EXERCISE 19.2

Evaluate the following integrals (1 - 44):

1. $\int (3x\sqrt{x} + 4\sqrt{x} + 5)\, dx$

Solution:

Given

$$\int (3x\sqrt{x} + 4\sqrt{x} + 5)\, dx$$

By Splitting, we get,

$$\Rightarrow \int ((3x\sqrt{x})dx + (4\sqrt{x})dx + 5dx)$$

$$\Rightarrow \int 3x\sqrt{x}dx + \int 4\sqrt{x}dx + \int 5dx$$

$$\Rightarrow \int 3x^{\frac{3}{2}}\, dx + \int 4x^{(\frac{1}{2})}\, dx + \int 5dx$$

By using the formula, $\int x^n\, dx = \frac{x^{n+1}}{n+1}$

$$\Rightarrow \frac{3x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{4x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \int 5dx$$

We know that

$$\int k dx = kx + c$$

$$\Rightarrow \frac{3x^{\frac{5}{2}}}{5/2} + \frac{4x^{\frac{3}{2}}}{3/2} + 5x + c$$

$$\Rightarrow \frac{6}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{3/2} + 5x + c$$

2. $\int (2^x + \frac{5}{x} - \frac{1}{x^{\frac{1}{3}}})\, dx$

Solution:

Given

$$\int \left(2^x + \frac{5}{x} - \frac{1}{x^{1/3}}\right) dx$$

By splitting given equation we get,

$$\Rightarrow \int 2^x dx + \int \left(\frac{5}{x}\right) dx - \int \frac{1}{x^{1/3}} dx$$

By using the formula,

$$\int a^x \, dx = \frac{a^x}{\log a}$$

$$\Rightarrow \frac{2^x}{\log 2} + 5 \int \left(\frac{1}{x}\right) dx - \int x^{-1/3} dx$$

Again by using the formula,

$$\int \left(\frac{1}{x}\right) dx = \log x$$

$$\Rightarrow \frac{2^x}{\log 2} + 5\log x - \int x^{-1/3} dx$$

By using the below formula, we get

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

$$\Rightarrow \frac{2^x}{\log 2} + 5\log x - \frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}$$

$$\Rightarrow \frac{2^x}{\log 2} + 5\log x - \frac{x^{\frac{2}{3}}}{2/3}$$

On simplifying we get

$$\Rightarrow \frac{2^x}{\log 2} + 5\log x - \frac{3}{2} x^{2/3} + c$$

3. $\int \{\sqrt{x}(ax^2 + bx + c)\}\ dx$

Solution:

Given

$$\int \{\sqrt{x}(ax^2 + bx + c)\}dx$$

Now by multiplying we get

$$\Rightarrow \int (\sqrt{x}ax^2 + \sqrt{x}bx + \sqrt{x}c)\, dx$$

By splitting the given equation, we get,

$$\Rightarrow a\int x^2 \times x^{\frac{1}{2}}dx + b\int x^1 \times x^{\frac{1}{2}}dx + c\int x^{1/2}dx$$

$$\Rightarrow a\int x^{\frac{5}{2}}dx + b\int x^{\frac{3}{2}}dx + c\int x^{\frac{1}{2}}dx$$

By using the formula

$$\int x^n\, dx = \frac{x^{n+1}}{n+1}$$

We get

$$\Rightarrow \frac{ax^{\frac{5}{2}+1}}{\frac{5}{2}+1} + \frac{bx^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{cx^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

On simplifying

$$\Rightarrow \frac{ax^{\frac{7}{2}}}{7/2} + \frac{bx^{\frac{5}{2}}}{5/2} + \frac{cx^{\frac{3}{2}}}{3/2} + c$$

4. $\int (2 - 3x)(3 + 2x)(1 - 2x)\ dx$

Solution:

Given,

$\int(2 - 3x)(3 + 2x)(1 - 2x)\, dx$

$= \int(6 + 4x - 9x - 6x^2)(1 - 2x)\, dx$

$= \int(6 - 5x - 6x^2)(1 - 2x)\, dx$

$= \int(6 - 5x - 6x^2 - 12x + 10x^2 + 12x^3)\, dx$

$= \int(6 - 17x + 4x^2 + 12x^3)\, dx$

Upon splitting the above, we have

$= \int 6\, dx - \int 17x\, dx + \int 4x^2\, dx + \int 12x^3\, dx$

On integrating using formula,

$\int x^n\, dx = x^{n+1}/n+1$

we get

$= 6x - 17/(1+1)\, x^{1+1} + 4/(2+1)\, x^{2+1} + 12/(3+1)\, x^{3+1} + c$

$= 6x - 17x^2/2 + 4x^3/3 + 3x^4 + c$

5. $\int \left(\frac{m}{x} + \frac{x}{m} + m^x + x^m + mx\right) dx$

Solution:

Given

$$\int \left(\frac{m}{x} + \frac{x}{m} + m^x + x^m + mx\right) dx$$

By Splitting, we get,

$$\Rightarrow \int \frac{m}{x}dx + \int \frac{x}{m}dx + \int x^m dx + \int m^x dx + \int mxdx$$

We have

$$\int \frac{1}{x}dx = \log x + c$$

By applying the above formula, we get

$$\Rightarrow m\log x + \frac{1}{m}\int xdx + \int x^m dx + \int m^x dx + \int mxdx$$

By using this, we have

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\Rightarrow m\log x + \frac{\frac{1}{m}x^{1+1}}{1+1} + \frac{x^{m+1}}{m+1} + \int m^x dx + \frac{mx^{1+1}}{1+1}$$

By using the formula,

$$\int a^x dx = \frac{a^x}{\log a}$$

$$\Rightarrow m\log x + \frac{\frac{1}{m}x^2}{2} + \frac{x^{m+1}}{m+1} + \frac{m^x}{\log m} + \frac{mx^2}{2} + c$$

6. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$

Solution:

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$$

By applying $(a - b)^2 = a^2 - 2ab + b^2$ we get

$$\Rightarrow \int \left((\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2(\sqrt{x})\left(\frac{1}{\sqrt{x}}\right)\right)dx$$

After computing or simplifying, we get

$$\Rightarrow \int \left(x+\frac{1}{x}-2\right)dx$$

By splitting the above equation, we get,

$$\Rightarrow \int xdx+\int \frac{1}{x}dx-2\int 1dx$$

Now integrate by using standard integration formulae, we get

$$\Rightarrow \frac{x^{1+1}}{1+1}+\log x-2x+c$$

$= \frac{1}{2} x^2 + \log |x| - 2x + c$

7. $\int \frac{(1+x)^3}{\sqrt{x}}\, dx$

Solution:

Given

$$\int \frac{(1+x)^3}{\sqrt{x}}dx$$

Now by applying this formula $(a + b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b$ we get

$$\Rightarrow \int \frac{1+x^3+3x^2\times 1+3\times 1^2\times x}{\sqrt{x}}dx$$

$$\Rightarrow \int \frac{1+x^3+3x^2+3x}{\sqrt{x}}dx$$

By splitting the above equation, we get,

$$\Rightarrow \int \frac{1}{\sqrt{x}}dx+\int \frac{x^3}{\sqrt{x}}dx+\int \frac{3x^2}{\sqrt{x}}dx+\int \frac{3x}{\sqrt{x}}dx$$

$$\Rightarrow \int x^{-\frac{1}{2}}dx+\int x^3\times x^{-\frac{1}{2}}dx+\int 3x^2\times x^{-\frac{1}{2}}dx+\int 3x\times x^{-\frac{1}{2}}dx$$

$$\Rightarrow \int x^{-\frac{1}{2}}dx+\int x^{\frac{5}{2}}dx+3\int x^{\frac{3}{2}}dx+3\int x^{\frac{1}{2}}dx$$

Again we have formula,

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

By applying the above formula we get

$$\Rightarrow \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + 3\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{3x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$\Rightarrow \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 2\sqrt{x} + \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + \frac{6}{5}x^{\frac{5}{2}} + C$$

8. $\int \left\{x^2 + e^{\log x} + \left(\frac{e}{2}\right)^x\right\} dx$

Solution:

Given

$$\int \left\{x^2 + e^{\log x} + \left(\frac{e}{2}\right)^x\right\} dx$$

By splitting the above equation, we get,

$$\Rightarrow \int x^2\,dx + \int e^{\log x}dx + \int \left(\frac{e}{2}\right)^x dx$$

By applying formula,

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

We get

$$\Rightarrow \frac{x^{2+1}}{2+1} + \int e^{\log_e x}dx + \int \left(\frac{e}{2}\right)^x dx$$

$$\Rightarrow \frac{x^3}{3} + \int x\ dx + \frac{1}{\log(\frac{e}{2})}\left(\frac{e}{2}\right)^x$$

$$\Rightarrow \frac{x^3}{3} + \int x dx + \frac{1}{\log(\frac{e}{2})}\left(\frac{e}{2}\right)^x$$

Integrating and simplifying we get

$$\Rightarrow \frac{x^3}{3} + \frac{x^2}{2} + \frac{\left(\frac{e}{2}\right)^x}{\log\left(\frac{e}{2}\right)} + c$$

9. $\int (x^e + e^x + e^e)\ dx$

Solution:

$$\int (x^e + e^x + e^e) dx$$

By splitting the above equation, we get,

$$\Rightarrow \int x^e dx + \int e^x dx + \int e^e dx$$

By using the below formula,

$$\int x^n\, dx = \frac{x^{n+1}}{n+1}$$

We can write as

$$\Rightarrow \frac{x^{e+1}}{e+1} + \int e^x dx + \int e^e dx$$

Again by applying the formula,

$$\int a^x\, dx = \frac{a^x}{\log a}$$

We get

$$\Rightarrow \frac{x^{e+1}}{e+1} + \frac{e^x}{\log_e e} + \int e^e dx$$

We know that,

$$\int kdx = kx + c$$

So substituting this we have

$$\Rightarrow \frac{x^{e+1}}{e+1} + \frac{e^x}{\log_e e} + e^e x + c$$

10. $\int \sqrt{x}\left(x^3 - \frac{2}{x}\right) dx$

Solution:

Given

$$\int \sqrt{x}\left(x^3 - \frac{2}{x}\right) dx$$

Multiplying throughout the bracket, we get,

$$\Rightarrow \int (x^{\frac{1}{2}} \times x^3 - x^{\frac{1}{2}} \times \frac{2}{x})dx$$

$$\Rightarrow \int (x^{\frac{1}{2}+3} - x^{\frac{1}{2}-1} \times 2)dx$$

Again by simplifying

$$\Rightarrow \int (x^{\frac{7}{2}} - 2x^{-\frac{1}{2}})dx$$

By multiplying,

$$\Rightarrow \int x^{\frac{7}{2}}dx - 2\int x^{-\frac{1}{2}}dx$$

We have

$$\int x^n\, dx = \frac{x^{n+1}}{n+1}$$

Applying the above formula, we get

$$\Rightarrow \frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1} - 2\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$\Rightarrow \frac{x^{\frac{9}{2}}}{\frac{9}{2}} - 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\Rightarrow \frac{2x^{\frac{9}{2}}}{9} - 4x^{\frac{1}{2}} + c$$

11. $\int \frac{1}{\sqrt{x}}\left(1+\frac{1}{x}\right) dx$

Solution:

Given

$$\int \frac{1}{\sqrt{x}}\left\{1+\frac{1}{x}\right\} dx$$

By multiplying $\frac{1}{\sqrt{x}}$ throughout the brackets,

$$\Rightarrow \int \left\{\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{x}} \times \frac{1}{x}\right\} dx$$

The above equation can be written as

$$\Rightarrow \int \left\{\frac{1}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{1}{2}}} \times \frac{1}{x}\right\} dx$$

$$\Rightarrow \int \left\{\frac{1}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{1}{2}+1}}\right\} dx$$

$$\Rightarrow \int \left\{\frac{1}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{3}{2}}}\right\} dx$$

By splitting them, we get,

$$\Rightarrow \int x^{-\frac{1}{2}} dx + \int x^{-\frac{3}{2}} dx$$

We have

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

By applying the above formula and integrating, we get

$$\Rightarrow \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + c$$

$$\Rightarrow \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$\Rightarrow 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + c$$

14. $\int \frac{(1+\sqrt{x})^2}{\sqrt{x}} dx$

Solution:

$$\int \frac{(1+\sqrt{x})^2}{\sqrt{x}} dx$$

By applying $(a + b)^2 = a^2 + b^2 + 2ab$ we get

$$\Rightarrow \int \frac{(1)^2 + (\sqrt{x})^2 + 2 \times 1 \times \sqrt{x}}{\sqrt{x}} dx$$

$$\Rightarrow \int \frac{1 + x + 2\sqrt{x}}{\sqrt{x}} dx$$

By splitting the above equation, we get,

$$\Rightarrow \int \left(\frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}} + \frac{2\sqrt{x}}{\sqrt{x}}\right) dx$$

$$\Rightarrow \int x^{-\frac{1}{2}}dx + \int x \times x^{-\frac{1}{2}}dx + 2\int dx$$

On simplifying and integrating

$$\Rightarrow \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \int x^{1-\frac{1}{2}}dx + 2x + c$$

$$\Rightarrow \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \int x^{\frac{1}{2}}dx + 2x + c$$

Now by integrating, we get

$$\Rightarrow 2x^{\frac{1}{2}} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 2x + c$$

$$\Rightarrow 2x^{\frac{1}{2}} + \frac{2x^{\frac{3}{2}}}{3} + 2x + c$$

15. $\int \sqrt{x}(3-5x)\,dx$

Solution:

Given

$$\int \sqrt{x}(3-5x)dx$$

By multiplying √x throughout the bracket we get,

$$\Rightarrow \int (3\sqrt{x} - 5x\sqrt{x})dx$$

$$\Rightarrow \int \left(3x^{\frac{1}{2}} - 5x^{1} \times x^{\frac{1}{2}}\right)dx$$

$$\Rightarrow \int (3x^{\frac{1}{2}} - 5x^{1+\frac{1}{2}})dx$$

$$\Rightarrow \int (3x^{\frac{1}{2}} - 5x^{\frac{3}{2}})dx$$

By splitting the above equation, we get,

$$\Rightarrow 3\int x^{\frac{1}{2}}dx - 5\int x^{\frac{3}{2}}dx$$

By using the formula and integrating

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\Rightarrow \frac{3x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{5x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + c$$

$$\Rightarrow \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{5x^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$\Rightarrow 2x^{\frac{3}{2}} - 2x^{\frac{5}{2}} + c$$

16. $\int \frac{(x+1)(x-2)}{\sqrt{x}}\,dx$

Solution:

Given

$$\int \frac{(x+1)(x-2)}{\sqrt{x}}dx$$

Multiplying the above equation, we get

$$\Rightarrow \int \frac{x^2-2x+x-2}{\sqrt{x}}dx$$

$$\Rightarrow \int \frac{x^2-x-2}{\sqrt{x}}dx$$

By splitting the above equation,

$$\Rightarrow \int \frac{x^2}{\sqrt{x}}dx - \int \frac{x}{\sqrt{x}}dx - \int \frac{2}{\sqrt{x}}dx$$

$$\Rightarrow \int x^2 \times x^{-\frac{1}{2}} dx - \int x \times x^{-\frac{1}{2}} dx - 2\int x^{-\frac{1}{2}} dx$$

$$\Rightarrow \int x^{2-\frac{1}{2}} dx - \int x^{1-\frac{1}{2}} dx - 2\int x^{-\frac{1}{2}} dx$$

$$\Rightarrow \int x^{\frac{3}{2}} dx - \int x^{\frac{1}{2}} dx - 2\int x^{-\frac{1}{2}} dx$$

We have the formula,

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

By applying the above formula we get

$$\Rightarrow \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{2x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$\Rightarrow \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\Rightarrow \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + c$$

17. $\int \frac{x^5 + x^{-2} + 2}{x^2} dx$

Solution:

Given

$$\int \frac{x^5 + x^{-2} + 2}{x^2} dx$$

By splitting the above equation, we get,

$$\Rightarrow \int \left(\frac{x^5}{x^2} + \frac{x^{-2}}{x^2} + \frac{2}{x^2}\right) dx$$

The above equation can be written as

$$\Rightarrow \int (x^5 \times x^{-2} + x^{-2} \times x^{-2} + 2 \times x^{-2})dx$$

On simplifying,

$$\Rightarrow \int (x^{5-2} + x^{-2-2} + 2x^{-2})dx$$

$$\Rightarrow \int (x^3 + x^{-4} + 2x^{-2})dx$$

Again by splitting the above equation, we get,

$$\Rightarrow \int x^3 dx + \int x^{-4} dx + 2\int x^{-2} dx$$

By applying the formula,

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

Now by integrating by using the formula,

$$\Rightarrow \frac{x^{3+1}}{3+1} + \frac{x^{-4+1}}{-4+1} + \frac{2x^{-2+1}}{-2+1} + c$$

$$\Rightarrow \frac{x^4}{4} + \frac{x^{-3}}{-3} + \frac{2x^{-1}}{-1} + c$$

20. $\int \frac{5x^4 + 12x^3 + 7x^2}{x^2 + x} dx$

Solution:

Given

$$\int \frac{5x^4 + 12x^3 + 7x^2}{x^2 + x} dx$$

Now spilt $12x^3$ into $7x^3$ and $5x^3$

$$\Rightarrow \int \frac{5x^4 + 7x^3 + 5x^3 + 7x^2}{x^2 + x} dx$$

Now common $5x^3$ from two elements $7x^2$ from other two elements,

$$\Rightarrow \int \frac{5x^3(x+1)+7x^2(x+1)}{x^2+x}dx$$

$$\Rightarrow \frac{\int (5x^3+7x^2)(x+1)}{x(x+1)}dx$$

$$\Rightarrow \int (5x^2+7x)dx$$

Now splitting the above equation, we get,

$$\Rightarrow \int 5x^2 dx + \int 7x\, dx$$

$$\Rightarrow \frac{5x^{2+1}}{2+1} + \frac{7x^{1+1}}{1+1} + c$$

$$\Rightarrow \frac{5x^3}{3} + \frac{7x^2}{2} + c$$