 / 3$
Let $\theta$ be the angle between the lines.
Then, by using the formula

$$
\begin{aligned}
\tan \theta & =\left[\left(m_{1}-m_{2}\right) /\left(1+m_{1} m_{2}\right)\right] \\
& =[(3-1 / 3) /(1+3(1 / 3))] \\
& =[((9-1) / 3) /(1+1)] \\
& =8 / 6 \\
& =4 / 3
\end{aligned}
$$

So,
$\theta=\tan ^{-1}(4 / 3)$
$\therefore$ The acute angle between the lines is $\tan ^{-1}(4 / 3)$.
2. Find the acute angle between the lines $2 x-y+3=0$ and $x+y+2=0$.

## Solution:

Given:
The equations of the lines are
$2 \mathrm{x}-\mathrm{y}+3=0$
$x+y+2=0$

Let $m_{1}$ and $m_{2}$ be the slopes of these lines.
$\mathrm{m}_{1}=2, \mathrm{~m}_{2}=-1$
Let $\theta$ be the angle between the lines.
Then, by using the formula

$$
\begin{aligned}
\tan \theta & =\left[\left(m_{1}-m_{2}\right) /\left(1+m_{1} m_{2}\right)\right] \\
& =[(2-(-1) /(1+(2)(-1))] \\
& =[3 /(1-2)] \\
& =3
\end{aligned}
$$

So,
$\theta=\tan ^{-1}$ (3)
$\therefore$ The acute angle between the lines is $\tan ^{-1}(3)$.
3. Prove that the points $(2,-1),(0,2),(2,3)$ and $(4,0)$ are the coordinates of the vertices of a parallelogram and find the angle between its diagonals.
Solution:
To prove:
The points $(2,-1),(0,2),(2,3)$ and $(4,0)$ are the coordinates of the vertices of a parallelogram
Let us assume the points, $\mathrm{A}(2,-1), \mathrm{B}(0,2), \mathrm{C}(2,3)$ and $\mathrm{D}(4,0)$ be the vertices. Now, let us find the slopes

Slope of AB $=[(2+1) /(0-2)]$

$$
=-3 / 2
$$

Slope of BC $=[(3-2) /(2-0)]$

$$
=1 / 2
$$

Slope of CD $=[(0-3) /(4-2)]$

$$
=-3 / 2
$$

Slope of DA $=[(-1-0) /(2-4)]$

$$
=1 / 2
$$

Thus, $A B$ is parallel to $C D$ and $B C$ is parallel to $D A$.
Hence proved, the given points are the vertices of a parallelogram.

Now, let us find the angle between the diagonals AC and BD.
Let $\mathrm{m}_{1}$ and $\mathrm{m}_{2}$ be the slopes of AC and BD , respectively.

$$
\begin{aligned}
\mathrm{m}_{1} & =[(3+1) /(2-2)] \\
& =\infty \\
\mathrm{m}_{2} & =[(0-2) /(4-0)] \\
& =-1 / 2
\end{aligned}
$$

Thus, the diagonal AC is parallel to the y -axis.
$\angle \mathrm{ODB}=\tan ^{-1}(1 / 2)$
In triangle MND,
$\angle \mathrm{DMN}=\pi / 2-\tan ^{-1}(1 / 2)$
$\therefore$ The angle between the diagonals is $\pi / 2-\tan ^{-1}(1 / 2)$.
4. Find the angle between the line joining the points $(2,0),(0,3)$ and the line $x+y=$ 1.

## Solution:

Given:
Points $(2,0),(0,3)$ and the line $x+y=1$.
Let us assume $\mathrm{A}(2,0), \mathrm{B}(0,3)$ be the given points.
Now, let us find the slopes
Slope of $\mathrm{AB}=\mathrm{m}_{1}$

$$
\begin{aligned}
& =[(3-0) /(0-2)] \\
& =-3 / 2
\end{aligned}
$$

Slope of the line $\mathrm{x}+\mathrm{y}=1$ is -1
$\therefore \mathrm{m}_{2}=-1$
Let $\theta$ be the angle between the line joining the points $(2,0),(0,3)$ and the line $\mathrm{x}+\mathrm{y}=$ $\tan \theta=\left|\left[\left(m_{1}-m_{2}\right) /\left(1+m_{1} m_{2}\right)\right]\right|$

$$
=[(-3 / 2+1) /(1+3 / 2)]
$$

$$
=1 / 5
$$

$\theta=\tan ^{-1}(1 / 5)$
$\therefore$ The acute angle between the line joining the points $(2,0),(0,3)$ and the line $\mathrm{x}+\mathrm{y}=1$ is $\tan ^{-1}(1 / 5)$.

## 5. If $\boldsymbol{\theta}$ is the angle which the straight line joining the points ( $\mathrm{x}_{1}, \mathrm{y}_{1}$ ) and ( $\mathrm{x}_{2}, \mathrm{y}_{2}$ )

subtends at the origin, provethat $\tan \theta=\frac{x_{2} y_{1}-x_{1} y_{2}}{x_{1} x_{2}+y_{1} y_{2}}$ and $\cos \theta=\frac{x_{1} y_{2}+y_{1} y_{2}}{\sqrt{x_{1}^{2}+y_{1}^{2}} \sqrt{x_{2}^{2}+y_{2}^{2}}}$

## Solution:

We need to prove:

$$
\tan \theta=\frac{x_{2} y_{1}-x_{1} y_{2}}{x_{1} x_{2}+y_{1} y_{2}} \text { and } \cos \theta=\frac{x_{1} x_{2}+y_{1} y_{2}}{\sqrt{x_{1}^{2}+y_{1}^{2}} \sqrt{x_{2}^{2}+y_{2}^{2}}} .
$$



Let us assume $\mathrm{A}\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ and $\mathrm{B}\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right)$ be the given points and O be the origin.
Slope of OA $=\mathrm{m}_{1}=\mathrm{y}_{1 \times 1}$
Slope of $O B=m_{2}=y_{2 \times 2}$
It is given that $\theta$ is the angle between lines OA and OB .

$$
\tan \theta=\left|\frac{\mathrm{m}_{1}-\mathrm{m}_{2}}{1+\mathrm{m}_{1} \mathrm{~m}_{2}}\right|
$$

Now, substitute the values, we get

$$
\begin{aligned}
& =\frac{\frac{y_{1}}{x_{1}}-\frac{y_{2}}{x_{2}}}{1+\frac{y_{1}}{x_{1}} \times \frac{y_{2}}{x_{2}}} \\
\tan \theta & =\frac{x_{2} y_{1}-\bar{x}_{1} y_{2}}{x_{1} x_{2}+y_{1} y_{2}}
\end{aligned}
$$

Now,
As we know that $\cos \theta=\sqrt{\frac{1}{1+\tan ^{2} \theta}}$ Now, substitute the values, we get

$$
\begin{aligned}
& \cos \theta=\frac{x_{1} x_{2}+y_{1} y_{2}}{\sqrt{\left(x_{2} y_{1}-x_{1} y_{2}\right)^{2}+\left(x_{1} x_{2}+y_{1} y_{2}\right)^{2}}} \\
& \cos \theta=\frac{x_{1} x_{2}+y_{1} y_{2}}{\sqrt{x_{1}^{2} y_{1}^{2}+x_{1}^{2} y_{2}^{2}+x_{1}^{2} x_{2}^{2}+y_{1}^{2} y_{2}^{2}}} \\
& \cos \theta=\frac{x_{1} x_{2}+y_{1} y_{2}}{\sqrt{x_{1}^{2}+y_{1}^{2}} \sqrt{x_{2}^{2}+y_{2}^{2}}}
\end{aligned}
$$

Hence proved.

## EXERCISE 23.14

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1. Find the values of $\alpha$ so that the point $P\left(\alpha^{2}, \alpha\right)$ lies inside or on the triangle formed by the lines $x-5 y+6=0, x-3 y+2=0$ and $x-2 y-3=0$.

## Solution:

Given:
$x-5 y+6=0, x-3 y+2=0$ and $x-2 y-3=0$ forming a triangle and point $P\left(\alpha^{2}, \alpha\right)$ lies inside or on the triangle
Let ABC be the triangle of sides $\mathrm{AB}, \mathrm{BC}$ and CA whose equations are $\mathrm{x}-5 \mathrm{y}+6=0$, $x-3 y+2=0$ and $x-2 y-3=0$, respectively.
On solving the equations, we get $\mathrm{A}(9,3), \mathrm{B}(4,2)$ and $\mathrm{C}(13,5)$ as the coordinates of the vertices.


It is given that point $\mathrm{P}\left(\alpha^{2}, \alpha\right)$ lies either inside or on the triangle. The three conditions are given below.
(i) A and P must lie on the same side of BC .
(ii) B and P must lie on the same side of AC .
(iii) C and P must lie on the same side of AB .

If A and P lie on the same side of BC , then
$(9-9+2)\left(\alpha^{2}-3 \alpha+2\right) \geq 0$
$(\alpha-2)(\alpha-1) \geq 0$
$\alpha \in(-\infty, 1] \cup[2, \infty) \ldots(1)$
If $B$ and $P$ lie on the same side of $A C$, then
$(4-4-3)\left(\alpha^{2}-2 \alpha-3\right) \geq 0$
$(\alpha-3)(\alpha+1) \leq 0$
$\alpha \in[-1,3] \ldots(2)$

If C and P lie on the same side of AB , then
$(13-25+6)\left(\alpha^{2}-5 \alpha+6\right) \geq 0$
$(\alpha-3)(\alpha-2) \leq 0$
$\alpha \in[2,3] \ldots$ (3)
From equations (1), (2) and (3), we get $\alpha \in[2,3]$
$\therefore \alpha \in[2,3]$
2. Find the values of the parameter a so that the point $(\mathbf{a}, 2)$ is an interior point of the triangle formed by the lines $x+y-4=0,3 x-7 y-8=0$ and $4 x-y-31=0$. Solutions:
Given:
$x+y-4=0,3 x-7 y-8=0$ and $4 x-y-31=0$ forming a triangle and point $(a, 2)$ is an interior point of the triangle
Let ABC be the triangle of sides $\mathrm{AB}, \mathrm{BC}$ and CA whose equations are $\mathrm{x}+\mathrm{y}-4=0$, $3 x-7 y-8=0$ and $4 x-y-31=0$, respectively.
On solving them, we get $\mathrm{A}(7,-3)$, B $(18 / 5,2 / 5)$ and $\mathrm{C}(209 / 25,61 / 25)$ as the coordinates of the vertices.
Let $P(a, 2)$ be the given point.


It is given that point $\mathrm{P}(\mathrm{a}, 2)$ lies inside the triangle. So, we have the following:
(i) A and P must lie on the same side of BC .
(ii) B and P must lie on the same side of AC .
(iii) C and P must lie on the same side of AB .

Thus, if A and P lie on the same side of BC , then
$21+21-8-3 a-14-8>0$
a $>22 / 3$
If $B$ and $P$ lie on the same side of $A C$, then

$$
\begin{align*}
& 4 \times \frac{18}{5}-\frac{2}{5}-31-4 a-2-31>0 \\
& a<33 / 4 \ldots(2) \tag{2}
\end{align*}
$$

If $C$ and $P$ lie on the same side of $A B$, then

$$
\begin{aligned}
& \frac{209}{25}+\frac{61}{25}-4-a+2-4>0 \\
& \frac{34}{5}-4-a+2-4>0 \\
& a>2 \ldots(3)
\end{aligned}
$$

From (1), (2) and (3), we get:
$\mathrm{A} \in(22 / 3,33 / 4)$
$\therefore \mathrm{A} \in(22 / 3,33 / 4)$
3. Determine whether the point $(-3,2)$ lies inside or outside the triangle whose sides are given by the equations $x+y-4=0,3 x-7 y+8=0,4 x-y-31=0$.

## Solution:

Given:
$\mathrm{x}+\mathrm{y}-4=0,3 \mathrm{x}-7 \mathrm{y}+8=0,4 \mathrm{x}-\mathrm{y}-31=0$ forming a triangle and point $(-3,2)$
Let $A B C$ be the triangle of sides $A B, B C$ and $C A$, whose equations $x+y-4=0,3 x-7 y$ $+8=0$ and $4 x-y-31=0$, respectively.
On solving them, we get $\mathrm{A}(7,-3), \mathrm{B}(2,2)$ and $\mathrm{C}(9,5)$ as the coordinates of the vertices. Let $\mathrm{P}(-3,2)$ be the given point.


The given point $\mathrm{P}(-3,2)$ will lie inside the triangle ABC , if (i) A and P lies on the same side of BC
(ii) B and P lies on the same side of AC
(iii) C and P lies on the same side of AB

Thus, if A and P lie on the same side of BC , then
$21+21+8-9-14+8>0$
$50 \times-15>0$
$-750>0$,
This is false
$\therefore$ The point $(-3,2)$ lies outside triangle ABC .

## EXERCISE 23.15

1. Find the distance of the point $(4,5)$ from the straight line $3 x-5 y+7=0$. Solution:
Given:
The line: $3 x-5 y+7=0$
Comparing $a x+b y+c=0$ and $3 x-5 y+7=0$, we get:
$\mathrm{a}=3, \mathrm{~b}=-5$ and $\mathrm{c}=7$
So, the distance of the point $(4,5)$ from the straight line $3 x-5 y+7=0$ is

$$
\begin{aligned}
d & =\left|\frac{a_{1}+\mathrm{by}_{1}+\mathrm{c}}{\sqrt{\mathrm{a}^{2}+\mathrm{b}^{2}}}\right| \\
& =\left|\frac{3 \times 4-5 \times 5+7}{\sqrt{3^{2}+\left(-5^{2}\right)}}\right| \\
& =\frac{6}{\sqrt{34}}
\end{aligned}
$$

$\therefore$ The required distance is $6 / \sqrt{ } 34$
2. Find the perpendicular distance of the line joining the points $(\cos \theta, \sin \theta)$ and $(\cos \phi, \sin \phi)$ from the origin.

## Solution:

Given:
The points $(\cos \theta, \sin \theta)$ and $(\cos \phi, \sin \phi)$ from the origin.
The equation of the line joining the points $(\cos \theta, \sin \theta)$ and $(\cos \phi, \sin \phi)$ is given below:

$$
\begin{aligned}
& y-\sin \theta=\frac{\sin \phi-\sin \theta}{\cos \phi-\cos \theta}(x-\cos \theta) \\
& (\cos \phi-\cos \theta) y-\sin \theta(\cos \phi-\cos \theta)=(\sin \phi-\sin \theta) x-(\sin \phi- \\
& \sin \theta) \cos \theta
\end{aligned}
$$

$$
(\sin \phi-\sin \theta) x-(\cos \phi-\cos \theta) y+\sin \theta \cos \phi-\sin \phi \cos \theta=0
$$

Let d be the perpendicular distance from the origin to the line

$$
(\sin \phi-\sin \theta) \mathrm{x}-(\cos \phi-\cos \theta) \mathrm{y}+\sin \theta \cos \phi-\sin \phi \cos \theta=0
$$

$$
\mathrm{d}=\left|\frac{\sin \theta-\phi}{\sqrt{(\sin \phi-\sin \theta)^{2}+(\cos \phi-\cos \theta)^{2}}}\right|
$$

$$
=\left|\frac{\sin \theta-\phi}{\sqrt{\sin ^{2} \phi+\sin ^{2} \theta-2 \sin \phi \sin \theta+\cos ^{2} \phi+\cos ^{2} \theta-2 \cos \phi \cos \theta}}\right|
$$

$$
\begin{aligned}
& =\left|\frac{\sin \theta-\phi}{\sqrt{\sin ^{2} \phi+\cos ^{2} \phi+\sin ^{2} \theta+\cos ^{2} \phi+\cos ^{2} \theta-2 \cos (\theta-\phi)}}\right| \\
& =\left|\frac{\frac{1}{\sqrt{2}}(\sin (\theta-\phi))}{\sqrt{1-\cos (\theta-\phi)}}\right| \\
& =\frac{1}{\sqrt{2}}\left|\frac{\sin (\theta-\phi)}{\sqrt{2 \sin ^{2}\left(\frac{\theta-\phi}{2}\right)}}\right| \\
& =\frac{1}{2}\left|\frac{2 \sin \left(\frac{\theta-\phi}{2}\right) \cos \left(\frac{\theta-\phi}{2}\right)}{\sin \left(\frac{\theta-\phi}{2}\right)}\right| \\
& =\cos \left(\frac{\theta-\phi}{2}\right)
\end{aligned}
$$

$\therefore$ The required distance is $\cos \left(\frac{\theta-\phi}{2}\right)$
3. Find the length of the perpendicular from the origin to the straight line joining the two points whose coordinates are $(a \cos \alpha, a \sin \alpha)$ and $(a \cos \beta, a \sin \beta)$.

## Solution:

Given:
Coordinates are $(\mathrm{a} \cos \alpha, \mathrm{a} \sin \alpha)$ and $(\mathrm{a} \cos \beta$, a $\sin \beta)$.
Equation of the line passing through $(a \cos \alpha, a \sin \alpha)$ and $(a \cos \beta$, $a \sin \beta)$ is

$$
\begin{aligned}
& y-a \sin \alpha=\frac{a \sin \beta-a \sin \alpha}{a \cos \beta-a \cos \alpha}(x-a \cos \alpha) \\
& y-a \sin \alpha=\frac{\sin \beta-\sin \alpha}{\cos \beta-\cos \alpha}(x-a \cos \alpha) \\
& y-a \sin \alpha=\frac{2 \cos \left(\frac{\beta+\alpha}{2}\right) \sin \left(\frac{\beta-\alpha}{2}\right)}{2 \sin \left(\frac{\beta+\alpha}{2}\right) \sin \left(\frac{\alpha-\beta}{2}\right)}(x-a \cos \alpha) \\
& y-a \sin \alpha=-\cot \left(\frac{\beta+\alpha}{2}\right)(x-a \cos \alpha) \\
& y-a \sin \alpha=-\cot \left(\frac{\alpha+\beta}{2}\right)(x-a \cos \alpha) \\
& x \cot \left(\frac{\alpha+\beta}{2}\right)+y-a \sin \alpha-a \cos \alpha \cot \left(\frac{\alpha+\beta}{2}\right)=0
\end{aligned}
$$

The distance of the line from the origin is

$$
\begin{aligned}
d & =\left|\frac{-a \sin \alpha-a \cos \alpha \cot \left(\frac{\alpha+\beta}{2}\right)}{\sqrt{\cot ^{2}\left(\frac{(\alpha+\beta)}{2}\right)+1}}\right| \\
d & =\left|\frac{-a \sin \alpha-a \cos \alpha \cot \left(\frac{\alpha+\beta}{2}\right)}{\sqrt{\operatorname{cosec}^{2}\left(\frac{(\alpha+\beta)}{2}\right)}}\right| \because \operatorname{cosec}^{2} \theta=1+\cot ^{2} \theta \\
& =a\left|\sin \left(\frac{\alpha+\beta}{2}\right) \sin \alpha+\cos \alpha \cos \left(\frac{\alpha+\beta}{2}\right)\right| \\
& =a\left|\sin \alpha \sin \left(\frac{\alpha+\beta}{2}\right)+\cos \alpha \cos \left(\frac{\alpha+\beta}{2}\right)\right| \\
& =a\left|\cos \left(\frac{\alpha+\beta}{2}-\alpha\right)\right|=a \cos \left(\frac{\beta-\alpha}{2}\right)
\end{aligned}
$$

$\therefore$ The required distance is $\cos \left(\frac{\beta-\alpha}{2}\right)$
4. Show that the perpendicular let fall from any point on the straight line $2 x+11 y-$ $5=0$ upon the two straight lines $24 x+7 y=20$ and $4 x-3 y-2=0$ are equal to each other.
Solution:
Given:
The lines $24 x+7 y=20$ and $4 x-3 y-2=0$
Let us assume, $\mathrm{P}(\mathrm{a}, \mathrm{b})$ be any point on $2 \mathrm{x}+11 \mathrm{y}-5=0$
So,

$$
\begin{gather*}
2 \mathrm{a}+11 \mathrm{~b}-5=0 \\
\mathrm{~b}=\frac{5-2 \mathrm{a}}{11} \ldots \ldots . \tag{1}
\end{gather*}
$$

Let $d_{1}$ and $d_{2}$ be the perpendicular distances from point $P$ on the lines $24 \mathrm{x}+7 \mathrm{y}=20$ and $4 \mathrm{x}-3 \mathrm{y}-2=0$, respectively.

$$
\begin{aligned}
\mathrm{d}_{1} & =\left|\frac{24 \mathrm{a}+7 \mathrm{~b}-20}{\sqrt{24^{2}+7^{2}}}\right|=\left|\frac{24 \mathrm{a}+7 \mathrm{~b}-20}{25}\right| \\
& =\left|\frac{24 \mathrm{a}+7 \times \frac{5-2 \mathrm{a}}{11}-20}{25}\right|
\end{aligned}
$$

From(1)
$\mathrm{d}_{1}=\left|\frac{50 \mathrm{a}-37}{55}\right|$
Similarly,

$$
\begin{aligned}
\mathrm{d}_{2} & =\left|\frac{4 a-3 b-2}{\sqrt{3^{2}+(-4)^{2}}}\right|=\left|\frac{4 a-3 \times \frac{5-2 a}{11}-2}{5}\right| \\
& =\left|\frac{44 a-15+6 a-22}{11 \times 5}\right|
\end{aligned}
$$

From (1)
$\mathrm{d}_{2}=\left|\frac{50 \mathrm{a}-37}{55}\right|$
$\therefore \mathrm{d}_{1}=\mathrm{d}_{2}$
Hence proved.
5. Find the distance of the point of intersection of the lines $2 x+3 y=21$ and $3 x-4 y+$ $11=0$ from the line $8 x+6 y+5=0$.

## Solution:

Given:
The lines $2 \mathrm{x}+3 \mathrm{y}=21$ and $3 \mathrm{x}-4 \mathrm{y}+11=0$
Solving the lines $2 x+3 y=21$ and $3 x-4 y+11=0$ we get:

$$
\frac{x}{33-84}=\frac{y}{-63-22}=\frac{1}{-8-9}
$$

$\mathrm{x}=3, \mathrm{y}=5$
So, the point of intersection of $2 x+3 y=21$ and $3 x-4 y+11=0$ is $(3,5)$.
Now, the perpendicular distance $d$ of the line $8 x+6 y+5=0$ from the point $(3,5)$ is

$$
\mathrm{d}=\left|\frac{24+30+5}{\sqrt{8^{2}+6^{2}}}\right|=\frac{59}{10}
$$

$\therefore$ The distance is $59 / 10$.

## EXERCISE 23.16

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## 1. Determine the distance between the following pair of parallel lines:

(i) $4 x-3 y-9=0$ and $4 x-3 y-24=0$
(ii) $8 x+15 y-34=0$ and $8 x+15 y+31=0$

## Solution:

(i) $4 x-3 y-9=0$ and $4 x-3 y-24=0$

Given:
The parallel lines are
$4 \mathrm{x}-3 \mathrm{y}-9=0 \ldots$ (1)
$4 x-3 y-24=0 \ldots$ (2)
Let d be the distance between the given lines.
So,

$$
\mathrm{d}=\left|\frac{-9+24}{\sqrt{4^{2}+(-3)^{2}}}\right|=\frac{15}{5}=3 \text { units }
$$

$\therefore$ The distance between givens parallel line is 3units.
(ii) $8 x+15 y-34=0$ and $8 x+15 y+31=0$

Given:
The parallel lines are
$8 x+15 y-34=0 \ldots$ (1)
$8 x+15 y+31=0 \ldots$ (2)
Let $d$ be the distance between the given lines.
So,

$$
\mathrm{d}=\left|\frac{-34-31}{\sqrt{8^{2}+15^{2}}}\right|=\frac{65}{17} \text { units }
$$

$\therefore$ The distance between givens parallel line is $65 / 17$ units.
2. The equations of two sides of a square are $5 x-12 y-65=0$ and $5 x-12 y+26=0$.

Find the area of the square.

## Solution:

Given:
Two side of square are $5 \mathrm{x}-12 \mathrm{y}-65=0$ and $5 \mathrm{x}-12 \mathrm{y}+26=0$
The sides of a square are
$5 x-12 y-65=0 \ldots$ (1)
$5 x-12 y+26=0 \ldots$ (2)
We observe that lines (1) and (2) are parallel.
So, the distance between them will give the length of the side of the square.

Let d be the distance between the given lines.
$\mathrm{d}=\left|\frac{-65-26}{\sqrt{5^{2}+(-12)^{2}}}\right|=\frac{91}{13}=7$
$\therefore$ Area of the square $=7^{2}=49$ square units

## 3. Find the equation of two straight lines which are parallel to $x+7 y+2=0$ and at unit distance from the point $(1,-1)$.

## Solution:

Given:
The equation is parallel to $x+7 y+2=0$ and at unit distance from the point $(1,-1)$ The equation of given line is
$x+7 y+2=0$
The equation of a line parallel to line $x+7 y+2=0$ is given below:
$x+7 y+\lambda=0 \ldots$ (2)
The line $\mathrm{x}+7 \mathrm{y}+\lambda=0$ is at a unit distance from the point $(1,-1)$.
So,

$$
1=\left|\frac{1-7+\lambda}{\sqrt{1+49}}\right|
$$

$\lambda-6= \pm 5 \sqrt{ } 2$
$\lambda=6+5 \sqrt{ } 2,6-5 \sqrt{ } 2$
now, substitute the value of $\lambda$ back in equation $x+7 y+\lambda=0$, we get
$x+7 y+6+5 \sqrt{ } 2=0$ and $x+7 y+6-5 \sqrt{ } 2$
$\therefore$ The required lines:
$x+7 y+6+5 \sqrt{ } 2=0$ and $x+7 y+6-5 \sqrt{ } 2$
4. Prove that the lines $2 x+3 y=19$ and $2 x+3 y+7=0$ are equidistant from the line $2 \mathrm{x}+3 \mathrm{y}=6$.

## Solution:

Given:
The lines $A, 2 x+3 y=19$ and $B, 2 x+3 y+7=0$ also a line C, $2 x+3 y=6$.
Let $d_{1}$ be the distance between lines $2 x+3 y=19$ and $2 x+3 y=6$, While $\mathrm{d}_{2}$ is the distance between lines $2 \mathrm{x}+3 \mathrm{y}+7=0$ and $2 \mathrm{x}+3 \mathrm{y}=6$

$$
\begin{aligned}
& \mathrm{d}_{1}=\left|\frac{-19-(-6)}{\sqrt{2^{2}+3^{2}}}\right| \text { and } \mathrm{d}_{2}=\left|\frac{7-(-6)}{\sqrt{2^{2}+3^{2}}}\right| \\
& \mathrm{d}_{1}=\left|-\frac{13}{\sqrt{13}}\right|=\sqrt{13} \text { and } \mathrm{d}_{2}=\left|\frac{13}{\sqrt{13}}\right|=\sqrt{13}
\end{aligned}
$$

Hence proved, the lines $2 \mathrm{x}+3 \mathrm{y}=19$ and $2 \mathrm{x}+3 \mathrm{y}+7=0$ are equidistant from the line 2 x $+3 y=6$
5. Find the equation of the line mid-way between the parallel lines $9 x+6 y-7=0$ and $3 x+2 y+6=0$.
Solution:
Given:
$9 x+6 y-7=0$ and $3 x+2 y+6=0$ are parallel lines
The given equations of the lines can be written as:
$3 x+2 y-7 / 3=0 \ldots$ (1)
$3 x+2 y+6=0$
Let the equation of the line midway between the parallel lines (1) and (2) be $3 x+2 y+\lambda=0$..
The distance between (1) and (3) and the distance between (2) and (3) are equal.

$$
\begin{align*}
& \left|\frac{-\frac{7}{3}-\lambda}{\sqrt{3^{2}+2^{2}}}\right|=\left|\frac{6-\lambda}{\sqrt{3^{2}+2^{2}}}\right|  \tag{3}\\
& \left|-\lambda+\frac{7}{3}\right|=|6-\lambda| \\
& 6-\lambda=\lambda+\frac{7}{3} \\
& \lambda=\frac{11}{6}
\end{align*}
$$

Now substitute the value of $\lambda$ back in equation $3 \mathrm{x}+2 \mathrm{y}+\lambda=0$, we get
$3 x+2 y+11 / 6=0$
By taking LCM
$18 \mathrm{x}+12 \mathrm{y}+11=0$
$\therefore$ The required equation of line is $18 \mathrm{x}+12 \mathrm{y}+11=0$

## EXERCISE 23.17

## 1. Prove that the area of the parallelogram formed by the lines

$a_{1} x+b_{1} y+c_{1}=0, a_{1} x+b_{1} y+d_{1}=0, a_{2} x+b_{2} y+c_{2}=0, a_{2} x+b_{2} y+d_{2}=0$
is $\left|\frac{\left(d_{1}-c_{1}\right)\left(d_{2}-c_{2}\right)}{a_{1} b_{2}-a_{2} b_{1}}\right|$ sq. units.

## Deduce the condition for these lines to form a rhombus.

## Solution:

Given:
The given lines are
$a_{1} x+b_{1} y+c_{1}=0 \ldots$ (1)
$\mathrm{a}_{1} \mathrm{x}+\mathrm{b}_{1} \mathrm{y}+\mathrm{d}_{1}=0$.
$\mathrm{a}_{2} \mathrm{x}+\mathrm{b}_{2} \mathrm{y}+\mathrm{c}_{2}=0$..
$\mathrm{a}_{2} \mathrm{x}+\mathrm{b}_{2} \mathrm{y}+\mathrm{d}_{2}=0$
Let us prove, the area of the parallelogram formed by the lines $a_{1} x+b_{1} y+c_{1}=0, a_{1} x+$ $b_{1} y+d_{1}=0, a_{2} x+b_{2} y+c_{2}=0, a_{2} x+b_{2} y+d_{2}=0$ is $\left|\frac{\left(d_{1}-c_{1}\right)\left(d_{2}-c_{2}\right)}{\left(a_{1} b_{2}-a_{2} b_{1}\right)}\right|$ sq. units.

The area of the parallelogram formed by the lines $a_{1} x+b_{1} y+c_{1}=0, a_{1} x+b_{1} y+d_{1}=0$, $\mathrm{a}_{2} \mathrm{x}+\mathrm{b}_{2} \mathrm{y}+\mathrm{c}_{2}=0$ and $\mathrm{a}_{2} \mathrm{x}+\mathrm{b}_{2} \mathrm{y}+\mathrm{d}_{2}=0$ is given below:
Area $=\left|\frac{\left(c_{1}-d_{1}\right)\left(c_{2}-d_{2}\right)}{\left|\begin{array}{ll}a_{1} & a_{2} \\ b_{1} & b_{2}\end{array}\right|}\right|$
Since, $\left|\begin{array}{ll}a_{1} & a_{2} \\ b_{1} & b_{2}\end{array}\right|=a_{1} b_{2}-a_{2} b_{1}$
$\therefore$ Area $=\left|\frac{\left(c_{1}-d_{1}\right)\left(c_{2}-d_{2}\right)}{\left(a_{1} b_{2}-a_{2} b_{1}\right)}\right|=\left|\frac{\left(d_{1}-c_{1}\right)\left(d_{2}-c_{2}\right)}{\left(a_{1} b_{2}-a_{2} b_{1}\right)}\right|$
If the given parallelogram is a rhombus, then the distance between the pair of parallel lines is equal.

$$
\therefore\left|\frac{\mathrm{c}_{1}-\mathrm{d}_{1}}{\sqrt{\mathrm{a}_{1}^{2}+\mathrm{b}_{1}^{2}}}\right|=\left|\frac{\mathrm{c}_{2}-\mathrm{d}_{2}}{\sqrt{\mathrm{a}_{2}^{2}+\mathrm{b}_{2}^{2}}}\right|
$$

Hence proved.
2. Prove that the area of the parallelogram formed by the lines $3 x-4 y+a=0,3 x-$ $4 y+3 a=0,4 x-3 y-a=0$ and $4 x-3 y-2 a=0$ is $2 a^{2} / 7$ sq. units.
Solution:
Given:
The given lines are
$3 x-4 y+a=0 \ldots$ (1)
$3 x-4 y+3 a=0 \ldots$ (2)
$4 \mathrm{x}-3 \mathrm{y}-\mathrm{a}=0 \ldots$ (3)
$4 x-3 y-2 a=0 \ldots$ (4)
Let us prove, the area of the parallelogram formed by the lines $3 x-4 y+a=0,3 x-4 y+$ $3 \mathrm{a}=0,4 \mathrm{x}-3 \mathrm{y}-\mathrm{a}=0$ and $4 \mathrm{x}-3 \mathrm{y}-2 \mathrm{a}=0$ is $2 \mathrm{a}^{2} / 7$ sq. units.
From above solution, we know that
Area of the parallelogram $=\left|\frac{\left(c_{1}-d_{1}\right)\left(c_{2}-d_{2}\right)}{\left(a_{1} b_{2}-a_{2} b_{1}\right)}\right|$
Area of the parallelogram $=\left|\frac{(a-3 a)(2 a-a)}{(-9+16)}\right|=\frac{2 a^{2}}{7}$ square units
Hence proved.
3. Show that the diagonals of the parallelogram whose sides are $1 x+m y+n=0,1 x+$ $\mathbf{m y}+\mathbf{n}^{\prime}=\mathbf{0}, \mathbf{m x}+\mathrm{ly}+\mathbf{n}=\mathbf{0}$ and $\mathbf{m x}+\mathrm{ly}+\mathbf{n}^{\prime}=\mathbf{0}$ include an angle $\boldsymbol{\pi} / \mathbf{2}$.

## Solution:

Given:
The given lines are
$1 \mathrm{x}+\mathrm{my}+\mathrm{n}=0 \ldots$ (1)
$m x+l y+n=0 \ldots$ (2)
$1 \mathrm{x}+\mathrm{my}+\mathrm{n}$ ' $=0 \ldots$ (3)
$m x+1 y+n=0 \ldots$ (4)
Let us prove, the diagonals of the parallelogram whose sides are $1 \mathrm{x}+\mathrm{my}+\mathrm{n}=0, \mathrm{~lx}+\mathrm{my}$ $+\mathrm{n}^{\prime}=0, \mathrm{mx}+\mathrm{ly}+\mathrm{n}=0$ and $\mathrm{mx}+\mathrm{ly}+\mathrm{n}^{\prime}=0$ include an angle $\pi / 2$.

By solving (1) and (2), we get
$B=\left(\frac{\mathrm{mn}^{\prime}-\ln }{\mathrm{l}^{2}-\mathrm{m}^{2}}, \frac{\mathrm{mn}-\mathrm{ln}^{\prime}}{\mathrm{l}^{2}-\mathrm{m}^{2}}\right)$
Solving (2) and (3), we get,
$\mathrm{C}=\left(-\frac{\mathrm{n}^{\prime}}{\mathrm{m}+1^{\prime}},-\frac{\mathrm{n}^{\prime}}{\mathrm{m}+1}\right)$
Solving (3) and (4), we get,
$\mathrm{D}=\left(\frac{\mathrm{mn}-\mathrm{ln}^{\prime}}{\mathrm{l}^{2}-\mathrm{m}^{2}}, \frac{\mathrm{mn}^{\prime}-\mathrm{ln}}{\mathrm{l}^{2}-\mathrm{m}^{2}}\right)$
Solving (1) and (4), we get,

$$
\mathrm{A}=\left(\frac{-\mathrm{n}}{\mathrm{~m}+1}, \frac{-\mathrm{n}}{\mathrm{~m}+1}\right)
$$

Let $\mathrm{m}_{1}$ and $\mathrm{m}_{2}$ be the slope of AC and BD .
Now,

$$
\mathrm{m}_{1}=\frac{\frac{-\mathrm{n}^{\prime}}{\mathrm{m}+1}+\frac{\mathrm{n}}{\mathrm{~m}+1}}{\frac{-\mathrm{n}^{\prime}}{\mathrm{m}+1}+\frac{\mathrm{n}}{\mathrm{~m}+1}}=1
$$

$$
m_{2}=\frac{\frac{m n^{\prime}-\ln }{1^{2}-m^{2}}-\frac{m n-n^{\prime}}{1^{2}-m^{2}}}{\frac{m n-n^{\prime}}{1^{2}-m^{2}}-\frac{m n^{\prime}-\ln }{1^{2}-m^{2}}}=-1
$$

$\therefore \mathrm{m}_{1} \mathrm{~m}_{2}=-1$
Hence proved.

## 1. Find the equation of the straight lines passing through the origin and making an angle of $45^{\circ}$ with the straight line $\sqrt{3} x+y=11$.

## Solution:

Given:
Equation passes through $(0,0)$ and make an angle of $45^{\circ}$ with the line $\sqrt{3 x}+y=11$. We know that, the equations of two lines passing through a point $\mathrm{x}_{1}, \mathrm{y}_{1}$ and making an angle $\alpha$ with the given line $y=m x+c$ are

$$
\mathrm{y}-\mathrm{y}_{1}=\frac{\mathrm{m} \pm \tan \alpha}{1 \mp \mathrm{mtan} \alpha}\left(\mathrm{x}-\mathrm{x}_{1}\right)
$$

Here,

$$
\mathrm{x}_{1}=0, \mathrm{y}_{1}=0, \alpha=45^{\circ} \text { and } \mathrm{m}=-\sqrt{ } 3
$$

So, the equations of the required lines are

$$
\begin{aligned}
y-0 & =\frac{-\sqrt{3}+\tan 45^{\circ}}{1+\sqrt{3} \tan 45^{\circ}}(x-0) \text { and } y-0 \\
& =\frac{-\sqrt{3}-\tan 45^{\circ}}{1-\sqrt{3} \tan 45^{\circ}}(x-0) \\
& =\frac{-\sqrt{3}+1}{1+\sqrt{3}} x \text { and } y=\frac{\sqrt{3}+1}{\sqrt{3}-1} x \\
& =-\frac{3+1-2 \sqrt{3}}{3-1} x \text { and } y=\frac{3+1+2 \sqrt{3}}{3-1} x \\
& =(\sqrt{3}-2) x \text { and } y=(\sqrt{3}+2) x
\end{aligned}
$$

$\therefore$ The equation of given line is $y=(\sqrt{3}-2) x$ and $y=(\sqrt{3}+2) x$

## 2. Find the equations to the straight lines which pass through the origin and are

 inclined at an angle of $75^{\circ}$ to the straight line $x+y+\sqrt{3}(y-x)=a$.
## Solution:

Given:
The equation passes through $(0,0)$ and make an angle of $75^{\circ}$ with the line $x+y+\sqrt{3}(y-$ $\mathrm{x})=\mathrm{a}$.
We know that the equations of two lines passing through a point $\mathrm{x}_{1}, \mathrm{y}_{1}$ and making an angle $\alpha$ with the given line $y=m x+c$ are
$y-y_{1}=\frac{m \pm \tan \alpha}{1 \mp m \tan \alpha}\left(x-x_{1}\right)$

Here, equation of the given line is,

$$
\begin{aligned}
& x+y+\sqrt{3}(y-x)=a \\
& (\sqrt{3}+1) y=(\sqrt{3}-1) x+a \\
& y=\frac{\sqrt{3}-1}{\sqrt{3}+1} x+\frac{a}{\sqrt{3}+1}
\end{aligned}
$$

Comparing this equation with $\mathrm{y}=\mathrm{mx}+\mathrm{c}$
We get,

$$
\begin{aligned}
& \mathrm{m}=\frac{\sqrt{3}-1}{\sqrt{3}+1} \\
& \therefore \mathrm{x}_{1}=0, \mathrm{y}_{1}=0, \alpha=75^{\circ}, \\
& \mathrm{m}=\frac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3} \text { and } \tan 75^{\circ}=2+\sqrt{3}
\end{aligned}
$$

So, the equations of the required lines are

$$
\begin{aligned}
& \begin{array}{l}
y-0=\frac{2-\sqrt{3}+\tan 75^{\circ}}{1-(2-\sqrt{3}) \tan 75^{\circ}}(x-0) \text { and } y-0 \\
\quad=\frac{2-\sqrt{3}-\tan 75^{\circ}}{1+(2-\sqrt{3}) \tan 75^{\circ}}(x-0) \\
y=\frac{2-\sqrt{3}+2+\sqrt{3}}{1-(2-\sqrt{3})(2+\sqrt{3})} x \text { and } y=\frac{2-\sqrt{3}-2-\sqrt{3}}{1+(2-\sqrt{3})(2+\sqrt{3})} x \\
y=\frac{4}{1-1} x \text { and } y=-\sqrt{3} \mathrm{x} \\
\mathrm{x}=0 \text { and } \sqrt{3} \mathrm{x}+\mathrm{y}=0
\end{array}
\end{aligned}
$$

$\therefore$ The equation of given line is $\mathrm{x}=0$ and $\sqrt{3} \mathrm{x}+\mathrm{y}=0$
3. Find the equations of straight lines passing through ( $2,-1$ ) and making an angle of $45^{\circ}$ with the line $6 x+5 y-8=0$.

## Solution:

Given:
The equation passes through $(2,-1)$ and make an angle of $45^{\circ}$ with the line $6 x+5 y-8=0$ We know that the equations of two lines passing through a point $\mathrm{x}_{1}, \mathrm{y}_{1}$ and making an angle $\alpha$ with the given line $\mathrm{y}=\mathrm{mx}+\mathrm{c}$ are

$$
\mathrm{y}-\mathrm{y}_{1}=\frac{\mathrm{m} \pm \tan \alpha}{1 \mp \mathrm{mtan} \alpha}\left(\mathrm{x}-\mathrm{x}_{1}\right)
$$

Here, equation of the given line is,
$6 x+5 y-8=0$
$5 y=-6 x+8$
$y=-6 x / 5+8 / 5$
Comparing this equation with $\mathrm{y}=\mathrm{mx}+\mathrm{c}$
We get, $m=-6 / 5$
Where, $\mathrm{x}_{1}=2, \mathrm{y}_{1}=-1, \alpha=45^{\circ}, \mathrm{m}=-6 / 5$
So, the equations of the required lines are

$$
\begin{aligned}
& y+1=\frac{\left(-\frac{6}{5}+\tan 45^{\circ}\right)}{\left(1+\frac{6}{5} \tan 45^{\circ}\right)}(x-2) \text { and } y+1=\frac{\left(-\frac{6}{5}-\tan 45^{\circ}\right)}{\left(1-\frac{6}{5} \tan 45^{\circ}\right)}(x-2) \\
& y+1=\frac{\left(-\frac{6}{5}+1\right)}{\left(1+\frac{6}{5}\right)}(x-2) \text { and } y+1=\frac{\left(-\frac{6}{5}-1\right)}{\left(1-\frac{6}{5}\right)}(x-2) \\
& y+1=-\frac{1}{11}(x-2) \text { and } y+1=-\frac{11}{-1}(x-2) \\
& x+11 y+9=0 \text { and } 11 x-y-23=0
\end{aligned}
$$

$\therefore$ The equation of given line is $\mathrm{x}+11 \mathrm{y}+9=0$ and $11 \mathrm{x}-\mathrm{y}-23=0$

## 4. Find the equations to the straight lines which pass through the point $(h, k)$ and

 are inclined at angle $\tan ^{-1} \mathrm{~m}$ to the straight line $\mathbf{y}=\mathbf{m x}+c$.
## Solution:

Given:
The equation passes through $(h, k)$ and make an angle of $\tan ^{-1} m$ with the line $y=m x+c$ We know that the equations of two lines passing through a point $\mathrm{x}_{1}, \mathrm{y}_{1}$ and making an angle $\alpha$ with the given line $\mathrm{y}=\mathrm{mx}+\mathrm{c}$ are
$\mathrm{m}^{\prime}=\mathrm{m}$
So,
$y-y_{1}=\frac{m \pm \tan \alpha}{1 \mp m \tan \alpha}\left(x-x_{1}\right)$
Here,
$\mathrm{x}_{1}=\mathrm{h}, \mathrm{y}_{1}=\mathrm{k}, \alpha=\tan ^{-1} \mathrm{~m}, \mathrm{~m}^{\prime}=\mathrm{m}$.
So, the equations of the required lines are

$$
\begin{aligned}
& y-k=\frac{m+m}{1-m^{2}}(x-h) \text { and } y-k=\frac{m-m}{1+m^{2}}(x-h) \\
& y-k=\frac{2 m}{1-m^{2}}(x-h) \text { and } y-k=0 \\
& (y-k)\left(1-m^{2}\right)=2 m(x-h) \text { and } y=k
\end{aligned}
$$

$\therefore$ The equation of given line is $(y-k)\left(1-m^{2}\right)=2 m(x-h)$ and $y=k$.
5. Find the equations to the straight lines passing through the point $(2,3)$ and inclined at an angle of $45^{\circ}$ to the lines $3 x+y-5=0$.

## Solution:

Given:
The equation passes through $(2,3)$ and make an angle of $45^{\circ}$ with the line $3 x+y-5=0$. We know that the equations of two lines passing through a point $\mathrm{x}_{1}, \mathrm{y}_{1}$ and making an angle $\alpha$ with the given line $y=m x+c$ are

$$
\mathrm{y}-\mathrm{y}_{1}=\frac{\mathrm{m} \pm \tan \alpha}{1 \mp \mathrm{~m} \tan \alpha}\left(\mathrm{x}-\mathrm{x}_{1}\right)
$$

Here,
Equation of the given line is,
$3 x+y-5=0$
$y=-3 x+5$
Comparing this equation with $\mathrm{y}=\mathrm{mx}+\mathrm{c}$ we get, $\mathrm{m}=-3$
$\mathrm{x}_{1}=2, \mathrm{y}_{1}=3, \alpha=45 \circ, \mathrm{~m}=-3$.
So, the equations of the required lines are

$$
\begin{aligned}
y-3 & =\frac{-3+\tan 45^{\circ}}{1+3 \tan 45^{\circ}}(x-2) \text { and } y-3=\frac{-3-\tan 45^{\circ}}{1-3 \tan 45^{\circ}}(x-2) \\
y-3 & =\frac{-3+1}{1+3}(x-2) \text { and } y-3=\frac{-3-1}{1-3}(x-2) \\
y-3 & =\frac{-1}{2}(x-2) \text { and } y-3=2(x-2) \\
x+2 y-8 & =0 \text { and } 2 x-y-1=0
\end{aligned}
$$

$\therefore$ The equation of given line is $\mathrm{x}+2 \mathrm{y}-8=0$ and $2 \mathrm{x}-\mathrm{y}-1=0$

1. Find the equation of a straight line through the point of intersection of the lines $4 x$ $-3 y=0$ and $2 x-5 y+3=0$ and parallel to $4 x+5 y+6=0$.

## Solution:

Given:
Lines $4 x-3 y=0$ and $2 x-5 y+3=0$ and parallel to $4 x+5 y+6=0$
The equation of the straight line passing through the points of intersection of $4 x-3 y=0$ and $2 x-5 y+3=0$ is given below:
$4 x-3 y+\lambda(2 x-5 y+3)=0$
$(4+2 \lambda) x+(-3-5 \lambda) y+3 \lambda=0$

$$
\mathrm{y}=\left(\frac{4+2 \lambda}{3+5 \lambda}\right) \mathrm{x}+\frac{3 \lambda}{(3+5 \lambda)}
$$

The required line is parallel to $4 x+5 y+6=0$ or, $y=-4 x / 5-6 / 5$

$$
\begin{aligned}
& \frac{4+2 \lambda}{3+5 \lambda}=-\frac{4}{5} \\
& \frac{4+2 \lambda}{3+5 \lambda}=-\frac{4}{5} \\
& \lambda=-16 / 15
\end{aligned}
$$

$\therefore$ The required equation is

$$
\begin{aligned}
& \left(4-\frac{32}{15}\right) x-\left(3-\frac{80}{15}\right) y-\frac{48}{15}=0 \\
& 28 x+35 y-48=0
\end{aligned}
$$

2. Find the equation of a straight line passing through the point of intersection of $x+$ $2 y+3=0$ and $3 x+4 y+7=0$ and perpendicular to the straight line $x-y+9=0$.
Solution:
Given:
$x+2 y+3=0$ and $3 x+4 y+7=0$
The equation of the straight line passing through the points of intersection of $x+2 y+3=$ 0 and $3 x+4 y+7=0$ is
$x+2 y+3+\lambda(3 x+4 y+7)=0$
$(1+3 \lambda) x+(2+4 \lambda) y+3+7 \lambda=0$
$y=-\left(\frac{1+3 \lambda}{2+4 \lambda}\right) x-\left(\frac{3+7 \lambda}{2+4 \lambda}\right)$
The required line is perpendicular to $\mathrm{x}-\mathrm{y}+9=0$ or, $\mathrm{y}=\mathrm{x}+9$
3. Find the equation of the line passing through the point of intersection of $2 x-7 y+$ $11=0$ and $x+3 y-8=0$ and is parallel to (i) $x=a x i s$ (ii) $y$-axis.
Solution:
Given:
The equations, $2 \mathrm{x}-7 \mathrm{y}+11=0$ and $\mathrm{x}+3 \mathrm{y}-8=0$
The equation of the straight line passing through the points of intersection of $2 x-7 y+$ $11=0$ and $x+3 y-8=0$ is given below:
$2 \mathrm{x}-7 \mathrm{y}+11+\lambda(\mathrm{x}+3 \mathrm{y}-8)=0$
$(2+\lambda) x+(-7+3 \lambda) y+11-8 \lambda=0$
(i) The required line is parallel to the x -axis. So, the coefficient of x should be zero.
$2+\lambda=0$
$\lambda=-2$
Now, substitute the value of $\lambda$ back in equation, we get
$0+(-7-6) y+11+16=0$
$13 \mathrm{y}-27=0$
$\therefore$ The equation of the required line is $13 y-27=0$
(ii) The required line is parallel to the $y$-axis. So, the coefficient of $y$ should be zero.
$-7+3 \lambda=0$
$\lambda=7 / 3$
Now, substitute the value of $\lambda$ back in equation, we get
$(2+7 / 3) x+0+11-8(7 / 3)=0$
$13 \mathrm{x}-23=0$
$\therefore$ The equation of the required line is $13 x-23=0$
4. Find the equation of the straight line passing through the point of intersection of $2 x+3 y+1=0$ and $3 x-5 y-5=0$ and equally inclined to the axes.
Solution:
Given:
The equations, $2 \mathrm{x}+3 \mathrm{y}+1=0$ and $3 \mathrm{x}-5 \mathrm{y}-5=0$
The equation of the straight line passing through the points of intersection of $2 x+3 y+1$
$=0$ and $3 x-5 y-5=0$ is
$2 \mathrm{x}+3 \mathrm{y}+1+\lambda(3 \mathrm{x}-5 \mathrm{y}-5)=0$
$(2+3 \lambda) x+(3-5 \lambda) y+1-5 \lambda=0$
$y=-[(2+3 \lambda) /(3-5 \lambda)]-[(1-5 \lambda) /(3-5 \lambda)]$
The required line is equally inclined to the axes. So, the slope of the required line is either 1 or -1 .

So,
$-[(2+3 \lambda) /(3-5 \lambda)]=1$ and $-[(2+3 \lambda) /(3-5 \lambda)]=-1$
$-2-3 \lambda=3-5 \lambda$ and $2+3 \lambda=3-5 \lambda$
$\lambda=5 / 2$ and $1 / 8$
Now, substitute the values of $\lambda$ in $(2+3 \lambda) x+(3-5 \lambda) y+1-5 \lambda=0$, we get the equations of the required lines as:
$(2+15 / 2) x+(3-25 / 2) y+1-25 / 2=0$ and $(2+3 / 8) x+(3-5 / 8) y+1-5 / 8=0$
$19 \mathrm{x}-19 \mathrm{y}-23=0$ and $19 \mathrm{x}+19 \mathrm{y}+3=0$
$\therefore$ The required equation is $19 x-19 y-23=0$ and $19 x+19 y+3=0$

## 5. Find the equation of the straight line drawn through the point of intersection of

 the lines $x+y=4$ and $2 x-3 y=1$ and perpendicular to the line cutting off intercepts 5,6 on the axes.Solution:
Given:
The lines $\mathrm{x}+\mathrm{y}=4$ and $2 \mathrm{x}-3 \mathrm{y}=1$
The equation of the straight line passing through the point of intersection of $x+y=4$ and $2 x-3 y=1$ is
$x+y-4+\lambda(2 x-3 y-1)=0$
$(1+2 \lambda) x+(1-3 \lambda) y-4-\lambda=0 \ldots$ (1)
$\mathrm{y}=-[(1+2 \lambda) /(1-3 \lambda)] \mathrm{x}+[(4+\lambda) /(1-3 \lambda)]$
The equation of the line with intercepts 5 and 6 on the axis is
$\mathrm{x} / 5+\mathrm{y} / 6=1$
So, the slope of this line is $-6 / 5$
The lines (1) and (2) are perpendicular.
$\therefore-6 / 5 \times[(-1+2 \lambda) /(1-3 \lambda)]=-1$
$\lambda=11 / 3$
Now, substitute the values of $\lambda$ in (1), we get the equation of the required line.
$(1+2(11 / 3)) x+(1-3(11 / 3)) y-4-11 / 3=0$
$(1+22 / 3) x+(1-11) y-4-11 / 3=0$
$25 \mathrm{x}-30 \mathrm{y}-23=0$
$\therefore$ The required equation is $25 \mathrm{x}-30 \mathrm{y}-23=0$

