

EXERCISE 19.14

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Evaluate the following integrals:

$$1. \int \frac{1}{a^2 - b^2x^2} dx$$

Solution:

Taking out b^2 as common from the given equation, we get

$$\begin{aligned} & \frac{1}{b^2} \int \frac{1}{\left(\frac{a^2}{b^2}\right) - x^2} dx \\ &= \frac{1}{b^2} \int \frac{1}{\left(\frac{a}{b}\right)^2 - x^2} dx = \frac{1}{b^2} \int \frac{1}{\left(\frac{a}{b}\right)^2 - x^2} dx \end{aligned}$$

On integrating above equation using

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{x+a}{a-x} \right| + c, \text{ we get}$$

$$= \frac{1}{b^2} \times \frac{1}{2\left(\frac{a}{b}\right)} \log \left| \frac{\frac{a}{b} + x}{\frac{a}{b} - x} \right| + c$$

On simplification we get

$$= \frac{1}{2ab} \log \left| \frac{a + bx}{a - bx} \right| + c$$

$$2. \int \frac{1}{a^2x^2 - b^2} dx$$

Solution:

Taking out a^2 as common from the given equation, we get

$$= \frac{1}{a^2} \int \frac{1}{x^2 - \frac{b^2}{a^2}} dx$$

On integrating above equation using

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c$$

$$= \frac{1}{a^2} \int \frac{1}{x^2 - \left(\frac{b}{a}\right)^2} dx = \frac{1}{a^2} * \frac{1}{2\left(\frac{b}{a}\right)} \log \left| \frac{x - \left(\frac{b}{a}\right)}{x + \frac{b}{a}} \right| + c$$

On simplification

$$= \frac{1}{2ab} \log \frac{ax-b}{ax+b} + c$$

$$3. \int \frac{1}{a^2x^2 + b^2} dx$$

Solution:

Taking out a^2 as common from the given equation, we get

$$= \frac{1}{a^2} \int \frac{1}{x^2 + \frac{b^2}{a^2}} dx$$

On integrating above equation using

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c, \text{ we get}$$

$$= \frac{1}{a^2} \int \frac{1}{x^2 + \left(\frac{b}{a}\right)^2} dx = \frac{1}{a^2} * \frac{1}{\left(\frac{b}{a}\right)} \tan^{-1} \left[\frac{x}{\frac{b}{a}} \right] + c$$

By simplifying we get

$$= \frac{1}{ab} \tan^{-1} \left(\frac{ax}{b} \right) + c$$

$$4. \int \frac{x^2 - 1}{x^2 + 4} dx$$

Solution:

Add and subtract 4 in the numerator of given equation, we get

$$= \int \frac{x^2 + 4 - 4 - 1}{x^2 + 4} = \int \frac{(x^2 + 4) - 4 - 1}{x^2 + 4} dx$$

Now separate the numerator terms, we get

$$= \int \frac{(x^2 + 4) - 5}{x^2 + 4} dx = \int \frac{(x^2 + 4)}{x^2 + 4} dx - \int \frac{5}{x^2 + 4} dx$$

On computing we get

$$= \int dx - \int \frac{5}{x^2 + 4} dx = \int dx - 5 \int \frac{1}{x^2 + 4} dx$$

We know $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$

$$= \int dx - 5 \int \frac{1}{x^2 + 2^2} dx = x - 5 \times \frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) + c$$

On integrating we get

$$= x - \frac{5}{2} \tan^{-1} \left(\frac{x}{2} \right) + c$$

5. $\int \frac{1}{\sqrt{1 + 4x^2}} dx$

Solution:

Let $I = \int \frac{1}{\sqrt{1 + 4x^2}} dx$

The above equation can be written as

$$= \int \frac{1}{\sqrt{1 + (2x)^2}} dx$$

Let $t = 2x$, then $dt = 2dx$ or $dx = dt/2$

Therefore,

$$\int \frac{1}{\sqrt{1 + (2x)^2}} dx = \frac{1}{2} \int \frac{dt}{\sqrt{1 + t^2}}$$

We know $\int \frac{1}{\sqrt{(a^2 + x^2)}} dx = \log|x + \sqrt{(a^2 + x^2)}| + c$

$$= \frac{1}{2} \log|t + \sqrt{1 + t^2}| + c$$

$$= \frac{1}{2} \log|2x + \sqrt{1 + 4x^2}| + c$$