EXERCISE 19.11

PAGE NO: 19.69

Evaluate the following integrals:

1. $\int \tan^3 x \sec^2 x \, dx$

Solution:

Let $I = \int \tan^3 x \sec^2 x \, dx$

Let tan x = t, then

$\Rightarrow \sec^2 x \, dx = dt$

Substituting the values of x

$\Rightarrow I = \int t^3 dt$

On integrating we get

$\Rightarrow I = \frac{t^4}{4} + c$

Substituting the value of t we get

$\Rightarrow I = \frac{\tan^4 x}{4} + c$

Therefore, $\int \tan^3 x \sec^2 x \, dx = \frac{\tan^4 x}{4} + c$

2. $\int \tan x \sec^4 x \, dx$

Solution:

Let $I = \int \tan x \sec^4 x \, dx$

The above equation can be written as

$\Rightarrow I = \int \tan x \sec^2 x \sec^2 x \, dx$

$\Rightarrow I = \int \tan x (1 + \tan^2 x) \sec^2 x \, dx$

$\Rightarrow I = \int (\tan x + \tan^3 x) \sec^2 x \, dx$

Let tan x = t, then

$\Rightarrow \sec^2 x \, dx = dt$

Substituting the values of x

$\Rightarrow I = \int (t + t^3) dt$

On integrating we get

$\Rightarrow I = \frac{t^2}{2} + \frac{t^4}{4} + c$

$\Rightarrow I = \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + c$

Therefore, $\int \tan x \sec^4 x \, dx = \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + c$

3. $\int \tan^5 x \sec^4 x \, dx$

Solution:

Let $I = \int \tan^5 x \sec^4 x \, dx$

The above equation can be written as

$\Rightarrow I = \int \tan^5 x \sec^2 x \sec^2 x \, dx$

Taking $\tan^5 x$ as common

$$\Rightarrow I = \int \tan^5 x\,(1 + \tan^2 x)\sec^2 x\,dx$$

On simplifying

$$\Rightarrow I = \int (\tan^5 x + \tan^7 x)\sec^2 x\,dx$$

Let tan x = t, then

$\Rightarrow \sec^2 x\,dx = dt$

Substituting the value of x

$$\Rightarrow I = \int (t^5 + t^7)dt$$

Integrating we get

$$\Rightarrow I = \frac{t^6}{6} + \frac{t^8}{8} + c$$

Substituting the values of t

$$\Rightarrow I = \frac{\tan^6 x}{6} + \frac{\tan^8 x}{8} + c$$

Therefore, $\int \tan^5 x \sec^4 x\,dx = \frac{\tan^6 x}{6} + \frac{\tan^8 x}{8} + c$

4. $\int \sec^6 x \tan x\,dx$

Solution:

Let $I = \int \sec^6 x \tan x\,dx$

The above equation can be written as

$$\Rightarrow I = \int \sec^5 x\,(\sec x \tan x)dx$$

Substituting, sec x = t ⇒ sec x tan x dx = dt

$$\Rightarrow I = \int t^5 dt$$

On integrating we get

$$\Rightarrow I = \frac{t^6}{6} + c$$

Now substituting the values of t we get

$$\Rightarrow I = \frac{\sec^6 x}{6} + c$$

Therefore, $\int \sec^6 x (\sec x \tan x) dx = \frac{\sec^6 x}{6} + c$

5. $\int \tan^5 x \, dx$

Solution:

Let $I = \int \tan^5 x \, dx$

The above equation can be written as

$$\Rightarrow I = \int \tan^2 x \tan^3 x \, dx$$

Using standard formula

$$\Rightarrow I = \int (\sec^2 x - 1) \tan^3 x \, dx$$

Splitting the above equation we get

$$\Rightarrow I = \int \tan^3 x \sec^2 x \, dx - \int \tan^3 x \, dx$$

$$\Rightarrow I = \int \tan^3 x \sec^2 x \, dx - \int (\sec^2 x - 1) \tan x \, dx$$

$$\Rightarrow I = \int \tan^3 x \sec^2 x \, dx - \int (\sec^2 x \tan x) dx + \int \tan x \, dx$$

Let tan x = t, then

$\Rightarrow \sec^2 x\, dx = dt$

$$\Rightarrow I = \int t^3 dt - \int t dt + \int \tan x\, dx$$

$$\Rightarrow I = \frac{t^4}{4} - \frac{t^2}{2} + \log|\sec x| + c$$

$$\Rightarrow I = \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log|\sec x| + c$$

Therefore, $\int \tan^5 x\, dx = \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log|\sec x| + c$

6. $\int \sqrt{\tan x}\, \sec^4 x\, dx$

Solution:

Let $I = \int \sqrt{\tan x}\, \sec^4 x\, dx$

The above equation can be written as

$$\Rightarrow I = \int \sqrt{\tan x}\, \sec^2 x \sec^2 x\, dx$$

Taking common

$$\Rightarrow I = \int \sqrt{\tan x}\, (1 + \tan^2 x) \sec^2 x\, dx$$

$$\Rightarrow I = \int (\tan^{\frac{1}{2}} x + \tan^{\frac{5}{2}} x) \sec^2 x\, dx$$

Let tan x = t, then

$\Rightarrow \sec^2 x\, dx = dt$

$$\Rightarrow I = \int \left(t^{\frac{1}{2}} + t^{\frac{5}{2}}\right) dt$$

On integrating we get

$$\Rightarrow I = \frac{2}{3}t^{\frac{3}{2}} + \frac{2}{7}t^{\frac{7}{2}} + c$$

Substituting the value of t

$$\Rightarrow I = \frac{2}{3}\tan^{\frac{3}{2}}x + \frac{2}{7}\tan^{\frac{7}{2}}x + c$$

Therefore, $\int \sqrt{\tan x}\sec^4 x\,dx = \frac{2}{3}\tan^{\frac{3}{2}}x + \frac{2}{7}\tan^{\frac{7}{2}}x + c$