

### EXERCISE 23.1

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### 1. Find the slopes of the lines which make the following angles with the positive direction of x - axis: (i) $-\pi/4$ (ii) $2\pi/3$ **Solution:** (i) $-\pi/4$ Let the slope of the line be 'm' Where, $m = \tan \theta$ So, the slope of Line is $m = tan (-\pi/4)$ = -1 $\therefore$ The slope of the line is -1. (ii) $2\pi/3$ Let the slope of the line be 'm' Where, $m = \tan \theta$ So, the slope of Line is $m = \tan(2\pi/3)$ $\tan\left(\frac{2\pi}{3}\right) = \tan\left(\pi - \frac{\pi}{3}\right)$ $\tan\left(\frac{2\pi}{3}\right) = \tan\left(-\frac{\pi}{3}\right)$ $\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

 $\therefore$  The slope of the line is  $-\sqrt{3}$ 

#### 2. Find the slopes of a line passing through the following points :

(i) (-3, 2) and (1, 4) (ii)  $(at^{2}_{1}, 2at_{1})$  and  $(at^{2}_{2}, 2at_{2})$ Solution: (i) (-3, 2) and (1, 4) By using the formula, Slope of line,  $m = \frac{y_{2} - y_{1}}{x_{2} - x_{1}}$ So, the slope of the line,  $m = \frac{4 - 2}{1 - (-3)}$  = 2/4= 1/2



: The slope of the line is  $\frac{1}{2}$ .

(ii)  $(at_1^2, 2at_1)$  and  $(at_2^2, 2at_2)$ By using the formula, Slope of line,  $m = \frac{y_2 - y_1}{x_2 - x_1}$ Now, substitute the values The slope of the line,  $m = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2}$   $= \frac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)}$   $= \frac{2a(t_2 - t_1)}{a(t_2 - t_1)t_2 + t_1}$  [Since,  $(a^{2-}b^2 = (a-b)(a+b)$ ]  $= \frac{2}{t_2 + t_1}$ 

 $\therefore$  The slope of the line is  $\overline{t_2 + t_1}$ 

**3.** State whether the two lines in each of the following are parallel, perpendicular or neither:

(i) Through (5, 6) and (2, 3); through (9, -2) and (6, -5) (ii) Through (9, 5) and (-1, 1); through (3, -5) and (8, -3) Solution: (i) Through (5, 6) and (2, 3); through (9, -2) and (6, -5) By using the formula, Slope of line,  $m = \frac{y_2 - y_1}{x_2 - x_1}$ The slope of the line whose Coordinates are (5, 6) and (2, 3)  $m_1 = \frac{3 - 6}{2 - 5}$   $= \frac{-3}{-3}$  = 1So,  $m_1 = 1$ The slope of the line whose Coordinates are (9, -2) and (6, -5)  $m_2 = \frac{-5 - (-2)}{6 - 9}$   $= \frac{-3}{-3}$ So,  $m_2 = 1$ 



Here,  $m_1 = m_2 = 1$ 

 $\therefore$  The lines are parallel to each other.

(ii) Through (9, 5) and (-1, 1); through (3, -5) and (8, -3)By using the formula, Slope of line,  $m = \frac{y_2 - y_1}{x_2 - x_1}$ The slope of the line whose Coordinates are (9, 5) and (-1, 1) $m_1 = \frac{1-5}{-1-9}$  $=\frac{-4}{-10}$ The slope of the line whose Coordinates are (3, -5) and (8, -3)  $m_2 = \frac{-3 - (-5)}{8 - 3}$ = 2/5= 2/5 $So, m_2 = 2/5$ Here,  $m_1 = m_2 = 2/5$ : The lines are parallel to each other. 4. Find the slopes of a line (i) which bisects the first quadrant angle (ii) which makes an angle of  $30^{\circ}$  with the positive direction of y - axis measured anticlockwise. Solution: (i) Which bisects the first quadrant angle? Given: Line bisects the first quadrant We know that, if the line bisects in the first quadrant, then the angle must be between line and the positive direction of x - axis. Since, angle =  $90/2 = 45^{\circ}$ By using the formula, The slope of the line,  $m = \tan \theta$ The slope of the line for a given angle is  $m = \tan 45^{\circ}$ 

So, m = 1

 $\therefore$  The slope of the line is 1.

(ii) Which makes an angle of  $30^{\circ}$  with the positive direction of y - axis measured



anticlockwise? Given: The line makes an angle of 30° with the positive direction of y – axis. We know that, angle between line and positive side of axis => 90° + 30° = 120° By using the formula, The slope of the line, m = tan  $\theta$ The slope of the line for a given angle is m = tan 120° So, m =  $-\sqrt{3}$  $\therefore$  The slope of the line is  $-\sqrt{3}$ .

#### 5. Using the method of slopes show that the following points are collinear:

(i) A (4, 8), B (5, 12), C (9, 28) (ii) A(16, -18), B(3, -6), C(-10, 6)Solution: (i) A (4, 8), B (5, 12), C (9, 28) By using the formula, The slope of the line =  $[y_2 - y_1] / [x_2 - x_1]$ So, The slope of line AB = [12 - 8] / [5 - 4]= 4 / 1The slope of line BC = [28 - 12] / [9 - 5]= 16 / 4= 4The slope of line CA = [8 - 28] / [4 - 9]= -20 / -5=4Here, AB = BC = CA $\therefore$  The Given points are collinear.

(ii) A(16, -18), B(3, -6), C(-10, 6) By using the formula, The slope of the line =  $[y_2 - y_1] / [x_2 - x_1]$ So, The slope of line AB = [-6 - (-18)] / [3 - 16]= 12 / -13

The slope of line BC = [6 - (-6)] / [-10 - 3]= 12 / -13



The slope of line CA = [6 - (-18)] / [-10 - 16]= 12 / -13 = 4 Here, AB = BC = CA

 $\therefore$  The Given points are collinear.





### EXERCISE 23.2

### PAGE NO: 23.17

## 1. Find the equation of the line parallel to x-axis and passing through (3, -5). Solution:

Given: A line which is parallel to x-axis and passing through (3, -5)By using the formula, The equation of line:  $[y - y_1 = m(x - x_1)]$ We know that the parallel lines have equal slopes And, the slope of x-axis is always 0 Then The slope of line, m = 0Coordinates of line are  $(x_1, y_1) = (3, -5)$ The equation of line  $= y - y_1 = m(x - x_1)$ Now, substitute the values, we get y - (-5) = 0(x - 3)y + 5 = 0 $\therefore$  The equation of line is y + 5 = 0

## 2. Find the equation of the line perpendicular to x-axis and having intercept -2 on x-axis.

#### Solution:

Given: A line which is perpendicular to x-axis and having intercept -2By using the formula, The equation of line:  $[y - y_1 = m(x - x_1)]$ We know that, the line is perpendicular to the x-axis, then x is 0 and y is -1. The slope of line is, m = y/x= -1/0

It is given that x-intercept is -2, so, y is 0. Coordinates of line are  $(x_1, y_1) = (-2, 0)$ The equation of line =  $y - y_1 = m(x - x_1)$ Now, substitute the values, we get y - 0 = (-1/0) (x - (-2))x + 2 = 0 $\therefore$  The equation of line is x + 2 = 0

# 3. Find the equation of the line parallel to x-axis and having intercept – 2 on y – axis.

Solution:



Given: A line which is parallel to x-axis and having intercept -2 on y - axis By using the formula, The equation of line:  $[y - y_1 = m(x - x_1)]$ The parallel lines have equal slopes, And, the slope of x-axis is always 0 Then The slope of line, m = 0 It is given that intercept is -2, on y - axis then Coordinates of line are  $(x_1, y_1) = (0, -2)$ The equation of line is  $y - y_1 = m(x - x_1)$ Now, substitute the values, we get y - (-2) = 0 (x - 0)y + 2 = 0 $\therefore$  The equation of line is y + 2 = 0

4. Draw the lines x = -3, x = 2, y = -2, y = 3 and write the coordinates of the vertices of the square so formed. Solution:

Given: x = -3, x = 2, y = -2 and y = 3



: The Coordinates of the square are: A(2, 3), B(2, -2), C(-3, 3), and D(-3, -2).

5. Find the equations of the straight lines which pass through (4, 3) and are respectively parallel and perpendicular to the x-axis. Solution:



Given: A line which is perpendicular and parallel to x-axis respectively and passing through (4, 3) By using the formula, The equation of line:  $[y - y_1 = m(x - x_1)]$ Let us consider,

<u>Case 1:</u> When Line is parallel to x-axis The parallel lines have equal slopes, And, the slope of x-axis is always 0, then The slope of line, m = 0Coordinates of line are  $(x_1, y_1) = (4, 3)$ The equation of line is  $y - y_1 = m(x - x_1)$ Now substitute the values, we get y - (3) = 0(x - 4)y - 3 = 0

<u>Case 2:</u> When line is perpendicular to x-axis The line is perpendicular to the x-axis, then x is 0 and y is – 1. The slope of the line is, m = y/x = -1/0Coordinates of line are  $(x_1, y_1) = (4, 3)$ The equation of line  $= y - y_1 = m(x - x_1)$ Now substitute the values, we get y - 3 = (-1/0) (x - 4)x = 4

: The equation of line when it is parallel to x - axis is y = 3 and it is perpendicular is x = 4.



### EXERCISE 23.3

### PAGE NO: 23.21

# 1. Find the equation of a line making an angle of 150° with the x-axis and cutting off an intercept 2 from y-axis. Solution:

## Given: A line which makes an angle of $150^{\circ}$ with the x-axis and cutting off an intercept at 2

By using the formula, The equation of a line is y = mx + cWe know that angle,  $\theta = 150^{\circ}$ The slope of the line,  $m = \tan \theta$ Where,  $m = \tan 150^{\circ}$  $= -1/\sqrt{3}$ 

Coordinate of y-intercept is (0, 2)The required equation of the line is y = mx + cNow substitute the values, we get  $y = -x/\sqrt{3} + 2$  $\sqrt{3}y - 2\sqrt{3} + x = 0$  $x + \sqrt{3}y = 2\sqrt{3}$  $\therefore$  The equation of line is  $x + \sqrt{3}y = 2\sqrt{3}$ 

#### 2. Find the equation of a straight line:

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(i) with slope 2 and y – intercept 3;
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(ii) with slope -1/3 and y - intercept -4.
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(iii) with slope -2 and intersecting the x-axis at a distance of 3 units to the left of origin.

#### Solution:

(i) With slope 2 and y – intercept 3 The slope is 2 and the coordinates are (0, 3)Now, the required equation of line is y = mx + cSubstitute the values, we get y = 2x + 3

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(ii) With slope -1/3 and y - intercept -4
The slope is -1/3 and the coordinates are (0, -4)
Now, the required equation of line is
y = mx + c
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Substitute the values, we get y = -1/3x - 43y + x = -12

(iii) With slope -2 and intersecting the x-axis at a distance of 3 units to the left of origin The slope is -2 and the coordinates are (-3, 0)

Now, the required equation of line is  $y - y_1 = m (x - x_1)$ 

Substitute the values, we get

y-0 = -2(x + 3)y = -2x - 62x + y + 6 = 0

### **3.** Find the equations of the bisectors of the angles between the coordinate axes. Solution:

There are two bisectors of the coordinate axes.

Their inclinations with the positive x-axis are  $45^{\circ}$  and  $135^{\circ}$ 

The slope of the bisector is  $m = \tan 45^{\circ}$  or  $m = \tan 135^{\circ}$ 

i.e., m = 1 or m = -1, c = 0

By using the formula, y = mx + c

Now, substitute the values of m and c, we get

 $\mathbf{y} = \mathbf{x} + \mathbf{0}$ 

x - y = 0 or y = -x + 0

$$\mathbf{x} + \mathbf{y} = \mathbf{0}$$

: The equation of the bisector is  $x \pm y = 0$ 

# 4. Find the equation of a line which makes an angle of $\tan^{-1}(3)$ with the x-axis and cuts off an intercept of 4 units on the negative direction of y-axis. Solution:

Given:

The equation which makes an angle of  $\tan^{-1}(3)$  with the x-axis and cuts off an intercept of 4 units on the negative direction of y-axis

By using the formula,

The equation of the line is y = mx + cHere, angle  $\theta = \tan^{-1}(3)$ 

So,  $\tan \theta = 3$ 

The slope of the line is, m = 3

And, Intercept in the negative direction of y-axis is (0, -4)

The required equation of the line is y = mx + c

Now, substitute the values, we get



y = 3x - 4

 $\therefore$  The equation of the line is y = 3x - 4.

## 5. Find the equation of a line that has y - intercept - 4 and is parallel to the line joining (2, -5) and (1, 2).

#### Solution:

Given: A line segment joining (2, -5) and (1, 2) if it cuts off an intercept - 4 from y-axis By using the formula, The equation of line is y = mx + CIt is given that, c = -4Slope of line joining  $(x_1 - x_2)$  and  $(y_1 - y_2)$ ,  $m = \frac{y_2 - y_1}{x_2 - x_1}$ So, Slope of line joining (2, -5) and (1, 2),  $m = \frac{2 - (-5)}{1 - 2} = \frac{7}{-1}$  m = -7The equation of line is y = mx + cNow, substitute the values, we get y = -7x - 4 y + 7x + 4 = 0 $\therefore$  The equation of line is y + 7x + 4 = 0.



### **EXERCISE 23.4**

### PAGE NO: 23.29

# 1. Find the equation of the straight line passing through the point (6, 2) and having slope -3.

#### Solution:

Given, A straight line passing through the point (6, 2) and the slope is -3By using the formula, The equation of line is  $[y - y_1 = m(x - x_1)]$ Here, the line is passing through (6, 2) It is given that, the slope of line, m = -3Coordinates of line are  $(x_1, y_1) = (6, 2)$ The equation of line  $= y - y_1 = m(x - x_1)$ Now, substitute the values, we get y - 2 = -3(x - 6)y - 2 = -3x + 18y + 3x - 20 = 0 $\therefore$  The equation of line is 3x + y - 20 = 0

# 2. Find the equation of the straight line passing through (-2, 3) and indicated at an angle of $45^{\circ}$ with the x – axis. Solution:

Given: A line which is passing through (-2, 3), the angle is 45°. By using the formula, The equation of line is  $[y - y_1 = m(x - x_1)]$ Here, angle,  $\theta = 45^\circ$ The slope of the line,  $m = \tan \theta$   $m = \tan 45^\circ$  = 1The line passing through  $(x_1, y_1) = (-2, 3)$ The required equation of line is  $y - y_1 = m(x - x_1)$ Now, substitute the values, we get y - 3 = 1(x - (-2)) y - 3 = x + 2 x - y + 5 = 0 $\therefore$ The equation of line is x - y + 5 = 0

## **3.** Find the equation of the line passing through (0, 0) with slope m Solution:



Given:

A straight line passing through the point (0, 0) and slope is m. By using the formula, The equation of line is  $[y - y_1 = m(x - x_1)]$ It is given that, the line is passing through (0, 0) and the slope of line, m = mCoordinates of line are  $(x_1, y_1) = (0, 0)$ The equation of line  $= y - y_1 = m(x - x_1)$ Now, substitute the values, we get y - 0 = m(x - 0)y = mx $\therefore$  The equation of line is y = mx.

## 4. Find the equation of the line passing through $(2, 2\sqrt{3})$ and inclined with x – axis at an angle of 75°.

#### Solution:

Given:

A line which is passing through  $(2, 2\sqrt{3})$ , the angle is 75°.

By using the formula,

The equation of line is  $[y - y_1 = m(x - x_1)]$ 

Here, angle,  $\theta = 75^{\circ}$ 

The slope of the line,  $m = \tan \theta$ 

 $m = \tan 75^{\circ}$ 

$$= 3.73 = 2 + \sqrt{3}$$

The line passing through  $(x_1, y_1) = (2, 2\sqrt{3})$ 

The required equation of the line is  $y - y_1 = m(x - x_1)$ 

Now, substitute the values, we get

 $y - 2\sqrt{3} = (2 + \sqrt{3}) (x - 2)$ 

 $y - 2\sqrt{3} = (2 + \sqrt{3})x - 7.46$ (2 +  $\sqrt{3}$ )x - y - 4 = 0

 $(2 + \sqrt{3})x - y - 4 = 0$  $\therefore$  The equation of the line is  $(2 + \sqrt{3})x - y - 4 = 0$ 

5. Find the equation of the straight line which passes through the point (1, 2) and makes such an angle with the positive direction of x - axis whose sine is 3/5. Solution:

A line which is passing through (1, 2) To Find: The equation of a straight line. By using the formula, The equation of line is  $[y - y_1 = m(x - x_1)]$ Here,  $\sin \theta = 3/5$ 



We know,  $\sin \theta = \text{perpendicular/hypotenuse}$ = 3/5So, according to Pythagoras theorem,  $(Hypotenuse)^{2} = (Base)^{2} + (Perpendicular)^{2}$  $(5)^{2} = (Base)^{2} + (3)^{2}$ (Base) =  $\sqrt{(25 - 9)}$  $(Base)^2 = \sqrt{16}$ Base = 4Hence,  $\tan \theta = \text{perpendicular/base}$ = 3/4The slope of the line,  $m = \tan \theta$ = 3/4The line passing through  $(x_1, y_1) = (1, 2)$ The required equation of line is  $y - y_1 = m(x - x_1)$ Now, substitute the values, we get  $y - 2 = (\frac{3}{4})(x - 1)$ 4y - 8 = 3x - 33x - 4y + 5 = 0 $\therefore$  The equation of line is 3x - 4y + 5 = 0



### EXERCISE 23.5

### PAGE NO: 23.35

**1.** Find the equation of the straight lines passing through the following pair of points: (i) (0, 0) and (2, -2)(ii) (a, b) and (a + c sin  $\alpha$ , b + c cos  $\alpha$ ) Solution: (i) (0, 0) and (2, -2)Given:  $(x_1, y_1) = (0, 0), (x_2, y_2) = (2, -2)$ The equation of the line passing through the two points (0, 0) and (2, -2) is By using the formula,  $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ Now, substitute the values, we get  $y - 0 = \frac{-2 - 0}{2 - 0} (x - 0)$  $\mathbf{v} = -\mathbf{x}$  $\therefore$  The equation of line is y = -x(ii) (a, b) and  $(a + c \sin \alpha, b + c \cos \alpha)$ Given:  $(x_1, y_1) = (a, b), (x_2, y_2) = (a + c \sin \alpha, b + c \cos \alpha)$ So, the equation of the line passing through the two points (0, 0) and (2, -2) is By using the formula,  $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ Now, substitute the values, we get  $y - b = \frac{b + c \cos \alpha - b}{a + c \sin \alpha - a} (x - a)$  $y - b = \cot \alpha (x - a)$  $\therefore$  The equation of line is  $y - b = \cot \alpha (x - a)$ 

2. Find the equations to the sides of the triangles the coordinates of whose angular points are respectively:

(i) (1, 4), (2, -3) and (-1, -2)
(ii) (0, 1), (2, 0) and (-1, -2)
Solution:
(i) (1, 4), (2, -3) and (-1, -2)
Given:



Points A (1, 4), B (2, -3) and C (-1, -2). Let us assume,  $m_1$ ,  $m_2$ , and  $m_3$  be the slope of the sides AB, BC and CA, respectively. So, The equation of the line passing through the two points  $(x_1, y_1)$  and  $(x_2, y_2)$ . Then,  $m_1 = \frac{-3-4}{2-1}$  $m_2 = \frac{-2+3}{-1-2}$  $m_3 = \frac{4+2}{1+1}$  $m_1 = -7$ ,  $m_2 = -1/3$  and  $m_3 = 3$ So, the equation of the sides AB, BC and CA are By using the formula,  $y - y_1 = m (x - x_1)$ => y - 4 = -7 (x - 1)y - 4 = -7x + 7y + 7x = 11, => y + 3 = (-1/3) (x - 2)3y + 9 = -x + 23y + x = -7x + 3y + 7 = 0 and => y + 2 = 3(x+1)y + 2 = 3x + 3y - 3x = 1So, we get y + 7x = 11, x + 3y + 7 = 0 and y - 3x = 1: The equation of sides are y + 7x = 11, x + 3y + 7 = 0 and y - 3x = 1(**ii**) (0, 1), (2, 0) and (-1, -2) Given: Points A (0, 1), B (2, 0) and C (-1, -2). Let us assume,  $m_1$ ,  $m_2$ , and  $m_3$  be the slope of the sides AB, BC and CA, respectively. So, The equation of the line passing through the two points  $(x_1, y_1)$  and  $(x_2, y_2)$ . Then,



 $m_1 = \frac{0-1}{2-0}$  $m_2 = \frac{-2-0}{-1-2},$  $m_3 = \frac{1+2}{1+0}$  $m_1 = -1/2$ ,  $m_2 = 2/3$  and  $m_3 = 3$ So, the equation of the sides AB, BC and CA are By using the formula,  $y - y_1 = m (x - x_1)$ => y - 1 = (-1/2) (x - 0)2y - 2 = -xx + 2y = 2 $\Rightarrow y - 0 = (2/3) (x - 2)$ 3y = 2x - 42x - 3y = 4=> y + 2 = 3(x+1)y + 2 = 3x + 3y - 3x = 1So, we get x + 2y = 2, 2x - 3y = 4 and y - 3x = 1: The equation of sides are x + 2y = 2, 2x - 3y = 4 and y - 3x = 1

# 3. Find the equations of the medians of a triangle, the coordinates of whose vertices are (-1, 6), (-3,-9) and (5, -8).

#### Solution:

Given:

A (-1, 6), B (-3, -9) and C (5, -8) be the coordinates of the given triangle. Let us assume: D, E, and F be midpoints of BC, CA and AB, respectively. So, the coordinates of D, E and F are





Median AD passes through A (-1, 6) and D (1, -17/2) So, by using the formula,

$$y - y_{1} = \left(\frac{y_{2} - y_{1}}{x_{2} - x_{1}}\right)(x - x_{1})$$
$$y - 6 = \frac{-\frac{17}{2} - 6}{1 + 1}(x + 1)$$
$$4y - 24 = -29x - 29$$
$$29x + 4y + 5 = 0$$

Similarly, Median BE passes through B (-3,-9) and E (2,-1) So, by using the formula,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$
$$y + 9 = \frac{-1 + 9}{2 + 3}(x + 3)$$
$$5y + 45 = 8x + 24$$
$$8x - 5y - 21 = 0$$

Similarly, Median CF passes through C (5,-8) and F(-2,-3/2) So, by using the formula,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$
  

$$y + 8 = \frac{-\frac{3}{2} + 8}{-2 - 5}(x - 5)$$
  

$$y + 8 = [(-3 + 16)/(2(-7))](x - 5)$$
  

$$y + 8 = (-13/14)(x - 5)$$
  

$$-14y - 112 = 13x - 65$$
  

$$13x + 14y + 47 = 0$$
  
∴ The equation of lines are:  $29x + 4y + 5 = 0$ ,  $8x - 5y - 21 = 0$  and  $13x + 14y + 47 = 0$   
https://byjus.com



# 4. Find the equations to the diagonals of the rectangle the equations of whose sides are x = a, x = a', y = b and y = b'.

#### Solution:

Given:

The rectangle formed by the lines x = a, x = a', y = b and y = b'

It is clear that, the vertices of the rectangle are A (a, b), B (a', b), C (a', b') and D (a, b'). The diagonal passing through A (a, b) and C (a', b') is

By using the formula,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$
$$y - b = \frac{b' - b}{a' - a}(x - a)$$
$$(a' - a)y - b(a' - a) = (b' - b)x - a(b' - b)$$
$$(a' - a) - (b' - b)x = ba' - ab'$$

Similarly, the diagonal passing through B (a', b) and D (a, b') is By using the formula,

$$y - y_{1} = \left(\frac{y_{2} - y_{1}}{x_{2} - x_{1}}\right)(x - x_{1})$$

$$y - b = \frac{b' - b}{a - a'}(x - a')$$

$$(a' - a)y - b(a - a') = (b' - b)x - a'(b' - b)$$

$$(a' - a) + (b' - b)x = a'b' - ab$$

$$\therefore \text{ The equation of diagonals are } y(a' - a) - x(b' - b) = a'b - ab' \text{ and } y(a' - a) + x(b' - b) = a'b' - ab$$

5. Find the equation of the side BC of the triangle ABC whose vertices are A (-1, -2), B (0, 1) and C (2, 0) respectively. Also, find the equation of the median through A (-1, -2). Solution:

Given:

The vertices of triangle ABC are A (-1, -2), B(0, 1) and C(2, 0).

Let us find the equation of median through A.

So, the equation of BC is

By using the formula,



$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$
$$y - 1 = \frac{0 - 1}{2 - 0}(x - 0)$$
$$y - 1 = \frac{-1}{2}(x - 0)$$
$$x + 2y - 2 = 0$$

Let D be the midpoint of median AD,

So,  $D\left(\frac{0+2}{2}, \frac{1+0}{2}\right) = \left(1, \frac{1}{2}\right)$ The equation of the median AD is By using the formula,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$
$$y + 2 = \frac{\frac{1}{2} + 2}{1 + 1}(x + 1)$$

 $\begin{array}{l} 4y+8=5x+5\\ 5x-4y-3=0 \end{array}$ 

: The equation of line BC is x + 2y - 2 = 0The equation of median is 5x - 4y - 3 = 0



### EXERCISE 23.6

### PAGE NO: 23.46

**1.** Find the equation to the straight line (i) cutting off intercepts 3 and 2 from the axes. (ii) cutting off intercepts -5 and 6 from the axes. Solution: (i) Cutting off intercepts 3 and 2 from the axes. Given: a = 3, b = 2Let us find the equation of line cutoff intercepts from the axes. By using the formula, The equation of the line is x/a + y/b = 1x/3 + y/2 = 1By taking LCM, 2x + 3y = 6 $\therefore$  The equation of line cut off intercepts 3 and 2 from the axes is 2x + 3y = 6(ii) Cutting off intercepts -5 and 6 from the axes. Given: a = -5, b = 6Let us find the equation of line cutoff intercepts from the axes. By using the formula, The equation of the line is x/a + y/b = 1x/-5 + y/6 = 1By taking LCM, 6x - 5y = -30 $\therefore$  The equation of line cut off intercepts 3 and 2 from the axes is 6x - 5y = -302. Find the equation of the straight line which passes through (1, -2) and cuts off equal intercepts on the axes. Solution: Given: A line passing through (1, -2)Let us assume, the equation of the line cutting equal intercepts at coordinates of length 'a' is By using the formula,

The equation of the line is x/a + y/b = 1

$$x/a + y/a = 1 x + y = a$$



The line x + y = a passes through (1, -2)Hence, the point satisfies the equation. 1 - 2 = aa = -1 $\therefore$  The equation of the line is x + y = -1

**3.** Find the equation to the straight line which passes through the point (5, 6) and has intercepts on the axes

a = b

Let us find the equation of line cutoff intercepts from the axes.

By using the formula,

The equation of the line is x/a + y/b = 1

x/a + y/a = 1

$$x + y = a$$

The line passes through the point (5, 6)

Hence, the equation satisfies the points.

5 + 6 = a

a = 11

: The equation of the line is x + y = 11

(ii) Equal in magnitude but opposite in sign

Given: b = -a

Let us find the equation of line cutoff intercepts from the axes.

By using the formula,

The equation of the line is x/a + y/b = 1

$$x/a + y/-a = 1$$

$$\mathbf{x} - \mathbf{y} = \mathbf{a}$$

The line passes through the point (5, 6)

Hence, the equation satisfies the points.

5 - 6 = a

a = -1

: The equation of the line is x - y = -1



# 4. For what values of a and b the intercepts cut off on the coordinate axes by the line ax + by + 8 = 0 are equal in length but opposite in signs to those cut off by the line 2x - 3y + 6 = 0 on the axes.

Solution:

Given:

Intercepts cut off on the coordinate axes by the line ax + by +8 = 0 ..... (i) And are equal in length but opposite in sign to those cut off by the line 2x - 3y + 6 = 0 .....(ii) We know that, the slope of two lines is equal The slope of the line (i) is -a/bThe slope of the line (ii) is 2/3So let us equate, -a/b = 2/3a = -2b/3

The length of the perpendicular from the origin to the line (i) is By using the formula,

$$d = \left| \frac{ax+by+d}{\sqrt{a^2+b^2}} \right|$$
$$d_1 = \left| \frac{a(0)+b(0)+8}{\sqrt{a^2+b^2}} \right|$$
$$= \frac{8\times3}{\sqrt{13b^2}}$$

The length of the perpendicular from the origin to the line (ii) is By using the formula,

$$d = \left| \frac{ax+by+d}{\sqrt{a^2+b^2}} \right|$$
$$d_2 = \left| \frac{2(0)-3(0)+6}{\sqrt{2^2+3^2}} \right|$$
It is given that,  $d_1 = d_2$ 

$$\frac{8 \times 3}{\sqrt{13b^2}} = \frac{6}{\sqrt{13}}$$
  
b = 4  
So, a = -2b/3  
= -8/3

 $\therefore$  The value of a is -8/3 and b is 4.



# 5. Find the equation to the straight line which cuts off equal positive intercepts on the axes and their product is 25.

Solution: Given: a = b and ab = 25Let us find the equation of the line which cutoff intercepts on the axes.  $\therefore a^2 = 25$  a = 5 [considering only positive value of intercepts] By using the formula, The equation of the line with intercepts a and b is x/a + y/b = 1 x/5 + y/5 = 1By taking LCM  $\therefore The equation of line is <math>x + y = 5$ 





### EXERCISE 23.7

#### PAGE NO: 23.53

**1.** Find the equation of a line for which (i)  $p = 5, \alpha = 60^{\circ}$ (ii) p = 4,  $\alpha = 150^{\circ}$ Solution: (i)  $p = 5, \alpha = 60^{\circ}$ Given:  $p = 5, \alpha = 60^{\circ}$ The equation of the line in normal form is given by Using the formula,  $x \cos \alpha + y \sin \alpha = p$ Now, substitute the values, we get  $x \cos 60^\circ + y \sin 60^\circ = 5$  $x/2 + \sqrt{3y/2} = 5$  $x + \sqrt{3}y = 10$ : The equation of line in normal form is  $x + \sqrt{3}y = 10$ . (ii)  $p = 4, \alpha = 150^{\circ}$ Given:  $p = 4, \alpha = 150^{\circ}$ The equation of the line in normal form is given by Using the formula,  $x \cos \alpha + y \sin \alpha = p$ Now, substitute the values, we get  $x \cos 150^{\circ} + y \sin 150^{\circ} = 4$  $\cos(180^\circ - \theta) = -\cos\theta$ ,  $\sin(180^\circ - \theta) = \sin\theta$  $x \cos(180^\circ - 30^\circ) + y \sin(180^\circ - 30^\circ) = 4$  $-x \cos 30^{\circ} + y \sin 30^{\circ} = 4$  $-\sqrt{3x/2} + y/2 = 4$  $-\sqrt{3}x + y = 8$ : The equation of line in normal form is  $-\sqrt{3x} + y = 8$ .

2. Find the equation of the line on which the length of the perpendicular segment from the origin to the line is 4 and the inclination of the perpendicular segment with the positive direction of x-axis is  $30^{\circ}$ .

Solution: Given:  $p = 4, \alpha = 30^{\circ}$ 



The equation of the line in normal form is given by Using the formula,  $x \cos \alpha + y \sin \alpha = p$ Now, substitute the values, we get  $x \cos 30^\circ + y \sin 30^\circ = 4$ 

 $x\sqrt{3/2} + y1/2 = 4$   $\sqrt{3x} + y = 8$ ∴ The equation of line in normal form is  $\sqrt{3x} + y = 8$ .

3. Find the equation of the line whose perpendicular distance from the origin is 4 units and the angle which the normal makes with the positive direction of x-axis is 15°.

Solution:

Given:

 $p = 4, \alpha = 15^{\circ}$ 

The equation of the line in normal form is given by

We know that,  $\cos 15^\circ = \cos (45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$ 

Cos (A - B) = cos A cos B + sin A sin BSo,

$$\cos 15 = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

And  $\sin 15 = \sin (45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$ Sin  $(A - B) = \sin A \cos B - \cos A \sin B$ So,

$$\sin 15 = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

Now, by using the formula, x  $\cos \alpha + y \sin \alpha = p$ Now, substitute the values, we get

$$\frac{\sqrt{3}+1}{2\sqrt{2}}x + \frac{\sqrt{3}-1}{2\sqrt{2}}y = 4$$

 $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 8\sqrt{2}$ 

: The equation of line in normal form is  $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 8\sqrt{2}$ .

4. Find the equation of the straight line at a distance of 3 units from the origin such that the perpendicular from the origin to the line makes an angle  $\alpha$  given by tan  $\alpha$  =



## 5/12 with the positive direction of x-axis. Solution:

Given: p = 3,  $\alpha = \tan^{-1} (5/12)$ So,  $\tan \alpha = 5/12$   $\sin \alpha = 5/13$   $\cos \alpha = 12/13$ The equation of the line in normal form is given by By using the formula,  $x \cos \alpha + y \sin \alpha = p$ Now, substitute the values, we get 12x/13 + 5y/13 = 3 12x + 5y = 39 $\therefore$  The equation of line in normal form is 12x + 5y = 39.

5. Find the equation of the straight line on which the length of the perpendicular from the origin is 2 and the perpendicular makes an angle  $\alpha$  with x-axis such that  $\sin \alpha = 1/3$ .

#### Solution:

Given: p = 2,  $\sin \alpha = 1/3$ We know that  $\cos \alpha = \sqrt{(1 - \sin^2 \alpha)}$   $= \sqrt{(1 - 1/9)}$  $= 2\sqrt{2/3}$ 

The equation of the line in normal form is given by

By using the formula,  $x \cos \alpha + y \sin \alpha = p$ Now, substitute the values, we get  $x2\sqrt{2/3} + y/3 = 2$   $2\sqrt{2x} + y = 6$  $\therefore$  The equation of line in normal form is  $2\sqrt{2x} + y = 6$ .



### EXERCISE 23.8

### PAGE NO: 23.65

**1.** A line passes through a point A (1, 2) and makes an angle of  $60^0$  with the x-axis and intercepts the line x + y = 6 at the point P. Find AP. Solution:

Given:

 $(x_1, y_1) = A (1, 2), \theta = 60^{\circ}$ Let us find the distance AP. By using the formula, The equation of the line is given by:

 $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$ Now, substitute the values, we get  $\frac{x-1}{\cos60^\circ} = \frac{y-2}{\sin60^\circ} = r$  $\frac{x-1}{\frac{1}{2}} = \frac{y-2}{\frac{\sqrt{2}}{2}} = r$ 

Here, r represents the distance of any point on the line from point A (1, 2). The coordinate of any point P on this line are  $(1 + r/2, 2 + \sqrt{3r/2})$  It is clear that, P lies on the line x + y = 6 So,

 $1 + \frac{r}{2} + 2 + \frac{\sqrt{3}}{2}r = 6$   $\frac{\sqrt{3}}{2}r + \frac{r}{2} = 3$   $r(\sqrt{3} + 1) = 6$   $r = \frac{6}{\sqrt{3} + 1} = 3(\sqrt{3} - 1)$  $\therefore \text{ The value of AP is } 3(\sqrt{3} - 1)$ 

2. If the straight line through the point P(3, 4) makes an angle  $\pi/6$  with the x-axis and meets the line 12x + 5y + 10 = 0 at Q, find the length PQ. Solution:

Given:

 $(x_1, y_1) = A (3, 4), \theta = \pi/6 = 30^{\circ}$ Let us find the length PQ. By using the formula, The equation of the line is given by:



 $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$ Now, substitute the values, we get  $\frac{x-3}{\cos 30^\circ} = \frac{y-4}{\sin 30^\circ} = r$  $\frac{x-3}{\frac{\sqrt{3}}{2}} - \frac{y-4}{\frac{1}{2}} = r$  $x - \sqrt{3} y + 4\sqrt{3} - 3 = 0$ Let PQ = rThen, the coordinate of Q are given by  $\frac{x-3}{\cos 30^\circ} = \frac{y-4}{\sin 30^\circ} = r$  $X = 3 + \frac{\sqrt{3}}{2}r, Y = 4 + \frac{r}{2}$ The coordinate of point Q is  $\left(3 + \frac{\sqrt{3}}{2}r, 4 + \frac{r}{2}\right)$ It is clear that, Q lies on the line 12x + 5y + 10 = 0So.  $12\left(3 + \frac{\sqrt{3}}{2}r\right) + 5\left(4 + \frac{r}{2}\right) + 10 = 0$  $66 + \frac{12\sqrt{3}+5}{2}r = 0$  $r = -\frac{132}{5+12\sqrt{3}}$  $PQ = |r| = \frac{132}{5 + 12\sqrt{3}}$  $\therefore$  The value of PQ is  $\overline{5+12\sqrt{3}}$ 

3. A straight line drawn through the point A (2, 1) making an angle  $\pi/4$  with positive x-axis intersects another line x + 2y + 1 = 0 in the point B. Find length AB. Solution:

Given:

 $(x_1, y_1) = A (2, 1), \theta = \pi/4 = 45^{\circ}$ Let us find the length AB. By using the formula, The equation of the line is given by:



 $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$ Now, substitute the values, we get  $\frac{x-2}{\cos 45^\circ} = \frac{y-1}{\sin 45^\circ} = r$   $\frac{x-2}{\frac{1}{\sqrt{2}}} = \frac{y-1}{\frac{1}{\sqrt{2}}} = r$  x - y - 1 = 0Let AB = r
Then, the coordinate of B is given by  $\frac{x-2}{\cos 45_\circ} = \frac{y-1}{\sin 45_\circ} = r$   $x = 2 + \frac{1}{\sqrt{2}}r, y = 1 + \frac{r}{\sqrt{2}}$ The coordinate of point B is  $\left(2 + \frac{1}{\sqrt{2}}r, 1 + \frac{r}{\sqrt{2}}\right)$ It is clear that, B lies on the line x + 2y + 1 = 0  $2 + \frac{1}{\sqrt{2}}r + 2\left(1 + \frac{r}{\sqrt{2}}\right) + 1 = 0$   $5 + \frac{3r}{\sqrt{2}}r = 0$   $r = \frac{5\sqrt{2}}{3}$   $\therefore$  The value of AB is  $\frac{5\sqrt{2}}{3}$ 

4. A line a drawn through A (4, -1) parallel to the line 3x - 4y + 1 = 0. Find the coordinates of the two points on this line which are at a distance of 5 units from A. Solution:

Given:

 $(x_1, y_1) = A(4, -1)$ 

Let us find Coordinates of the two points on this line which are at a distance of 5 units from A.

Given: Line 3x - 4y + 1 = 0 4y = 3x + 1 y = 3x/4 + 1/4Slope  $\tan \theta = 3/4$ So,  $\sin \theta = 3/5$  $\cos \theta = 4/5$ 



The equation of the line passing through A (4, -1) and having slope <sup>3</sup>/<sub>4</sub> is By using the formula,

The equation of the line is given by:

 $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$ Now, substitute the values, we get  $\frac{x-4}{\frac{4}{5}} = \frac{y+1}{\frac{3}{5}}$ 3x - 4y = 16

Here,  $AP = r = \pm 5$ Thus, the coordinates of P are given by  $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$ 

$$\frac{x-4}{\frac{4}{5}} = \frac{y+1}{\frac{3}{5}} = r$$

$$x = \frac{4r}{5} + 4 \text{ and } y = \frac{3r}{5} - 1$$

$$x = \frac{4(\pm 5)}{5} + 4 \text{ and } y = \frac{3(\pm 5)}{5} - 1$$

$$x = \pm 4 + 4 \text{ and } y = \pm 3 - 1$$

$$x = 8, 0 \text{ and } y = 2, -4$$

: The coordinates of the two points at a distance of 5 units from A are (8, 2) and (0, -4).

1

# 5. The straight line through $P(x_1, y_1)$ inclined at an angle $\theta$ with the x-axis meets the line ax + by + c = 0 in Q. Find the length of PQ. Solution:

Given:

The equation of the line that passes through  $P(x_1, y_1)$  and makes an angle of  $\theta$  with the x-axis.

Let us find the length of PQ.

We know that,

```
\frac{\mathbf{x} - \mathbf{x}_1}{\cos \theta} = \frac{\mathbf{y} - \mathbf{y}_1}{\sin \theta}
```

Let PQ = r

Then, the coordinates of Q are given by

By using the formula,

The equation of the line is given by:



 $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$  $x = x_1 + r\cos\theta, y = y_1 + r\sin\theta$ Thus, the coordinates of Q are  $(x_1 + r\cos\theta, y_1 + r\sin\theta)$ It is clear that, Q lies on the line ax + by + c = 0. So,  $a(x_1 + r\cos\theta) + b(y_1 + r\sin\theta) + c = 0$  $r = PQ = \left| \frac{ax_1 + by_1 + c}{a \cos \theta + b \sin \theta} \right|$  $\therefore \text{ The value of PQ is } \begin{vmatrix} \frac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta} \end{vmatrix}$ 



### EXERCISE 23.9

### PAGE NO: 23.72

**1.** Reduce the equation  $\sqrt{3x + y} + 2 = 0$  to: (i) slope - intercept form and find slope and y - intercept; (ii) Intercept form and find intercept on the axes (iii) The normal form and find p and  $\alpha$ . Solution: (i) Given:  $\sqrt{3x + y} + 2 = 0$  $y = -\sqrt{3}x - 2$ This is the slope intercept form of the given line.  $\therefore$  The slope = - $\sqrt{3}$  and y - intercept = -2 (ii) Given:  $\sqrt{3x + y} + 2 = 0$  $\sqrt{3x} + y = -2$ Divide both sides by -2, we get  $\sqrt{3x/-2} + \frac{y}{-2} = 1$ : The intercept form of the given line. Here, x - intercept =  $-2/\sqrt{3}$  and y - intercept = -2

(iii) Given:  

$$\sqrt{3x} + y + 2 = 0$$
  
 $-\sqrt{3x} - y = 2$   
 $-\frac{\sqrt{3x}}{\sqrt{(-\sqrt{3})^2 + (-1)^2}} - \frac{y}{\sqrt{(-\sqrt{3})^2 + (-1)^2}} = \frac{2}{\sqrt{(-\sqrt{3})^2 + (-1)^2}}$ 

Divide both sides by  $\sqrt{(\operatorname{coefficient of } x)^2 + (\operatorname{coefficient of } y)^2}$ 

 $-\frac{\sqrt{3}x}{2} - \frac{y}{2} = 1$ This is the normal form of the given line. So, p = 1 cos  $\alpha = -\sqrt{3}/2$  and sin  $\alpha = -1/2$  $\therefore$  p = 1 and  $\alpha = 210$ 

2. Reduce the following equations to the normal form and find p and  $\alpha$  in each case: (i)  $x + \sqrt{3}y - 4 = 0$ (ii)  $x + y + \sqrt{2} = 0$ Solution: (i)  $x + \sqrt{3}y - 4 = 0$ 



 $x + \sqrt{3}y = 4$ 

$$\frac{x}{\sqrt{1^2 + (\sqrt{3})^2}} + \frac{\sqrt{3}y}{\sqrt{1^2 + (\sqrt{3})^2}} = \frac{4}{\sqrt{1^2 + (\sqrt{3})^2}}$$

Divide both sides by  $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$ 

$$\frac{x}{2} + \frac{\sqrt{3}y}{2} = 2$$

The normal form of the given line, where p = 2,  $\cos \alpha = 1/2$  and  $\sin \alpha = \sqrt{3}/2$  $\therefore p = 2$  and  $\alpha = \pi/3$ 

(ii) 
$$x + y + \sqrt{2} = 0$$
  
 $-x - y = \sqrt{2}$   
 $\frac{-x}{\sqrt{(-1)^2 + (-1)^2}} + \frac{y}{\sqrt{(-1)^2 + (-1)^2}} = \frac{\sqrt{2}}{\sqrt{(-1)^2 + (-1)^2}}$ 

Divide both sides by  $\sqrt{(\operatorname{coefficient of } x)^2 + (\operatorname{coefficient of } y)^2}$ 

$$-\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = 1$$

The normal form of the given line, where p = 1,  $\cos \alpha = -1/\sqrt{2}$  and  $\sin \alpha = -1/\sqrt{2}$  $\therefore p = 1$  and  $\alpha = 225^{\circ}$ 

# 3. Put the equation x/a + y/b = 1 the slope intercept form and find its slope and y - intercept.

Solution:

Given: the equation is x/a + y/b = 1We know that, General equation of line y = mx + c. bx + ay = abay = -bx + aby = -bx/a + bThe slope intercept form of the given line.  $\therefore$  Slope = - b/a and y - intercept = b

4. Reduce the lines 3x - 4y + 4 = 0 and 2x + 4y - 5 = 0 to the normal form and hence find which line is nearer to the origin. Solution:

Given:



The normal forms of the lines 3x - 4y + 4 = 0 and 2x + 4y - 5 = 0. Let us find, in given normal form of a line, which is nearer to the origin. -3x + 4y = 4

$$-\frac{3 x}{\sqrt{(-3)^2 + (4)^2}} + 4\frac{y}{\sqrt{(-3)^2 + (4)^2}} = \frac{4}{\sqrt{(-3)^2 + (4)^2}}$$

Divide both sides by  $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$  $-\frac{3}{5}x + \frac{4}{5}y = \frac{4}{5}$ .....(1)

Now 
$$2x + 4y = -5$$
  
 $-2x - 4y = 5$   
 $-\frac{2x}{\sqrt{(-2)^2 + (-4)^2}} - 4\frac{y}{\sqrt{(-2)^2 + (-4)^2}} = \frac{5}{\sqrt{(-2)^2 + (-4)^2}}$ 

Divide both sides by  $\sqrt{(\operatorname{coefficient of } x)^2 + (\operatorname{coefficient of } y)^2}$ 

$$-\frac{2}{2\sqrt{5}}x - \frac{4}{2\sqrt{5}}y = \frac{5}{2\sqrt{5}} \dots \dots (2)$$

From equations (1) and (2): 45 < 525 $\therefore$  The line 3x - 4y + 4 = 0 is nearer to the origin.

5. Show that the origin is equidistant from the lines 4x + 3y + 10 = 0; 5x - 12y + 26 = 0 and 7x + 24y = 50.

#### Solution:

Given:

The lines 4x + 3y + 10 = 0; 5x - 12y + 26 = 0 and 7x + 24y = 50.

We need to prove that, the origin is equidistant from the lines 4x + 3y + 10 = 0; 5x - 12y + 26 = 0 and 7x + 24y = 50.

Let us write down the normal forms of the given lines.

First line: 4x + 3y + 10 = 0

$$-4x - 3y = 10$$

$$-\frac{4x}{\sqrt{(-4)^2 + (-3)^2}} - 3\frac{y}{\sqrt{(-4)^2 + (-3)^2}} = \frac{10}{\sqrt{(-4)^2 + (-3)^2}}$$

Divide both sides by  $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$ 

$$-\frac{4}{5}x - \frac{3}{5}y = 2$$
  
So, p = 2



Second line: 5x - 12y + 26 = 0 -5x + 12y = 26  $-\frac{5x}{\sqrt{(-5)^2 + (12)^2}} + 12\frac{y}{\sqrt{(-5)^2 + (12)^2}} = \frac{26}{\sqrt{(-5)^2 + (12)^2}}$ Divide both sides by  $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$   $-\frac{5}{13}x + \frac{12}{13}y = 2$ So, p = 2 Third line: 7x + 24y = 50  $\frac{7x}{\sqrt{(7)^2 + (24)^2}} + 24\frac{y}{\sqrt{(7)^2 + (24)^2}} = \frac{50}{\sqrt{(7)^2 + (24)^2}}$ Divide both sides by  $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$   $\frac{7}{25}x + \frac{24}{25}y = 2$ So, p = 2  $\therefore$  The origin is equidistant from the given lines.


### EXERCISE 23.10

### PAGE NO: 23.77

**1.** Find the point of intersection of the following pairs of lines: (i) 2x - y + 3 = 0 and x + y - 5 = 0(ii) bx + ay = ab and ax + by = abSolution: (i) 2x - y + 3 = 0 and x + y - 5 = 0Given: The equations of the lines are as follows:  $2x - y + 3 = 0 \dots (1)$  $x + y - 5 = 0 \dots (2)$ Let us find the point of intersection of pair of lines. By solving (1) and (2) using cross - multiplication method, we get  $\frac{x}{5-3} = \frac{y}{3+10} = \frac{1}{2+1}$  $\frac{x}{2} = \frac{y}{13} = \frac{1}{3}$ x = 2/3 and y = 13/3 $\therefore$  The point of intersection is (2/3, 13/3) (ii) bx + ay = ab and ax + by = abGiven: The equations of the lines are as follows: bx + ay - ab = 0...(1) $ax + by = ab \Rightarrow ax + by - ab = 0 \dots (2)$ Let us find the point of intersection of pair of lines. By solving (1) and (2) using cross - multiplication method, we get  $\frac{x}{-a^2b + ab^2} = \frac{y}{-a^2b + ab^2} = \frac{1}{b^2 - a^2}$  $\frac{x}{ab(b-a)} = \frac{y}{ab(b-a)} = \frac{1}{(a+b)(b-a)}$  $x = \frac{ab}{a+b}$  and  $y = \frac{ab}{a+b}$  $\therefore$  The point of intersection is (ab/a+b, ab/a+b)

2. Find the coordinates of the vertices of a triangle, the equations of whose sides are: (i) x + y - 4 = 0, 2x - y + 3 = 0 and x - 3y + 2 = 0(ii)  $y(t_1 + t_2) = 2x + 2at_1t_2$ ,  $y(t_2 + t_3) = 2x + 2at_2t_3$  and,  $y(t_3 + t_1) = 2x + 2at_1t_3$ . Solution:



(i) x + y - 4 = 0, 2x - y + 3 = 0 and x - 3y + 2 = 0Given: x + y - 4 = 0, 2x - y + 3 = 0 and x - 3y + 2 = 0Let us find the point of intersection of pair of lines.  $x + y - 4 = 0 \dots (1)$   $2x - y + 3 = 0 \dots (2)$   $x - 3y + 2 = 0 \dots (3)$ By solving (1) and (2) using cross - multiplication method, we get

$$\frac{x}{3-4} = \frac{y}{-8-3} = \frac{1}{-1-2}$$
  
x = 1/3, y = 11/3

Solving (1) and (3) using cross - multiplication method, we get

 $\frac{x}{2-12} = \frac{y}{-4-2} = \frac{1}{-3-1}$ x = 5/2, y = 3/2

Similarly, solving (2) and (3) using cross - multiplication method, we get

 $\frac{x}{-2+9} = \frac{y}{3-4} = \frac{1}{-6+1}$ x = -7/5, y = 1/5

: The coordinates of the vertices of the triangle are (1/3, 11/3), (5/2, 3/2) and (-7/5, 1/5)

(ii)  $y(t_1 + t_2) = 2x + 2at_1t_2$ ,  $y(t_2 + t_3) = 2x + 2at_2t_3$  and,  $y(t_3 + t_1) = 2x + 2at_1t_3$ . Given:  $y(t_1 + t_2) = 2x + 2at_1t_2$ ,  $y(t_2 + t_3) = 2x + 2at_2t_3$  and  $y(t_3 + t_1) = 2x + 2at_1t_3$ Let us find the point of intersection of pair of lines.  $2x - y(t_1 + t_2) + 2at_1t_2 = 0 \dots (1)$   $2x - y(t_2 + t_3) + 2at_2t_3 = 0 \dots (2)$   $2x - y(t_3 + t_1) + 2at_1t_3 = 0 \dots (3)$ By solving (1) and (2) using cross - multiplication method, we get



$$\begin{aligned} \frac{x}{-(t_1 + t_2) \times 2at_2t_3 + (t_2 + t_3)2at_1t_2} &= \frac{-y}{4at_2t_3 - 4at_1t_2} \\ &= \frac{1}{-2(t_2 + t_3) + 2(t_1 + t_2)} \\ x &= \frac{-(t_1 + t_2) \times 2at_2t_3 + (t_2 + t_3)2at_1t_2}{-2(t_2 + t_3) + 2(t_1 + t_2)} &= at_2^2 \\ y &= -\frac{4at_2t_3 - 4at_1t_2}{-2(t_2 + t_3) + 2(t_1 + t_2)} &= 2at_2 \end{aligned}$$

Solving (1) and (3) using cross - multiplication method, we get

$$\frac{x}{-(t_1 + t_2) \times 2at_1t_3 + (t_3 + t_1)2at_1t_2} = \frac{-y}{4at_1t_3 - 4at_1t_2}$$
$$= \frac{1}{-2(t_3 + t_1) + 2(t_1 + t_2)}$$
$$x = \frac{-(t_1 + t_2) \times 2at_1t_3 + (t_3 + t_1)2at_1t_2}{-2(t_3 + t_1) + 2(t_1 + t_2)} = at_1^2$$
$$y = -\frac{4at_1t_3 - 4at_1t_2}{-2(t_3 + t_1) + 2(t_1 + t_2)} = 2at_1$$

Solving (2) and (3) using cross - multiplication method, we get  

$$\frac{x}{-(t_2 + t_3) \times 2at_1t_3 + (t_3 + t_1)2at_2t_3} = \frac{-y}{4at_1t_3 - 4at_2t_3}$$

$$= \frac{1}{-2(t_3 + t_1) + 2(t_2 + t_3)}$$

$$x = \frac{-(t_2 + t_3) \times 2at_1t_3 + (t_3 + t_1)2at_2t_3}{-2(t_3 + t_1) + 2(t_2 + t_3)} = at_3^2$$

$$y = -\frac{4at_1t_3 - 4at_2t_3}{-2(t_3 + t_1) + 2(t_2 + t_3)} = 2at_3$$

: The coordinates of the vertices of the triangle are  $(at_1^2, 2at_1)$ ,  $(at_2^2, 2at_2)$  and  $(at_3^2, 2at_3)$ .

3. Find the area of the triangle formed by the lines  $y = m_1 x + c_1$ ,  $y = m_2 x + c_2$  and x = 0Solution: Given:

 $y = m_1 x + c_1 \dots (1)$  $y = m_2 x + c_2 \dots (2)$ 



 $x = 0 \dots (3)$ 

In triangle ABC, let equations (1), (2) and (3) represent the sides AB, BC and CA, respectively.

Solving (1) and (2), we get  $x = \frac{c_2 - c_1}{m_1 - m_2}, y = \frac{m_1 c_2 - m_2 c_1}{m_1 - m_2}$ Thus, AB and BC intersect at B  $\left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1 c_2 - m_2 c_1}{m_1 - m_2}\right)$ Solving (1) and (3):  $x = 0, y = c_1$ Thus, AB and CA intersect at A 0,c\_1. Similarly, solving (2) and (3):  $x = 0, y = c_2$ Thus, BC and CA intersect at C 0,c\_2.  $\therefore$  Area of triangle ABC  $= \frac{1}{2} \begin{vmatrix} 0 & c_1 & 1 \\ 0 & c_2 & 1 \\ \frac{c_2 - c_1}{m_1 - m_2} & \frac{m_1 c_2 - m_2 c_1}{m_1 - m_2} & 1 \end{vmatrix}$   $= \frac{1}{2} \left( \frac{c_2 - c_1}{m_1 - m_2} \right) (c_1 - c_2)$  $= \frac{\frac{1}{2} (c_1 - c_2)^2}{m_2 - m_1}$ 

4. Find the equations of the medians of a triangle, the equations of whose sides are: 3x + 2y + 6 = 0, 2x - 5y + 4 = 0 and x - 3y - 6 = 0Solution:

Given:

 $3x + 2y + 6 = 0 \dots (1)$   $2x - 5y + 4 = 0 \dots (2)$   $x - 3y - 6 = 0 \dots (3)$ Let us assume, in triangle ABC, let equations (1), (2) and (3) represent the sides AB, BC and CA, respectively. Solving equations (1) and (2), we get x = -2, y = 0

Thus, AB and BC intersect at B (-2, 0).



Now, solving (1) and (3), we get x = - 6/11, y = - 24/11 Thus, AB and CA intersect at A (-6/11, -24/11)

Similarly, solving (2) and (3), we get x = -42, y = -16Thus, BC and CA intersect at C (-42, -16).

Now, let D, E and F be the midpoints the sides BC, CA and AB, respectively. Then, we have:

$$D = \left(\frac{-2-42}{2}, \frac{0-16}{2}\right) = (-22, -8)$$

$$E = \left(\frac{-\frac{6}{11}-42}{2}, \frac{-\frac{24}{11}-16}{2}\right) = \left(-\frac{234}{11}, -\frac{100}{11}\right)$$

$$F = \left(\frac{-\frac{6}{11}-2}{2}, \frac{-\frac{24}{11}+0}{2}\right) = \left(-\frac{14}{11}, -\frac{12}{11}\right)$$
Now, the equation of the median AD is
$$y + \frac{24}{11} = \frac{-8 + \frac{24}{11}}{-22 + \frac{6}{11}} \left(x + \frac{6}{11}\right)$$

Now, the equation of the median AD is

$$y + \frac{24}{11} = \frac{-8 + \frac{24}{11}}{-22 + \frac{6}{11}} \left( x + \frac{6}{11} \right)$$
  
16x-59y-120=0

The equation of median CF is

y + 16 = 
$$\frac{-\frac{12}{11} + 16}{-\frac{14}{11} + 42}$$
(x + 42)  
41 x - 112 y - 70 = 0

And, the equation of the median BE is

$$y - 0 = \frac{-\frac{100}{11} - 0}{-\frac{234}{11} + 2}(x + 2)$$
  
25x - 53y + 50 = 0

: The equations of the medians of a triangle are: 41x - 112y - 70 = 0,



16x - 59y - 120 = 0, 25x - 53y + 50 = 0

## 5. Prove that the lines $y = \sqrt{3x + 1}$ , y = 4 and $y = -\sqrt{3x + 2}$ form an equilateral triangle.

**Solution:** 

Given:  $y = \sqrt{3x + 1}$ .....(1) y = 4.....(2)  $y = -\sqrt{3x + 2}$ .....(3) Let us assume in triangle ABC, let equations (1), (2) and (3) represent the sides AB, BC and CA, respectively.

By solving equations (1) and (2), we get  $x = \sqrt{3}$ , y = 4

Thus, AB and BC intersect at  $B(\sqrt{3},4)$ 

Now, solving equations (1) and (3), we get  $x = 1/2\sqrt{3}$ , y = 3/2Thus, AB and CA intersect at A (1/2 $\sqrt{3}$ , 3/2)

Similarly, solving equations (2) and (3), we get  $x = -2/\sqrt{3}$ , y = 4Thus, BC and AC intersect at C (-2/ $\sqrt{3}$ ,4)

Now, we have:

$$AB = \sqrt{\left(\frac{1}{2\sqrt{3}} - \sqrt{3}\right)^2 + \left(\frac{3}{2} - 4\right)^2} = \frac{5}{\sqrt{3}}$$
$$BC = \sqrt{\left(\frac{1}{2\sqrt{3}} + \frac{2}{\sqrt{3}}\right)^2 + \left(\frac{3}{2} - 4\right)^2} = \frac{5}{\sqrt{3}}$$
$$AC = \sqrt{\left(\frac{1}{2\sqrt{3}} + \frac{2}{\sqrt{3}}\right)^2 + \left(\frac{3}{2} - 4\right)^2} = \frac{5}{\sqrt{3}}$$

Hence proved, the given lines form an equilateral triangle.



### EXERCISE 23.11

### PAGE NO: 23.83

**1.** Prove that the following sets of three lines are concurrent: (i) 15x - 18y + 1 = 0, 12x + 10y - 3 = 0 and 6x + 66y - 11 = 0(ii) 3x - 5y - 11 = 0, 5x + 3y - 7 = 0 and x + 2y = 0Solution: (i) 15x - 18y + 1 = 0, 12x + 10y - 3 = 0 and 6x + 66y - 11 = 0Given:  $15x - 18y + 1 = 0 \dots$  (i) 12x + 10y - 3 = 0 ..... (ii) 6x + 66y - 11 = 0 ..... (iii) Now, consider the following determinant: -18115 = 15(-110+198) + 18(-132+18) + 1(792-60)12 10 6 66 -11=> 1320 - 2052 + 732 = 0Hence proved, the given lines are concurrent. (ii) 3x - 5y - 11 = 0, 5x + 3y - 7 = 0 and x + 2y = 0Given:  $3x - 5y - 11 = 0 \dots$  (i) 5x + 3y - 7 = 0 ..... (ii) x + 2y = 0 ..... (iii) Now, consider the following determinant: 3 - 5 -7 = 3 × 14 + 5 × 7 - 11 × 7 = 0 5 3 1 2

Hence, the given lines are concurrent.

### 2. For what value of $\lambda$ are the three lines 2x - 5y + 3 = 0, $5x - 9y + \lambda = 0$ and x - 2y + 1 = 0 concurrent?

#### Solution:

Given:

 $2x - 5y + 3 = 0 \dots (1)$   $5x - 9y + \lambda = 0 \dots (2)$  $x - 2y + 1 = 0 \dots (3)$ 

It is given that the three lines are concurrent. Now, consider the following determinant:



$$\begin{array}{c|c} 2 & -5 & 3 \\ 5 & -9 & \lambda \\ 1 & -2 & 1 \end{array} = 0 \\ 2(-9+2\lambda) + 5(5-\lambda) + 3(-10+9) = 0 \\ -18+4\lambda + 25 - 5\lambda - 3 = 0 \\ \lambda = 4 \\ \therefore \text{ The value of } \lambda \text{ is } 4. \end{array}$$

3. Find the conditions that the straight lines  $y = m_1x + c_1$ ,  $y = m_2x + c_2$  and  $y = m_3x + c_3$  may meet in a point.

#### Solution:

Given:

 $m_1 x - y + c_1 = 0 \dots (1)$   $m_2 x - y + c_2 = 0 \dots (2)$  $m_3 x - y + c_3 = 0 \dots (3)$ 

It is given that the three lines are concurrent. Now, consider the following determinant:

 $\begin{array}{c|cccc} \vdots & \begin{vmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ m_3 & -1 & c_3 \end{vmatrix} = 0$ 

 $m_1(-c_3 + c_2) + 1(m_2c_3 - m_3c_2) + c_1(-m_2 + m_3) = 0$ 

 $m_1(c_2-c_3) + m_2(c_3-c_1) + m_3(c_1-c_2) = 0$ 

: The required condition is  $m_1(c_2-c_3) + m_2(c_3-c_1) + m_3(c_1-c_2) = 0$ 

4. If the lines  $p_1x + q_1y = 1$ ,  $p_2x + q_2y = 1$  and  $p_3x + q_3y = 1$  be concurrent, show that the points  $(p_1, q_1)$ ,  $(p_2, q_2)$  and  $(p_3, q_3)$  are collinear. Solution:

Given:

p<sub>1</sub>x + q<sub>1</sub>y = 1 p<sub>2</sub>x + q<sub>2</sub>y = 1 p<sub>3</sub>x + q<sub>3</sub>y = 1 The given lines can be written as follows: p<sub>1</sub> x + q<sub>1</sub> y - 1 = 0 ... (1) p<sub>2</sub> x + q<sub>2</sub> y - 1 = 0 ... (2) p<sub>3</sub> x + q<sub>3</sub> y - 1 = 0 ... (3) It is given that the three lines are concurrent.

Now, consider the following determinant:



 $\begin{vmatrix} p_1 & q_1 & -1 \\ p_2 & q_2 & -1 \\ p_3 & q_3 & -1 \end{vmatrix} = 0$  $lp_3 q_3$  $-\begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_2 & q_2 & 1 \end{vmatrix} = 0$ lp<sub>3</sub> q<sub>3</sub>  $\begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \end{vmatrix} = 0$  $lp_3$ **q**<sub>3</sub>

Hence proved, the given three points,  $(p_1, q_1)$ ,  $(p_2, q_2)$  and  $(p_3, q_3)$  are collinear.

### 5. Show that the straight lines $L_1 = (b + c)x + ay + 1 = 0$ , $L_2 = (c + a)x + by + 1 = 0$ ming APF and $L_3 = (a + b)x + cy + 1 = 0$ are concurrent. Solution:

Given:

 $L_1 = (b + c)x + ay + 1 = 0$  $L_2 = (c + a)x + by + 1 = 0$  $L_3 = (a + b)x + cy + 1 = 0$ The given lines can be written as follows:  $(b + c) x + ay + 1 = 0 \dots (1)$  $(c + a) x + by + 1 = 0 \dots (2)$  $(a + b) x + cy + 1 = 0 \dots (3)$ Consider the following determinant.

|b+ca1| c + a b 1 la + b c 1

Let us apply the transformation  $C_1 \rightarrow C_1 + C_2$ , we get

```
\begin{vmatrix} b + c & a & 1 \\ c + a & b & 1 \\ a + b & c & 1 \end{vmatrix} = \begin{vmatrix} a + b + c & a & 1 \\ c + a + b & b & 1 \\ a + b + c & c & 1 \end{vmatrix}
\begin{vmatrix} b + c & a & 1 \\ c + a & b & 1 \\ a + b & c & 1 \end{vmatrix} = (a + b + c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}
|b+ca1|
 c + a b 1 = 0
```

Hence proved, the given lines are concurrent.



### EXERCISE 23.12

### P&GE NO: 23.92

**1.** Find the equation of a line passing through the point (2, 3) and parallel to the line 3x - 4y + 5 = 0. Solution: Given: The equation is parallel to 3x - 4y + 5 = 0 and pass through (2, 3) The equation of the line parallel to 3x - 4y + 5 = 0 is  $3x - 4y + \lambda = 0,$ Where,  $\lambda$  is a constant. It passes through (2, 3). Substitute the values in above equation, we get  $3(2) - 4(3) + \lambda = 0$  $6 - 12 + \lambda = 0$  $\lambda = 6$ Now, substitute the value of  $\lambda = 6$  in  $3x - 4y + \lambda = 0$ , we get 3x - 4y + 6 $\therefore$  The required line is 3x - 4y + 6 = 0. 2. Find the equation of a line passing through (3, -2) and perpendicular to the line x

### 2. Find the equation of a line passing through (3, -2) and perpendicular to the line x - 3y + 5 = 0.

Solution: Given: The equation is perpendicular to x - 3y + 5 = 0 and passes through (3,-2) The equation of the line perpendicular to x - 3y + 5 = 0 is  $3x + y + \lambda = 0$ , Where,  $\lambda$  is a constant. It passes through (3, -2). Substitute the values in above equation, we get  $3 (3) + (-2) + \lambda = 0$   $9 - 2 + \lambda = 0$   $\lambda = -7$ Now, substitute the value of  $\lambda = -7$  in  $3x + y + \lambda = 0$ , we get 3x + y - 7 = 0 $\therefore$  The required line is 3x + y - 7 = 0.

## **3.** Find the equation of the perpendicular bisector of the line joining the points (1, 3) and (3, 1).

Solution:



Given:

A (1, 3) and B (3, 1) be the points joining the perpendicular bisector Let C be the midpoint of AB. So, coordinates of C = [(1+3)/2, (3+1)/2] = (2, 2)Slope of AB = [(1-3) / (3-1)] = -1Slope of the perpendicular bisector of AB = 1 Thus, the equation of the perpendicular bisector of AB is given as, y - 2 = 1(x - 2) y = x x - y = 0 $\therefore$  The required equation is y = x.

### 4. Find the equations of the altitudes of a $\triangle$ ABC whose vertices are A (1, 4), B (-3, 2) and C (-5, -3).

### Solution:

Given:

The vertices of  $\triangle$ ABC are A (1, 4), B (-3, 2) and C (-5, -3). Now let us find the slopes of  $\triangle$ ABC.



Slope of AB = 
$$[(2 - 4) / (-3 - 1)]$$
  
=  $\frac{1}{2}$ 

Slope of BC = 
$$[(-3 - 2) / (-5+3)]$$
  
=  $5/2$ 

Slope of CA = [(4 + 3) / (1 + 5)]= 7/6 Thus, we have:



Chapter

Slope of CF = -2Slope of AD = -2/5Slope of BE = -6/7Hence, Equation of CF is: y + 3 = -2(x + 5)y + 3 = -2x - 102x + y + 13 = 0

Equation of AD is: y - 4 = (-2/5) (x - 1) 5y - 20 = -2x + 22x + 5y - 22 = 0

Equation of BE is: y - 2 = (-6/7) (x + 3) 7y - 14 = -6x - 18 6x + 7y + 4 = 0 $\therefore$  The required equations are 2x + y + 13 = 0, 2x + 5y - 22 = 0, 6x + 7y + 4 = 0.

# 5. Find the equation of a line which is perpendicular to the line $\sqrt{3x - y + 5} = 0$ and which cuts off an intercept of 4 units with the negative direction of y-axis. Solution:

Given:

The equation is perpendicular to  $\sqrt{3x} - y + 5 = 0$  equation and cuts off an intercept of 4 units with the negative direction of y-axis.

The line perpendicular to  $\sqrt{3x - y} + 5 = 0$  is  $x + \sqrt{3y} + \lambda = 0$ 

It is given that the line  $x + \sqrt{3}y + \lambda = 0$  cuts off an intercept of 4 units with the negative direction of the y-axis.

This means that the line passes through (0,-4). So,

Let us substitute the values in the equation  $x + \sqrt{3}y + \lambda = 0$ , we get  $0 - \sqrt{3}(4) + \lambda = 0$ 

$$0 - \sqrt{3} (4) + \lambda$$

 $\lambda = 4\sqrt{3}$ 

Now, substitute the value of  $\lambda$  back, we get

 $\mathbf{x} + \sqrt{3}\mathbf{y} + 4\sqrt{3} = 0$ 

: The required equation of line is  $x + \sqrt{3}y + 4\sqrt{3} = 0$ .

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RD Sharma Solutions for Class 11 Maths Chapter 23 – The Straight Lines



### EXERCISE 23.13

### PAGE NO: 23.99

**1.** Find the angles between each of the following pairs of straight lines: (i) 3x + y + 12 = 0 and x + 2y - 1 = 0(ii) 3x - y + 5 = 0 and x - 3y + 1 = 0Solution: (i) 3x + y + 12 = 0 and x + 2y - 1 = 0Given: The equations of the lines are  $3x + y + 12 = 0 \dots (1)$  $x + 2y - 1 = 0 \dots (2)$ Let  $m_1$  and  $m_2$  be the slopes of these lines.  $m_1 = -3, m_2 = -1/2$ Let  $\theta$  be the angle between the lines. Then, by using the formula  $\tan \theta = \left[ (m_1 - m_2) / (1 + m_1 m_2) \right]$  $= \left[ \left( -3 + 1/2 \right) / \left( 1 + 3/2 \right) \right]$ = 1So,  $\theta = \pi/4 \text{ or } 45^{\circ}$  $\therefore$  The acute angle between the lines is 45° (ii) 3x - y + 5 = 0 and x - 3y + 1 = 0Given: The equations of the lines are  $3x - y + 5 = 0 \dots (1)$  $x - 3y + 1 = 0 \dots (2)$ Let  $m_1$  and  $m_2$  be the slopes of these lines.  $m_1 = 3, m_2 = 1/3$ Let  $\theta$  be the angle between the lines. Then, by using the formula  $\tan \theta = \left[ (m_1 - m_2) / (1 + m_1 m_2) \right]$ = [(3 - 1/3) / (1 + 3(1/3))]= [((9 - 1)/3)/(1 + 1)]= 8/6= 4/3So.  $\theta = \tan^{-1}(4/3)$  $\therefore$  The acute angle between the lines is tan<sup>-1</sup> (4/3).



### 2. Find the acute angle between the lines 2x - y + 3 = 0 and x + y + 2 = 0. Solution:

Given: The equations of the lines are  $2x - y + 3 = 0 \dots (1)$   $x + y + 2 = 0 \dots (2)$ Let  $m_1$  and  $m_2$  be the slopes of these lines.  $m_1 = 2, m_2 = -1$ Let  $\theta$  be the angle between the lines. Then, by using the formula  $\tan \theta = [(m_1 - m_2) / (1 + m_1 m_2)]$  = [(2 - (-1) / (1 + (2)(-1))]] = [3/(1 - 2)] = 3So,  $\theta = \tan^{-1}(3)$ 

### $\therefore$ The acute angle between the lines is $\tan^{-1}(3)$ .

# 3. Prove that the points (2, -1), (0, 2), (2, 3) and (4, 0) are the coordinates of the vertices of a parallelogram and find the angle between its diagonals. Solution:

To prove:

The points (2, -1), (0, 2), (2, 3) and (4, 0) are the coordinates of the vertices of a parallelogram

Let us assume the points, A (2, -1), B (0, 2), C (2, 3) and D (4, 0) be the vertices. Now, let us find the slopes

Slope of AB = [(2+1) / (0-2)]= -3/2Slope of BC = [(3-2) / (2-0)]=  $\frac{1}{2}$ Slope of CD = [(0-3) / (4-2)]= -3/2

Slope of DA = 
$$[(-1-0) / (2-4)]$$
  
=  $\frac{1}{2}$ 

Thus, AB is parallel to CD and BC is parallel to DA.

Hence proved, the given points are the vertices of a parallelogram.



Now, let us find the angle between the diagonals AC and BD. Let  $m_1$  and  $m_2$  be the slopes of AC and BD, respectively.  $m_1 = [(3+1) / (2-2)]$  $= \infty$ 

 $m_2 = [(0-2) / (4-0)] = -1/2$ 

Thus, the diagonal AC is parallel to the y-axis.  $\angle ODB = \tan^{-1} (1/2)$ 

In triangle MND,  $\angle DMN = \pi/2 - \tan^{-1} (1/2)$  $\therefore$  The angle between the diagonals is  $\pi/2 - \tan^{-1} (1/2)$ .

## 4. Find the angle between the line joining the points (2, 0), (0, 3) and the line x + y = 1.

### Solution:

Given: Points (2, 0), (0, 3) and the line x + y = 1. Let us assume A (2, 0), B (0, 3) be the given points. Now, let us find the slopes Slope of AB = m<sub>1</sub> = [(3-0) / (0-2)] = -3/2

Slope of the line x + y = 1 is -1  $\therefore m_2 = -1$ 

Let  $\theta$  be the angle between the line joining the points (2, 0), (0, 3) and the line  $x + y = \tan \theta = \left| \left[ (m_1 - m_2) / (1 + m_1 m_2) \right] \right|_{x = 1}^{x = 1} - \left[ (3/2 + 1) / (1 + 3/2) \right]$ 

$$= [(-3/2 + 1) / (1 + 3/2)]$$
  
= 1/5  
tor<sup>-1</sup> (1/5)

 $\theta = \tan^{-1} (1/5)$ 

: The acute angle between the line joining the points (2, 0), (0, 3) and the line x + y = 1 is  $\tan^{-1} (1/5)$ .



#### 5. If $\theta$ is the angle which the straight line joining the points $(x_1, y_1)$ and $(x_2, y_2)$

subtends at the origin, prove that 
$$\tan \theta = \frac{x_2 y_1 - x_1 y_2}{x_1 x_2 + y_1 y_2}$$
 and  $\cos \theta = \frac{x_1 y_2 + y_1 y_2}{\sqrt{x_1^2 + y_1^2} \sqrt{x_2^2 + y_2^2}}$ 

### Solution:

We need to prove:

$$\tan \theta = \frac{x_2 y_1 - x_1 y_2}{x_1 x_2 + y_1 y_2} \text{ and } \cos \theta = \frac{x_1 x_2 + y_1 y_2}{\sqrt{x_1^2 + y_1^2} \sqrt{x_2^2 + y_2^2}}.$$



Let us assume A  $(x_1, y_1)$  and B  $(x_2, y_2)$  be the given points and O be the origin. Slope of OA =  $m_1 = y_{1x1}$ Slope of OB =  $m_2 = y_{2x2}$ 

It is given that  $\theta$  is the angle between lines OA and OB.



 $\begin{aligned} \tan\theta &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\ \text{Now, substitute the values, we get} \\ &= \frac{\frac{y_1}{x_1} - \frac{y_2}{x_2}}{1 + \frac{y_1}{x_1} \times \frac{y_2}{x_2}} \\ \tan\theta &= \frac{x_2 y_1 - x_1 y_2}{x_1 x_2 + y_1 y_2} \\ \text{Now,} \\ \text{As we know that } \cos\theta &= \sqrt{\frac{1}{1 + \tan^2 \theta}} \\ \text{Now, substitute the values, we get} \\ \cos\theta &= \frac{x_1 x_2 + y_1 y_2}{\sqrt{(x_2 y_1 - x_1 y_2)^2 + (x_1 x_2 + y_1 y_2)^2}} \\ \cos\theta &= \frac{x_1 x_2 + y_1 y_2}{\sqrt{x_1^2 y_1^2 + x_1^2 y_2^2 + x_1^2 x_2^2 + y_1^2 y_2^2}} \\ \cos\theta &= \frac{x_1 x_2 + y_1 y_2}{\sqrt{x_1^2 + y_1^2 \sqrt{x_2^2 + y_2^2}}}. \end{aligned}$ Hence proved.





### EXERCISE 23.14

### PAGE NO: 23.102

1. Find the values of  $\alpha$  so that the point P( $\alpha^2$ ,  $\alpha$ ) lies inside or on the triangle formed by the lines x - 5y + 6 = 0, x - 3y + 2 = 0 and x - 2y - 3 = 0. Solution:

Given:

x - 5y + 6 = 0, x - 3y + 2 = 0 and x - 2y - 3 = 0 forming a triangle and point P( $\alpha^2$ ,  $\alpha$ ) lies inside or on the triangle

Let ABC be the triangle of sides AB, BC and CA whose equations are x - 5y + 6 = 0, x - 3y + 2 = 0 and x - 2y - 3 = 0, respectively.

On solving the equations, we get A (9, 3), B (4, 2) and C (13, 5) as the coordinates of the vertices.



It is given that point P ( $\alpha^2$ ,  $\alpha$ ) lies either inside or on the triangle. The three conditions are given below.

(i) A and P must lie on the same side of BC.

(ii) B and P must lie on the same side of AC.

(iii) C and P must lie on the same side of AB.

If A and P lie on the same side of BC, then  $(9-9+2)(\alpha^2 - 3\alpha + 2) \ge 0$   $(\alpha - 2)(\alpha - 1) \ge 0$  $\alpha \in (-\infty, 1] \cup [2, \infty) \dots (1)$ 

If B and P lie on the same side of AC, then  $(4-4-3)(\alpha^2-2\alpha-3) \ge 0$   $(\alpha-3)(\alpha+1) \le 0$  $\alpha \in [-1, 3] \dots (2)$ 



If C and P lie on the same side of AB, then  $(13-25+6)(\alpha^2-5\alpha+6) \ge 0$   $(\alpha-3)(\alpha-2) \le 0$  $\alpha \in [2,3] \dots (3)$ 

From equations (1), (2) and (3), we get  $\alpha \in [2, 3]$  $\therefore \alpha \in [2, 3]$ 

2. Find the values of the parameter a so that the point (a, 2) is an interior point of the triangle formed by the lines x + y - 4 = 0, 3x - 7y - 8 = 0 and 4x - y - 31 = 0. Solutions:

Given:

x + y - 4 = 0, 3x - 7y - 8 = 0 and 4x - y - 31 = 0 forming a triangle and point (a, 2) is an interior point of the triangle

Let ABC be the triangle of sides AB, BC and CA whose equations are x + y - 4 = 0, 3x - 7y - 8 = 0 and 4x - y - 31 = 0, respectively.

On solving them, we get A (7, -3), B (18/5, 2/5) and C (209/25, 61/25) as the coordinates of the vertices.

Let P (a, 2) be the given point.



It is given that point P (a, 2) lies inside the triangle. So, we have the following:

(i) A and P must lie on the same side of BC.

(ii) B and P must lie on the same side of AC.

(iii) C and P must lie on the same side of AB.

Thus, if A and P lie on the same side of BC, then 21 + 21 - 8 - 3a - 14 - 8 > 0



 $a > 22/3 \dots (1)$ 

If B and P lie on the same side of AC, then  $4 \times \frac{18}{5} - \frac{2}{5} - 31 - 4a - 2 - 31 > 0$  $a < 33/4 \dots (2)$ 

If C and P lie on the same side of AB, then  $\frac{209}{25} + \frac{61}{25} - 4 - a + 2 - 4 > 0$   $\frac{34}{5} - 4 - a + 2 - 4 > 0$   $a > 2 \dots (3)$ From (1), (2) and (3), we get:  $A \in (22/3, 33/4)$  $\therefore A \in (22/3, 33/4)$ 

3. Determine whether the point (-3, 2) lies inside or outside the triangle whose sides are given by the equations x + y - 4 = 0, 3x - 7y + 8 = 0, 4x - y - 31 = 0. Solution:

Given:

x + y - 4 = 0, 3x - 7y + 8 = 0, 4x - y - 31 = 0 forming a triangle and point (-3, 2) Let ABC be the triangle of sides AB, BC and CA, whose equations x + y - 4 = 0, 3x - 7y + 8 = 0 and 4x - y - 31 = 0, respectively.

On solving them, we get A (7, -3), B (2, 2) and C (9, 5) as the coordinates of the vertices. Let P (-3, 2) be the given point.



The given point P (-3, 2) will lie inside the triangle ABC, if (i) A and P lies on the same side of BC



(ii) B and P lies on the same side of AC(iii) C and P lies on the same side of AB

Thus, if A and P lie on the same side of BC, then 21 + 21 + 8 - 9 - 14 + 8 > 0  $50 \times -15 > 0$  -750 > 0, This is false  $\therefore$  The point (-3, 2) lies outside triangle ABC.





### EXERCISE 23.15

### PAGE NO: 23.107

1. Find the distance of the point (4, 5) from the straight line 3x - 5y + 7 = 0. Solution:

Given: The line: 3x - 5y + 7 = 0Comparing ax + by + c = 0 and 3x - 5y + 7 = 0, we get: a = 3, b = -5 and c = 7So, the distance of the point (4, 5) from the straight line 3x - 5y + 7 = 0 is

$$d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

$$= \left| \frac{3 \times 4 - 5 \times 5 + 7}{\sqrt{3^2 + (-5^2)}} \right|$$
$$= \frac{6}{\sqrt{34}}$$

 $\therefore$  The required distance is  $6/\sqrt{34}$ 

## 2. Find the perpendicular distance of the line joining the points $(\cos \theta, \sin \theta)$ and $(\cos \phi, \sin \phi)$ from the origin.

**Solution:** Given:

The points  $(\cos \theta, \sin \theta)$  and  $(\cos \phi, \sin \phi)$  from the origin.

The equation of the line joining the points ( $\cos \theta$ ,  $\sin \theta$ ) and ( $\cos \phi$ ,  $\sin \phi$ ) is given below:

 $y - \sin\theta = \frac{\sin\phi - \sin\theta}{\cos\phi - \cos\theta}(x - \cos\theta)$  $(\cos\phi - \cos\theta)y - \sin\theta(\cos\phi - \cos\theta) = (\sin\phi - \sin\theta)x - (\sin\phi - \sin\theta)\cos\theta$ 

 $(\sin\varphi - \sin\theta)x - (\cos\varphi - \cos\theta)y + \sin\theta\cos\varphi - \sin\varphi\cos\theta = 0$ 

Let d be the perpendicular distance from the origin to the line  $(\sin\phi - \sin\theta)x - (\cos\phi - \cos\theta)y + \sin\theta\cos\phi - \sin\phi\cos\theta = 0$ 

$$d = \left| \frac{\sin\theta - \phi}{\sqrt{(\sin\phi - \sin\theta)^2 + (\cos\phi - \cos\theta)^2}} \right|$$
$$= \left| \frac{\sin\theta - \phi}{\sqrt{\sin^2\phi + \sin^2\theta - 2\sin\phi\sin\theta + \cos^2\phi + \cos^2\theta - 2\cos\phi\cos\theta}} \right|$$



$$= \left| \frac{\sin\theta - \phi}{\sqrt{\sin^2 \phi + \cos^2 \phi + \sin^2 \theta + \cos^2 \phi + \cos^2 \theta - 2\cos(\theta - \phi)}} \right|$$
$$= \left| \frac{\frac{1}{\sqrt{2}} (\sin(\theta - \phi))}{\sqrt{1 - \cos(\theta - \phi)}} \right|$$
$$= \frac{1}{\sqrt{2}} \left| \frac{\sin(\theta - \phi)}{\sqrt{2\sin^2(\frac{\theta - \phi}{2})}} \right|$$
$$= \frac{1}{2} \left| \frac{2\sin(\frac{\theta - \phi}{2})\cos(\frac{\theta - \phi}{2})}{\sin(\frac{\theta - \phi}{2})} \right|$$
$$= \cos\left(\frac{\theta - \phi}{2}\right)$$

 $\therefore$  The required distance is  $\cos\left(\frac{\theta-\varphi}{2}\right)$ 

# 3. Find the length of the perpendicular from the origin to the straight line joining the two points whose coordinates are (a $\cos \alpha$ , a $\sin \alpha$ ) and (a $\cos \beta$ , a $\sin \beta$ ). Solution:

Given:

Coordinates are  $(a \cos \alpha, a \sin \alpha)$  and  $(a \cos \beta, a \sin \beta)$ . Equation of the line passing through  $(a \cos \alpha, a \sin \alpha)$  and  $(a \cos \beta, a \sin \beta)$  is

$$y - a\sin\alpha = \frac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha}(x - a\cos\alpha)$$

$$y - a\sin\alpha = \frac{\sin\beta - \sin\alpha}{\cos\beta - \cos\alpha}(x - a\cos\alpha)$$

$$y - a\sin\alpha = \frac{2\cos\left(\frac{\beta + \alpha}{2}\right)\sin\left(\frac{\beta - \alpha}{2}\right)}{2\sin\left(\frac{\beta + \alpha}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)}(x - a\cos\alpha)$$

$$y - a\sin\alpha = -\cot\left(\frac{\beta + \alpha}{2}\right)(x - a\cos\alpha)$$

$$y - a\sin\alpha = -\cot\left(\frac{\alpha + \beta}{2}\right)(x - a\cos\alpha)$$

$$x\cot\left(\frac{\alpha + \beta}{2}\right) + y - a\sin\alpha - a\cos\alpha\cot\left(\frac{\alpha + \beta}{2}\right) = 0$$
The distance of the line from the origin is



$$d = \left| \frac{-\operatorname{asin}\alpha - \operatorname{acos}\alpha \operatorname{cot}\left(\frac{\alpha + \beta}{2}\right)}{\sqrt{\operatorname{cot}^2\left(\frac{(\alpha + \beta)}{2}\right) + 1}} \right|$$
  
$$d = \left| \frac{-\operatorname{asin}\alpha - \operatorname{acos}\alpha \operatorname{cot}\left(\frac{\alpha + \beta}{2}\right)}{\sqrt{\operatorname{cosec}^2\left(\frac{(\alpha + \beta)}{2}\right)}} \right| \because \operatorname{cosec}^2\theta = 1 + \operatorname{cot}^2\theta$$
  
$$= a \left| \sin\left(\frac{\alpha + \beta}{2}\right) \sin\alpha + \cos\alpha \cos\left(\frac{\alpha + \beta}{2}\right) \right|$$
  
$$= a \left| \sin\alpha \sin\left(\frac{\alpha + \beta}{2}\right) + \cos\alpha \cos\left(\frac{\alpha + \beta}{2}\right) \right|$$
  
$$= a \left| \cos\left(\frac{\alpha + \beta}{2} - \alpha\right) \right| = a \cos\left(\frac{\beta - \alpha}{2}\right)$$
  
$$\therefore \text{ The required distance is } a \cos\left(\frac{\beta - \alpha}{2}\right)$$

4. Show that the perpendicular let fall from any point on the straight line 2x + 11y - 15 = 0 upon the two straight lines 24x + 7y = 20 and 4x - 3y - 2 = 0 are equal to each other.

### **Solution:**

Given:

The lines 24x + 7y = 20 and 4x - 3y - 2 = 0Let us assume, P(a, b) be any point on 2x + 11y - 5 = 0So, 111 0

$$2a + 11b - 5 = 0$$

$$b = \frac{5-2a}{11} \dots \dots \dots (1)$$

Let  $d_1$  and  $d_2$  be the perpendicular distances from point P on the lines 24x + 7y = 20 and 4x - 3y - 2 = 0, respectively.

$$d_{1} = \left| \frac{24a + 7b - 20}{\sqrt{24^{2} + 7^{2}}} \right| = \left| \frac{24a + 7b - 20}{25} \right|$$
$$= \left| \frac{24a + 7 \times \frac{5 - 2a}{11} - 20}{25} \right|$$

From(1)

L



$$d_1 = \left| \frac{50a - 37}{55} \right|$$
  
Similarly,

$$d_{2} = \left| \frac{4a - 3b - 2}{\sqrt{3^{2} + (-4)^{2}}} \right| = \left| \frac{4a - 3 \times \frac{5 - 2a}{11} - 2}{5} \right|$$
$$= \left| \frac{44a - 15 + 6a - 22}{11 \times 5} \right|$$
From (1)

 $d_2 = \left| \frac{50a - 37}{55} \right|$  $\therefore d_1 = d_2$ Hence proved.

5. Find the distance of the point of intersection of the lines 2x + 3y = 21 and 3x - 4y + 11 = 0 from the line 8x + 6y + 5 = 0.

### Solution:

Given:

The lines 2x + 3y = 21 and 3x - 4y + 11 = 0Solving the lines 2x + 3y = 21 and 3x - 4y + 11 = 0 we get:

 $\frac{x}{33 - 84} = \frac{y}{-63 - 22} = \frac{1}{-8 - 9}$ x = 3, y = 5

So, the point of intersection of 2x + 3y = 21 and 3x - 4y + 11 = 0 is (3, 5).

Now, the perpendicular distance d of the line 8x + 6y + 5 = 0 from the point (3, 5) is

$$d = \left| \frac{24 + 30 + 5}{\sqrt{8^2 + 6^2}} \right| = \frac{59}{10}$$

 $\therefore$  The distance is 59/10.



### EXERCISE 23.16

### PAGE NO: 23.114

1. Determine the distance between the following pair of parallel lines: (i) 4x - 3y - 9 = 0 and 4x - 3y - 24 = 0(ii) 8x + 15y - 34 = 0 and 8x + 15y + 31 = 0Solution: (i) 4x - 3y - 9 = 0 and 4x - 3y - 24 = 0Given: The parallel lines are  $4x - 3y - 9 = 0 \dots (1)$   $4x - 3y - 24 = 0 \dots (2)$ Let d be the distance between the given lines. So,  $d = \left| \frac{-9 + 24}{\sqrt{4^2 + (-3)^2}} \right| = \frac{15}{5} = 3$  units  $\therefore$  The distance between givens parallel line is 3units. (ii) 8x + 15y - 34 = 0 and 8x + 15y + 31 = 0Given:

The parallel lines are

 $8x + 15y - 34 = 0 \dots (1)$ 

 $8x + 15y + 31 = 0 \dots (2)$ 

Let d be the distance between the given lines.

So, d =  $\left| \frac{-34 - 31}{\sqrt{8^2 + 15^2}} \right| = \frac{65}{17}$  units

 $\therefore$  The distance between givens parallel line is 65/17 units.

### 2. The equations of two sides of a square are 5x - 12y - 65 = 0 and 5x - 12y + 26 = 0. Find the area of the square.

### Solution:

Given:

Two side of square are 5x - 12y - 65 = 0 and 5x - 12y + 26 = 0

The sides of a square are

 $5x - 12y - 65 = 0 \dots (1)$ 

 $5x - 12y + 26 = 0 \dots (2)$ 

We observe that lines (1) and (2) are parallel.

So, the distance between them will give the length of the side of the square.



Let d be the distance between the given lines.

d = 
$$\left| \frac{-65 - 26}{\sqrt{5^2 + (-12)^2}} \right| = \frac{91}{13} = 7$$

: Area of the square  $= 7^2 = 49$  square units

# 3. Find the equation of two straight lines which are parallel to x + 7y + 2 = 0 and at unit distance from the point (1, -1). Solution:

Given:

The equation is parallel to x + 7y + 2 = 0 and at unit distance from the point (1, -1) The equation of given line is

 $x + 7y + 2 = 0 \dots (1)$ 

The equation of a line parallel to line x + 7y + 2 = 0 is given below:

 $x + 7y + \lambda = 0 \dots (2)$ 

The line  $x + 7y + \lambda = 0$  is at a unit distance from the point (1, -1). So.

 $1 = \left| \frac{1-7+\lambda}{\sqrt{1+49}} \right|$ 

 $\lambda - 6 = \pm 5\sqrt{2}$ 

 $\lambda = 6 + 5\sqrt{2}, 6 - 5\sqrt{2}$ 

now, substitute the value of  $\lambda$  back in equation  $x + 7y + \lambda = 0$ , we get  $x + 7y + 6 + 5\sqrt{2} = 0$  and  $x + 7y + 6 - 5\sqrt{2}$ 

: The required lines:

 $x + 7y + 6 + 5\sqrt{2} = 0$  and  $x + 7y + 6 - 5\sqrt{2}$ 

## 4. Prove that the lines 2x + 3y = 19 and 2x + 3y + 7 = 0 are equidistant from the line 2x + 3y = 6.

Solution:

Given:

The lines A, 2x + 3y = 19 and B, 2x + 3y + 7 = 0 also a line C, 2x + 3y = 6. Let d<sub>1</sub> be the distance between lines 2x + 3y = 19 and 2x + 3y = 6, While d<sub>2</sub> is the distance between lines 2x + 3y + 7 = 0 and 2x + 3y = 6

$$d_{1} = \left| \frac{-19 - (-6)}{\sqrt{2^{2} + 3^{2}}} \right| \text{ and } d_{2} = \left| \frac{7 - (-6)}{\sqrt{2^{2} + 3^{2}}} \right|$$
$$d_{1} = \left| -\frac{13}{\sqrt{13}} \right| = \sqrt{13} \text{ and } d_{2} = \left| \frac{13}{\sqrt{13}} \right| = \sqrt{13}$$

Hence proved, the lines 2x + 3y = 19 and 2x + 3y + 7 = 0 are equidistant from the line 2x + 3y = 6



### 5. Find the equation of the line mid-way between the parallel lines 9x + 6y - 7 = 0and 3x + 2y + 6 = 0.

### Solution:

Given: 9x + 6y - 7 = 0 and 3x + 2y + 6 = 0 are parallel lines The given equations of the lines can be written as:  $3x + 2y - 7/3 = 0 \dots (1)$   $3x + 2y + 6 = 0 \dots (2)$ Let the equation of the line midway between the parallel lines (1) and (2) be

 $3x + 2y + \lambda = 0 \dots (3)$ 

The distance between (1) and (3) and the distance between (2) and (3) are equal.

$$\begin{vmatrix} -\frac{7}{3} - \lambda \\ \sqrt{3^2 + 2^2} \end{vmatrix} = \begin{vmatrix} 6 - \lambda \\ \sqrt{3^2 + 2^2} \end{vmatrix}$$
$$\begin{vmatrix} -\lambda + \frac{7}{3} \end{vmatrix} = |6 - \lambda|$$
$$6 - \lambda = \lambda + \frac{7}{3}$$
$$\lambda = \frac{11}{6}$$

Now substitute the value of  $\lambda$  back in equation  $3x + 2y + \lambda = 0$ , we get 3x + 2y + 11/6 = 0By taking LCM 18x + 12y + 11 = 0

: The required equation of line is 18x + 12y + 11 = 0



### EXERCISE 23.17

### PAGE NO: 23.117

1. Prove that the area of the parallelogram formed by the lines  $a_1x + b_1y + c_1 = 0$ ,  $a_1x + b_1y + d_1 = 0$ ,  $a_2x + b_2y + c_2 = 0$ ,  $a_2x + b_2y + d_2 = 0$ 

$$\frac{(d_1 - c_1)(d_2 - c_2)}{a_1 b_2 - a_2 b_1}$$

is  $| a_1 b_2 a_2 b_1 |$  sq. units. Deduce the condition for these lines to form a rhombus.

**Solution:** Given:

The given lines are

 $a_{1}x + b_{1}y + c_{1} = 0 \dots (1)$   $a_{1}x + b_{1}y + d_{1} = 0 \dots (2)$   $a_{2}x + b_{2}y + c_{2} = 0 \dots (3)$  $a_{2}x + b_{2}y + d_{2} = 0 \dots (4)$ 

Let us prove, the area of the parallelogram formed by the lines  $a_1x + b_1y + c_1 = 0$ ,  $a_1x + |(d_1 - c_1)(d_2 - c_2)|$ 

 $b_1y + d_1 = 0$ ,  $a_2x + b_2y + c_2 = 0$ ,  $a_2x + b_2y + d_2 = 0$  is  $\left|\frac{(d_1 - c_1)(d_2 - c_2)}{(a_1b_2 - a_2b_1)}\right|$  sq. units.

The area of the parallelogram formed by the lines  $a_1x + b_1y + c_1 = 0$ ,  $a_1x + b_1y + d_1 = 0$ ,  $a_2x + b_2y + c_2 = 0$  and  $a_2x + b_2y + d_2 = 0$  is given below:

Area =  $\begin{vmatrix} (c_1 - d_1)(c_2 - d_2) \\ | a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$ 

Since,  $\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$ 

$$\therefore \text{Area} = \left| \frac{(c_1 - d_1)(c_2 - d_2)}{(a_1 b_2 - a_2 b_1)} \right| = \left| \frac{(d_1 - c_1)(d_2 - c_2)}{(a_1 b_2 - a_2 b_1)} \right|$$

If the given parallelogram is a rhombus, then the distance between the pair of parallel lines is equal.

$$\frac{c_1 - d_1}{\sqrt{a_1^2 + b_1^2}} = \frac{c_2 - d_2}{\sqrt{a_2^2 + b_2^2}}$$

Hence proved.



2. Prove that the area of the parallelogram formed by the lines 3x - 4y + a = 0, 3x -4y + 3a = 0, 4x - 3y - a = 0 and 4x - 3y - 2a = 0 is  $2a^{2}/7$  sq. units. Solution:

Given: The given lines are  $3x - 4y + a = 0 \dots (1)$  $3x - 4y + 3a = 0 \dots (2)$  $4x - 3y - a = 0 \dots (3)$  $4x - 3y - 2a = 0 \dots (4)$ Let us prove, the area of the parallelogram formed by the lines 3x - 4y + a = 0, 3x -3a = 0, 4x - 3y - a = 0 and 4x - 3y - 2a = 0 is  $2a^2/7$  sq. units. From above solution, we know that

Area of the parallelogram =  $\left|\frac{(c_1 - d_1)(c_2 - d_2)}{(a_1b_2 - a_2b_1)}\right|$ 

Area of the parallelogram = 
$$\left|\frac{(c_1 - d_1)(c_2 - d_2)}{(a_1b_2 - a_2b_1)}\right|$$
  
Area of the parallelogram =  $\left|\frac{(a - 3a)(2a - a)}{(-9 + 16)}\right| = \frac{2a^2}{7}$  square units

Hence proved.

3. Show that the diagonals of the parallelogram whose sides are lx + my + n = 0, lx +my + n' = 0, mx + ly + n = 0 and mx + ly + n' = 0 include an angle  $\pi/2$ . Solution:

Given:

The given lines are  $lx + my + n = 0 \dots (1)$  $mx + ly + n' = 0 \dots (2)$  $lx + my + n' = 0 \dots (3)$  $mx + ly + n = 0 \dots (4)$ 

Let us prove, the diagonals of the parallelogram whose sides are lx + my + n = 0, lx + my+n'=0, mx + ly + n = 0 and mx + ly + n' = 0 include an angle  $\pi/2$ .

By solving (1) and (2), we get  $\mathbf{B} = \left(\frac{\mathbf{mn'} - \mathbf{ln}}{\mathbf{l^2} - \mathbf{m^2}}, \frac{\mathbf{mn} - \mathbf{ln'}}{\mathbf{l^2} - \mathbf{m^2}}\right)$ 

Solving (2) and (3), we get,

$$C = \left( -\frac{n'}{m+l'} - \frac{n'}{m+l} \right)$$

Solving (3) and (4), we get,



$$D = \left(\frac{mn - ln'}{l^2 - m^2}, \frac{mn' - ln}{l^2 - m^2}\right)$$

Solving (1) and (4), we get,

$$A = \left(\frac{-n}{m+1}, \frac{-n}{m+1}\right)$$

Let  $m_1$  and  $m_2$  be the slope of AC and BD. Now,

$$m_{1} = \frac{\frac{-n'}{m+1} + \frac{n}{m+1}}{\frac{-n'}{m+1} + \frac{n}{m+1}} = 1$$

$$m_{2} = \frac{\frac{mn' - \ln}{l^{2} - m^{2}} - \frac{mn - \ln'}{l^{2} - m^{2}}}{\frac{mn - \ln'}{l^{2} - m^{2}} - \frac{mn' - \ln}{l^{2} - m^{2}}} = -1$$

$$\therefore m_{1}m_{2} = -1$$
Hence proved.



### EXERCISE 23.18

PAGE NO: 23.124

### 1. Find the equation of the straight lines passing through the origin and making an angle of $45^{\circ}$ with the straight line $\sqrt{3x + y} = 11$ . Solution:

Given:

Equation passes through (0, 0) and make an angle of  $45^{\circ}$  with the line  $\sqrt{3x + y} = 11$ . We know that, the equations of two lines passing through a point  $x_1, y_1$  and making an angle  $\alpha$  with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan \alpha}{1 \mp m \tan \alpha} (x - x_1)$$

Here,

 $x_1 = 0, y_1 = 0, \alpha = 45^\circ$  and  $m = -\sqrt{3}$ So the equations of the required lines are

$$y - 0 = \frac{-\sqrt{3} + \tan 45^{\circ}}{1 + \sqrt{3} \tan 45^{\circ}} (x - 0) \text{ and } y - 0$$
  
$$= \frac{-\sqrt{3} - \tan 45^{\circ}}{1 - \sqrt{3} \tan 45^{\circ}} (x - 0)$$
  
$$= \frac{-\sqrt{3} + 1}{1 + \sqrt{3}} x \text{ and } y = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} x$$
  
$$= -\frac{3 + 1 - 2\sqrt{3}}{3 - 1} x \text{ and } y = \frac{3 + 1 + 2\sqrt{3}}{3 - 1} x$$
  
$$= (\sqrt{3} - 2)x \text{ and } y = (\sqrt{3} + 2)x$$

: The equation of given line is  $y = (\sqrt{3} - 2)x$  and  $y = (\sqrt{3} + 2)x$ 

# 2. Find the equations to the straight lines which pass through the origin and are inclined at an angle of 75° to the straight line $x + y + \sqrt{3}(y - x) = a$ . Solution:

Given:

The equation passes through (0,0) and make an angle of 75° with the line  $x + y + \sqrt{3}(y - x) = a$ .

We know that the equations of two lines passing through a point  $x_1, y_1$  and making an angle  $\alpha$  with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan \alpha}{1 \mp m \tan \alpha} (x - x_1)$$



Here, equation of the given line is,  $x + y + \sqrt{3}(y - x) = a$  $(\sqrt{3} + 1)y = (\sqrt{3} - 1)x + a$  $y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}x + \frac{a}{\sqrt{3} + 1}$ Comparing this equation with y = mx + cWe get,  $m = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$  $\therefore x_1 = 0, y_1 = 0, \alpha = 75^{\circ}$ .  $m = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$  and  $\tan 75^\circ = 2 + \sqrt{3}$ So, the equations of the required lines are  $y - 0 = \frac{2 - \sqrt{3} + \tan 75^{\circ}}{1 - (2 - \sqrt{3})\tan 75^{\circ}}(x - 0) \text{ and } y - 0$  $= \frac{2 - \sqrt{3} - \tan 75^{\circ}}{1 + (2 - \sqrt{3})\tan 75^{\circ}} (x - 0)$  $y = \frac{2 - \sqrt{3} + 2 + \sqrt{3}}{1 - (2 - \sqrt{3})(2 + \sqrt{3})}x$  and  $y = \frac{2 - \sqrt{3} - 2 - \sqrt{3}}{1 + (2 - \sqrt{3})(2 + \sqrt{3})}x$  $y = \frac{4}{1 - 1}x$  and  $y = -\sqrt{3}x$ x = 0 and  $\sqrt{3}x + y = 0$ : The equation of given line is x = 0 and  $\sqrt{3}x + y = 0$ 

### 3. Find the equations of straight lines passing through (2, -1) and making an angle of $45^{\circ}$ with the line 6x + 5y - 8 = 0. Solution:

Given:

The equation passes through (2,-1) and make an angle of  $45^{\circ}$  with the line 6x + 5y - 8 = 0We know that the equations of two lines passing through a point  $x_1$ ,  $y_1$  and making an angle  $\alpha$  with the given line y = mx + c are



 $y - y_{1} = \frac{m \pm \tan \alpha}{1 \mp \max \alpha} (x - x_{1})$ Here, equation of the given line is, 6x + 5y - 8 = 05y = -6x + 8y = -6x/5 + 8/5Comparing this equation with y = mx + cWe get, m = -6/5Where,  $x_{1} = 2$ ,  $y_{1} = -1$ ,  $\alpha = 45^{\circ}$ , m = -6/5So, the equations of the required lines are  $y + 1 = \frac{\left(-\frac{6}{5} + \tan 45^{\circ}\right)}{\left(1 + \frac{6}{5} \tan 45^{\circ}\right)} (x - 2)$  and  $y + 1 = \frac{\left(-\frac{6}{5} - \tan 45^{\circ}\right)}{\left(1 - \frac{6}{5} \tan 45^{\circ}\right)} (x - 2)$  $y + 1 = \frac{\left(-\frac{6}{5} + 1\right)}{\left(1 + \frac{6}{5}\right)} (x - 2)$  and  $y + 1 = \frac{\left(-\frac{6}{5} - 1\right)}{\left(1 - \frac{6}{5}\right)} (x - 2)$  $y + 1 = -\frac{1}{11} (x - 2)$  and  $y + 1 = -\frac{11}{-11} (x - 2)$ x + 11y + 9 = 0 and 11x - y - 23 = 0 $\therefore$  The equation of given line is x + 11y + 9 = 0 and 11x - y - 23 = 0

### 4. Find the equations to the straight lines which pass through the point (h, k) and are inclined at angle $\tan^{-1} m$ to the straight line y = mx + c. Solution:

Given:

The equation passes through (h, k) and make an angle of  $\tan^{-1}$  m with the line y = mx + cWe know that the equations of two lines passing through a point  $x_1$ ,  $y_1$  and making an angle  $\alpha$  with the given line y = mx + c are

m' = m So,

$$y - y_1 = \frac{m \pm \tan \alpha}{1 \mp m \tan \alpha} (x - x_1)$$

Here,

 $x_1 = h$ ,  $y_1 = k$ ,  $\alpha = \tan^{-1} m$ , m' = m. So, the equations of the required lines are



$$y - k = \frac{m + m}{1 - m^2}(x - h) \text{ and } y - k = \frac{m - m}{1 + m^2}(x - h)$$
$$y - k = \frac{2m}{1 - m^2}(x - h) \text{ and } y - k = 0$$
$$(y - k)(1 - m^2) = 2m(x - h) \text{ and } y = k$$

: The equation of given line is  $(y - k)(1 - m^2) = 2m(x - h)$  and y = k.

#### 5. Find the equations to the straight lines passing through the point (2, 3) and inclined at an angle of $45^{\circ}$ to the lines 3x + y - 5 = 0. Solution:

Given:

The equation passes through (2, 3) and make an angle of  $45^{\circ}$  with the line 3x + y - 5 = 0. We know that the equations of two lines passing through a point  $x_1, y_1$  and making an angle  $\alpha$  with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan \alpha}{1 \mp m \tan \alpha} (x - x_1)$$

Here.

Equation of the given line is,

3x + y - 5 = 0

y = -3x + 5

Comparing this equation with y = mx + c we get, m = -3 $x_1 = 2, y_1 = 3, \alpha = 45$ °, m = -3.

So, the equations of the required lines are  $y - 3 = \frac{-3 + \tan 45^\circ}{1 + 3\tan 45^\circ} (x - 2)$  and  $y - 3 = \frac{-3 - \tan 45^\circ}{1 - 3\tan 45^\circ} (x - 2)$  $y - 3 = \frac{-3 + 1}{1 + 3}(x - 2)$  and  $y - 3 = \frac{-3 - 1}{1 - 3}(x - 2)$  $y - 3 = \frac{-1}{2}(x - 2)$  and y - 3 = 2(x - 2)x + 2y - 8 = 0 and 2x - y - 1 = 0: The equation of given line is x + 2y - 8 = 0 and 2x - y - 1 = 0



### EXERCISE 23.19

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### 1. Find the equation of a straight line through the point of intersection of the lines 4x - 3y = 0 and 2x - 5y + 3 = 0 and parallel to 4x + 5y + 6 = 0.

#### Solution:

Given:

Lines 4x - 3y = 0 and 2x - 5y + 3 = 0 and parallel to 4x + 5y + 6 = 0

The equation of the straight line passing through the points of intersection of 4x - 3y = 0and 2x - 5y + 3 = 0 is given below:

$$4x - 3y + \lambda (2x - 5y + 3) = 0$$
  
(4 + 2\lambda)x + (-3 - 5\lambda)y + 3\lambda = 0  
$$y = \binom{4 + 2\lambda}{3} + \frac{3\lambda}{3}$$

$$y = \left(\frac{1}{3+5\lambda}\right)x + \frac{1}{(3+5\lambda)}$$

The required line is parallel to 4x + 5y + 6 = 0 or, y = -4x/5 - 6/5

$$\frac{4+2\lambda}{3+5\lambda} = -\frac{4}{5}$$

$$\frac{4+2\lambda}{3+5\lambda} = -\frac{4}{5}$$

 $\lambda = -16/15$ 

$$\therefore$$
 The required equation is

$$\left(4 - \frac{32}{15}\right)x - \left(3 - \frac{80}{15}\right)y - \frac{48}{15} = 0$$
  
28x + 35y - 48 = 0

2. Find the equation of a straight line passing through the point of intersection of x + 2y + 3 = 0 and 3x + 4y + 7 = 0 and perpendicular to the straight line x - y + 9 = 0. Solution:

Given:

x + 2y + 3 = 0 and 3x + 4y + 7 = 0

The equation of the straight line passing through the points of intersection of x + 2y + 3 = 0 and 3x + 4y + 7 = 0 is

$$x + 2y + 3 + \lambda(3x + 4y + 7) = 0$$
  
(1 + 3\lambda)x + (2 + 4\lambda)y + 3 + 7\lambda = 0  
$$y = -\left(\frac{1+3\lambda}{2+4\lambda}\right)x - \left(\frac{3+7\lambda}{2+4\lambda}\right)$$

The required line is perpendicular to x - y + 9 = 0 or, y = x + 9


## 3. Find the equation of the line passing through the point of intersection of 2x - 7y + 11 = 0 and x + 3y - 8 = 0 and is parallel to (i) x = axis (ii) y-axis. Solution:

Given:

The equations, 2x - 7y + 11 = 0 and x + 3y - 8 = 0The equation of the straight line passing through the points of intersection of 2x - 7y + 11 = 0 and x + 3y - 8 = 0 is given below:

 $2x - 7y + 11 + \lambda(x + 3y - 8) = 0$ (2 + \lambda)x + (-7 + 3\lambda)y + 11 - 8\lambda = 0

(i) The required line is parallel to the x-axis. So, the coefficient of x should be zero.  $2 + \lambda = 0$   $\lambda = -2$ Now, substitute the value of  $\lambda$  back in equation, we get 0 + (-7 - 6)y + 11 + 16 = 0 13y - 27 = 0 $\therefore$  The equation of the required line is 13y - 27 = 0

(ii) The required line is parallel to the y-axis. So, the coefficient of y should be zero.  $-7 + 3\lambda = 0$   $\lambda = 7/3$ Now, substitute the value of  $\lambda$  back in equation, we get (2 + 7/3)x + 0 + 11 - 8(7/3) = 0 13x - 23 = 0 $\therefore$  The equation of the required line is 13x - 23 = 0

## 4. Find the equation of the straight line passing through the point of intersection of 2x + 3y + 1 = 0 and 3x - 5y - 5 = 0 and equally inclined to the axes. Solution:

Given:

The equations, 2x + 3y + 1 = 0 and 3x - 5y - 5 = 0

The equation of the straight line passing through the points of intersection of 2x + 3y + 1 = 0 and 3x - 5y - 5 = 0 is 2x + 3y + 1 + 3(3x - 5y - 5) = 0

$$2x + 3y + 1 + \lambda(3x - 5y - 5) = 0$$
  
(2 + 3\lambda)x + (3 - 5\lambda)y + 1 - 5\lambda = 0

 $y = -[(2 + 3\lambda) / (3 - 5\lambda)] - [(1 - 5\lambda) / (3 - 5\lambda)]$ 

The required line is equally inclined to the axes. So, the slope of the required line is either 1 or -1.



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So,

-  $[(2 + 3\lambda) / (3 - 5\lambda)] = 1$  and -  $[(2 + 3\lambda) / (3 - 5\lambda)] = -1$ -2 -  $3\lambda = 3 - 5\lambda$  and  $2 + 3\lambda = 3 - 5\lambda$  $\lambda = 5/2$  and 1/8

Now, substitute the values of  $\lambda$  in  $(2 + 3\lambda)x + (3 - 5\lambda)y + 1 - 5\lambda = 0$ , we get the equations of the required lines as:

(2 + 15/2)x + (3 - 25/2)y + 1 - 25/2 = 0 and (2 + 3/8)x + (3 - 5/8)y + 1 - 5/8 = 019x - 19y - 23 = 0 and 19x + 19y + 3 = 0  $\therefore$  The required equation is 19x - 19y - 23 = 0 and 19x + 19y + 3 = 0

5. Find the equation of the straight line drawn through the point of intersection of the lines x + y = 4 and 2x - 3y = 1 and perpendicular to the line cutting off intercepts 5, 6 on the axes.

Solution:

Given:

The lines x + y = 4 and 2x - 3y = 1

The equation of the straight line passing through the point of intersection of x + y = 4 and 2x - 3y = 1 is

 $x + y - 4 + \lambda(2x - 3y - 1) = 0$ 

 $(1+2\lambda)x + (1-3\lambda)y - 4 - \lambda = 0 \dots (1)$ 

 $y = - [(1 + 2\lambda) / (1 - 3\lambda)]x + [(4 + \lambda) / (1 - 3\lambda)]$ 

The equation of the line with intercepts 5 and 6 on the axis is

 $x/5 + y/6 = 1 \dots (2)$ 

So, the slope of this line is -6/5

The lines (1) and (2) are perpendicular.

 $\therefore -6/5 \times [(-1+2\lambda) / (1 - 3\lambda)] = -1$   $\lambda = 11/3$ Now, substitute the values of  $\lambda$  in (1), we get the equation of the required line. (1 + 2(11/3))x + (1 - 3(11/3))y - 4 - 11/3 = 0 (1 + 22/3)x + (1 - 11)y - 4 - 11/3 = 0 25x - 30y - 23 = 0 $\therefore$  The required equation is 25x - 30y - 23 = 0