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Evaluate the following integrals:

1. $\int e^{ax} \cos bx \, dx$

Solution:

Let $I = e^{ax} \cos bx \, dx$

Integrating by parts, we get

$$I = e^{ax} \frac{\sin bx}{b} - a \int e^{ax} \frac{\sin bx}{b} \, dx$$

Taking 1/b as common and a/b as common we get

$$= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \int e^{ax} \sin bx \, dx$$

Now again by using integration by parts, we get

$$= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \left[-e^{ax} \frac{\cos bx}{b} + a \int e^{ax} \frac{\cos bx}{b} \, dx \right]$$

$$= \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} \int e^{ax} \cos bx \, dx$$

By computing,

$$I = \frac{e^{ax}}{b^2} [b \sin bx + a \cos bx] - \frac{a^2}{b^2} I + c$$

$$= \frac{e^{ax}}{a^2 + b^2} [b \sin bx + a \cos bx] + c$$

2. $\int e^{ax} \sin(bx + c) \, dx$

Solution:

Let $I = \int e^{ax} \sin(bx + c)\, dx$

$$= -e^{ax}\frac{\cos(bx + c)}{b} + \int ae^{ax}\frac{\cos(bx + c)}{b}dx$$

Now taking common

$$= -\frac{1}{b}e^{ax}\cos(bx + c) + \frac{a}{b}\int e^{ax}\cos(bx + c)$$

On integrating we get

$$I = \frac{e^{ax}}{b^2}\{a\sin(bx + c) - b\cos(bx + c)\} - \frac{a^2}{b^2}I + c_1$$

By computing the above equation can be written as

$$I = \left\{\frac{a^2 + b^2}{b^2}\right\} - \frac{e^{ax}}{b^2}\{a\sin(bx + c) - b\cos(bx + c)\} + c_1$$

$$= \frac{e^{ax}}{a^2 + b^2}\{a\sin(bx + c) - b\cos(bx + c)\} + c_1$$

3. $\int \cos(\log x)\, dx$

Solution:

Let $I = \int \cos(\log x)\, dx$

Let log x=t

$$\frac{1}{x}dx = dt$$

dx = x dt

$$= \int e^t \cos t\, dt$$

We know that, $\int e^{ax}\cos x\, dx = \frac{e^{ax}}{a^2 + b^2}$ **{a cos bx + b sin bx}**

Hence, a=1, b=1

So, $I = \frac{e^t}{2}[\cos t + \sin t] + c$

Hence,

$$\int \cos(\log x)\, dx = \frac{e^{\log x}}{2}\{\cos(\log x) + \sin(\log x)\} + c$$

$$I = \frac{x}{2}\{\cos(\log x) + \sin(\log x)\} + c$$

4. $\int e^{2x} \cos(3x+4)\, dx$

Solution:

Let $I = \int e^{2x} \cos(3x+4)\, dx$

Integrating by parts

$$I = e^{2x}\frac{\sin(3x+4)}{3} - \int 2e^{2x}\frac{\sin(3x+4)}{3}dx$$

$$= \frac{1}{3}e^{2x}\sin(3x+4) - \frac{2}{3}\int e^{2x}\sin(3x+4)\, dx$$

$$= \frac{1}{3}e^{2x}\sin(3x+4) - \frac{2}{3}\left\{-e^{2x}\frac{\cos(3x+4)}{3} + \int 2e^{2x}\frac{\cos(3x+4)}{3}dx\right\}$$

$$I = \frac{1}{3}e^{2x}\sin(3x+4) + \frac{2}{9}e^{2x}\cos(3x+4) - \frac{4}{9} I$$

Hence,

$$I = \frac{e^{2x}}{13}[2\cos(3x+4) + 3\sin(3x+4)] + c$$

5. $\int e^{2x} \sin x \cos x\, dx$

Solution:

Let $I = \int e^{2x} \sin x \cos x dx$

$= \frac{1}{2} \int e^{2x} 2 \sin x \cos x dx$

$= \frac{1}{2} \int e^{2x} \sin 2x \, dx$

We know that,

$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \sin bx - b \cos bx\} + c$

$= \frac{e^{2x}}{8} \{2 \sin 2x - 2 \cos 2x\} + c$

$I = \frac{1}{2} \frac{e^{2x}}{8} \{2 \sin 2x - 2 \cos 2x\} + c$

$I = \frac{e^{2x}}{8} \{\sin 2x - \cos 2x\} + c$