EXERCISE 19.3

PAGE NO: 19.23

1. $\int (2x-3)^5 + \sqrt{3x+2}\, dx$

Solution:

Let I = $\int (2x-3)^5 + \sqrt{3x+2}$

Then,

I = $\int (2x-3)^5 + (3x+2)^{\frac{1}{2}}$

Now by integrating the above equation, we get

= $\frac{(2x-3)^{5+1}}{2(5+1)} + \frac{(3x+2)^{\frac{1}{2}+1}}{3(\frac{1}{2}+1)}$

= $\frac{(2x-3)^6}{2(6)} + \frac{(3x+2)^{\frac{3}{2}}}{3(\frac{3}{2})}$

= $\frac{(2x-3)^6}{12} + \frac{2(3x+2)^{\frac{3}{2}}}{9}$

Hence, I = $\frac{(2x-3)^6}{12} + \frac{2(3x+2)^{\frac{3}{2}}}{9}$ +C

2. $\int \frac{1}{(7x-5)^3} + \frac{1}{\sqrt{5x-4}}\, dx$

Solution:

Let I = $\int \frac{1}{(7x-5)^3} + \frac{1}{\sqrt{5x-4}} dx$ then,

I= $\int (7x-5)^{-3} + (5x-4)^{-\frac{1}{2}}$

Integrating the above equation, we get

$= \frac{(7x-5)^{-3+1}}{7(-3+1)} + \frac{(5x-4)^{-\frac{1}{2}+1}}{5\left(-\frac{1}{2}+1\right)}$

$= \frac{(7x-5)^{-2}}{-14} + \frac{(5x-4)^{\frac{1}{2}}}{5\left(\frac{1}{2}\right)}$

Hence, I = $-\frac{1}{14}(7x-5)^{-2} + \frac{2}{5}\sqrt{5x-4}$ +C

3. $\int \frac{1}{2-3x} + \frac{1}{\sqrt{3x-2}}\,dx$

Solution:

Let I = $\int \frac{1}{2-3x} + \frac{1}{\sqrt{3x-2}}dx$

I= $\int \frac{1}{2-3x} + \frac{1}{\sqrt{3x-2}}dx$

We know $\int \frac{1}{x}dx = \log|x|$

By applying the above formula we get

$= \frac{\log|2-3x|}{-3} + \frac{2}{3}(3x-2)^{\frac{1}{2}}$

$= -\frac{1}{3}\log|2x-3| + \frac{2}{3}\sqrt{3x-2} + C$

4. $\int \frac{x+3}{(x+1)^4}\,dx$

Solution:

Let,

I = $\int \frac{x+3}{(x+1)^4}dx$

Splitting the above given equation

$= \int \frac{x+1}{(x+1)^4} dx + \int \frac{2}{(x+1)^4} dx$

$= \int \frac{1}{(x+1)^3} dx + \int \frac{2}{(x+1)^4} dx$

The above equation can be written as

$= \int (x+1)^{-3}\, dx + \int 2(x+1)^{-4}\, dx$

Integrating the above equation we get

$= \frac{[x+1]^{-3+1}}{-3+1} + \frac{2(x+1)^{-4+1}}{-4+1}$

$= \frac{[x+1]^{-2}}{-2} + \frac{2(x+1)^{-3}}{-3}$

Hence, I = $-\frac{1}{2(x+1)^2} - \frac{2}{3(x+1)^3} + C$

5. $\int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx$

Solution:

Let I = $\int \frac{1}{\sqrt{x+1}+\sqrt{x}} dx$

Now multiply with the conjugate, we get

$= \int \frac{1}{\sqrt{x+1}+\sqrt{x}} \cdot \frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}} dx$

$= \int \frac{\sqrt{x+1}-\sqrt{x}}{x+1-x} dx$

On simplification we get

$= \int \sqrt{x+1} - \sqrt{x}\, dx$

The above equation can be written as

$= \int (x+1)^{\frac{1}{2}} - x^{\frac{1}{2}}$

On integrating we get

$= \frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}}$

Hence I= $\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}(x)^{\frac{3}{2}}$ +C

6. $\int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}} dx$

Solution:

Let I = $\int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}}$ dx

Now, multiply with the conjugate, we get

$= \int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}} \times \frac{(\sqrt{2x+3}-\sqrt{2x-3})}{\sqrt{2x+3}-\sqrt{2x-3}} dx$

$= \int \frac{(\sqrt{2x+3}-\sqrt{2x-3})}{(\sqrt{2x+3})^2-(\sqrt{2x-3})^2} dx$

$= \int \frac{(\sqrt{2x+3}-\sqrt{2x-3})}{2x+3-2x+3} dx$

Om simplifying or computing we get

$= \int \frac{\sqrt{2x+3}}{6} dx - \int \frac{\sqrt{2x-3}}{6} dx$

Taking 1/6 as common

$= \frac{1}{6}\int (2x+3)^{\frac{1}{2}} dx - \frac{1}{6}\int (2x-3)^{\frac{1}{2}} dx$

On integrating we get

$= \frac{1}{6}\left(\frac{2x+3}{2}\right)^{\frac{1}{2}+1} - \frac{1}{6}\left[\frac{2x-3}{2}\right]^{\frac{1}{2}+1}$

$= \frac{1}{6}\left(\frac{2x+3}{2\times\frac{3}{2}}\right)^{\frac{3}{2}} - \frac{1}{6}\left(\frac{2x-3}{2\times\frac{3}{2}}\right)^{\frac{3}{2}}$

Hence, I= $\frac{1}{18}(2x+3)^{\frac{3}{2}} - \frac{1}{18}(2x-3)^{\frac{3}{2}} + C$

7. $\int \frac{2x}{(2x+1)^2}\, dx$

Solution:

Let I = $\int \frac{2x}{(2x+1)^2} dx$

Now by splitting the above equation we get

$= \int \frac{2x+1}{(2x+1)^2} - \frac{1}{(2x+1)^2} dx$

The above equation can be written as

$= \int \frac{1}{(2x+1)} - (2x+1)^{-2} dx$

On integrating we get

$= \frac{1}{2}\log|2x+1| - \frac{(2x+1)^{-2+1}}{-2+1(2)}$

$= \frac{1}{2}\log|2x+1| - \frac{(2x+1)^{-1}}{-2}$

Hence, I= $\frac{1}{2}\log|2x+1| + \frac{1}{2(2x+1)}$+C

8. $\int \frac{1}{\sqrt{x+a}+\sqrt{x+b}}\, dx$

Solution:

Let I = $\int \frac{1}{\sqrt{x+a}+\sqrt{x+b}}$ dx

Now, multiply with conjugate, we get

$= \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{(\sqrt{x+a}-\sqrt{x+b})}{\sqrt{x+a}-\sqrt{(x+b)}} dx$

$= \int \frac{(\sqrt{x+a}-\sqrt{x+b})}{(\sqrt{x+a})^2-\sqrt{(x+b)}^2} dx$

On computing, we get

$= \int \frac{(\sqrt{x+a}-\sqrt{x+b})}{a-b} dx$

On integrating the above equation we get

$= \frac{1}{a-b}\left[\frac{2}{3}(x+a)^{\frac{3}{2}} - \frac{2}{3}(x+b)^{\frac{3}{2}}\right]$

Hence, I= $\frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + C$

9. $\int \sin x \sqrt{1+\cos 2x}\, dx$

Solution:

Let I = $\int \sin x \sqrt{(1+\cos 2x)} dx$

$= \int \sin x \sqrt{(1+\cos 2x)} dx$

By substituting the formula, we get

$= \int \sin x \sqrt{2\cos^2 x} dx$

$= \int \sin x \sqrt{2} \cos x\, dx$

$= \sqrt{2} \int \sin x \cos x\, dx$

Now, multiply and Divide by 2 we get,

$= \frac{\sqrt{2}}{2} \int 2 \sin x \cos x\, dx$

$= \frac{\sqrt{2}}{2}\int \sin 2x\, dx$

On integrating

$= \frac{\sqrt{2}}{2}\frac{-\cos 2x}{2}$

Hence, I= $-\frac{1}{2\sqrt{2}}\cos 2x$ +C