

EXERCISE 19.15

PAGE NO: 19.86

$$1. \int \frac{1}{4x^2 + 12x + 5} dx$$

Solution:

Let

$$I = \int \frac{1}{4x^2 + 12x + 5} dx$$

Taking out $\frac{1}{4}$ as common, then we get

$$= \frac{1}{4} \int \frac{1}{x^2 + 3x + \frac{5}{4}} dx$$

Adding and subtracting $(\frac{3}{2})^2$ to the denominator

$$= \frac{1}{4} \int \frac{1}{x^2 + 2x \times \frac{3}{2} + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{5}{4}} dx$$

The above equation can be written as

$$= \frac{1}{4} \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - 1} dx$$

Let

$$\left(x + \frac{3}{2}\right) = t \quad \dots (i)$$

$$\Rightarrow dx = dt$$

So, substituting the t values we get

$$I = \frac{1}{4} \int \frac{1}{t^2 - (1)^2} dt$$

$$I = \frac{1}{4} \times \frac{1}{2 \times 1} \log \left| \frac{t-1}{t+1} \right| + c$$

$$[\text{since, } \int \frac{1}{x^2 - (a)^2} dx = \frac{1}{2 \times a} \log \left| \frac{x-a}{x+a} \right| + c]$$

$$I = \frac{1}{8} \log \left| \frac{x+\frac{3}{2}-1}{x+\frac{3}{2}+1} \right| + c \quad [\text{Using (i)}]$$

$$2. \int \frac{1}{x^2 - 10x + 34} dx$$

Solution:

Let

$$I = \int \frac{1}{x^2 - 10x + 34} dx$$

$$I = \int \frac{1}{x^2 - 10x + 34} dx$$

Adding and subtracting 5^2 to both sides

$$= \int \frac{1}{x^2 - 2x \times 5 + (5)^2 - (5)^2 + 34} dx$$

The above equation can be written as

$$= \int \frac{1}{(x-5)^2 + 9} dx$$

$$\text{Let } (x-5) = t \dots (i)$$

$$\Rightarrow dx = dt$$

So, substituting the values of t we get

$$I = \int \frac{1}{t^2 + (3)^2} dt$$

$$I = \frac{1}{3} \tan^{-1} \left(\frac{t}{3} \right) + c$$

$$[\text{since, } \int \frac{1}{x^2 + (a)^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c]$$

$$I = \frac{1}{3} \tan^{-1} \left(\frac{x-5}{3} \right) + c \quad [\text{Using (i)}]$$

$$I = \frac{1}{3} \tan^{-1} \left(\frac{x-5}{3} \right) + c$$

$$3. \int \frac{1}{1+x-x^2} dx$$

Solution:

$$\text{Let } I = \int \frac{1}{1+x-x^2} dx = \int \frac{1}{-(x^2-x-1)} dx$$

The above equation can be written as

$$= \int \frac{1}{-(x^2-x-1)} dx$$

Add and subtract $\frac{1}{4}$ to both sides

$$= \int \frac{1}{-(x^2-x-\frac{1}{4}-1+\frac{1}{4})} dx$$

The above equation can be written as

$$= \int \frac{1}{-\left(\left(x-\frac{1}{2}\right)^2 - \frac{5}{4}\right)} dx$$

On computing we get

$$= \int \frac{1}{\left(\left(\frac{\sqrt{5}}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2\right)} dx$$

$$I = \frac{1}{2 \times \frac{\sqrt{5}}{2}} \log \left| \frac{\frac{\sqrt{5}}{2} + (x-\frac{1}{2})}{\frac{\sqrt{5}}{2} - (x-\frac{1}{2})} \right| + c$$

By using, $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$

$$I = \frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{5} + 2x - 1}{\sqrt{5} - 2x + 1} \right| + c$$

$$I = \frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{5} - 1 + 2x}{\sqrt{5} + 1 - 2x} \right| + c$$

4. $\int \frac{1}{2x^2 - x - 1} dx$

Solution:

Let $I = \int \frac{1}{2x^2 - x - 1} dx$

Taking out $\frac{1}{2}$ as common we get

$$= \frac{1}{2} \int \frac{1}{x^2 - \frac{x}{2} - \frac{1}{2}} dx$$

Again adding and subtracting $(\frac{1}{4})^2$ to the denominator we get

$$= \frac{1}{2} \int \frac{1}{x^2 - 2x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - \frac{1}{2}} dx$$

The above equation can be written as

$$= \frac{1}{2} \int \frac{1}{\left(x - \frac{1}{4}\right)^2 - \frac{9}{16}} dx$$

Let $\left(x - \frac{1}{4}\right) = t$ (i)

$$\Rightarrow dx = dt$$

$$I = \frac{1}{2} \int \frac{1}{t^2 - \left(\frac{3}{4}\right)^2} dt$$

So,

$$I = \frac{1}{2} \times \frac{1}{2 \times \frac{3}{4}} \log \left| \frac{t - \frac{3}{4}}{t + \frac{3}{4}} \right| + c$$

$$[\text{since, } \int \frac{1}{x^2 - (a)^2} dx = \frac{1}{2 \times a} \log \left| \frac{x - a}{x + a} \right| + c]$$

$$I = \frac{1}{3} \log \left| \frac{x - \frac{1}{4}}{x + \frac{1}{4}} \right| + c \quad [\text{Using (i)}]$$

$$I = \frac{1}{3} \log \left| \frac{2x - 2}{2x + 1} \right| + c$$

$$5. \int \frac{1}{x^2 + 6x + 13} dx$$

Solution:

In the denominator we have, and it can be written as

$$x^2 + 6x + 13 = x^2 + 6x + 3^2 - 3^2 + 13$$

The above equation can be written as

$$= (x + 3)^2 + 4$$

Substituting these values we get

$$\text{So, } \int \frac{1}{x^2 + 6x + 13} dx = \int \frac{1}{(x+3)^2 + 2^2} dx$$

Let $x+3 = t$

Then $dx = dt$

$$\int \frac{1}{(t)^2 + 2^2} dt = \frac{1}{2} \tan^{-1} \frac{t}{2} + c$$

$$[\text{since, } \int \frac{1}{x^2 + (a)^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c]$$

$$\frac{1}{2} \tan^{-1} \frac{x+3}{2} + c$$