EXERCISE 19.32 PAGE NO: 19.196

Evaluate the following integrals:

1. $\int \frac{1}{(x-1)\sqrt{x+2}} dx$

Solution:

Assume $x + 2 = t^2$

$dx = 2tdt$

Now,

$$\int \frac{2dt}{(t^2-3)}$$

Using identity $\int \frac{dz}{(z)^2-1} = \frac{1}{2}\log\left|\frac{z-1}{z+1}\right| + c$

$$= \frac{1}{\sqrt{3}}\log\left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + c$$

$$= \frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{(x+2)}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c$$

2. $\int \frac{1}{(x-1)\sqrt{2x+3}} dx$

Solution:

Assume $2x + 3 = t^2$

$dx = t\,dt$

$$\int \frac{dt}{\frac{t^2-3}{2}-1}$$

$$\int \frac{2dt}{(t^2-5)}$$

Using identity $\int \frac{dz}{(z)^2-1} = \frac{1}{2}\log\left|\frac{z-1}{z+1}\right| + c$

$$\frac{1}{\sqrt{5}}\log\left|\frac{t-\sqrt{5}}{t+\sqrt{5}}\right|+c$$

$$\frac{1}{\sqrt{5}}\log\left|\frac{\sqrt{(2x+3)}\ -\sqrt{5}}{\sqrt{2x+3}+\sqrt{5}}\right|+c$$

3. $\int \frac{x+1}{(x-1)\sqrt{x+2}}dx$

Solution:

The given equation can be written as

$$\int \frac{(x-1)+2}{(x-1)\sqrt{x+2}}dx$$

Now splitting the integral in two parts

$$\int \frac{(x-1)}{(x-1)\sqrt{x+2}}dx+\int \frac{2}{(x-1)\sqrt{x+2}}dx$$

For the first part using identity $\int x^n dx=\frac{x^{n+1}}{n+1}$

$2\sqrt{x+2}$

For the second part

Assume $x+2=t^2$

$dx = 2\ t\ dt$

$$\int \frac{4dt}{(t^2-3)}$$

Using identity $\int \frac{dz}{(z)^2-1}=\frac{1}{2}\log\left|\frac{z-1}{z+1}\right|+c$

$$\frac{2}{\sqrt{3}}\log\left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right|+c$$

$$\frac{2}{\sqrt{3}}\log\left|\frac{\sqrt{(x+2)}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right|+c$$

Hence integral is

$$2\sqrt{x+2}+\frac{2}{\sqrt{3}}\log\left|\frac{\sqrt{(x+2)}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right|+c$$

4. $\int \frac{x^2}{(x-1)\sqrt{x+2}}dx$

Solution:

The given equation can be written as

$$\int \frac{(x^2-1)+1}{(x-1)\sqrt{x+2}}dx$$

$$\int \frac{(x^2-1)}{(x-1)\sqrt{x+2}}dx+\int \frac{1}{(x-1)\sqrt{x+2}}dx$$

$$\int \frac{(x+1)}{\sqrt{x+2}}dx+\int \frac{1}{(x-1)\sqrt{x+2}}dx=\int \frac{(x+2)-1}{\sqrt{x+2}}dx+\int \frac{1}{(x-1)\sqrt{x+2}}dx$$

$$\int \frac{(1)}{\sqrt{x+2}}dx+\int \sqrt{x+2}\,dx+\int \frac{1}{(x-1)\sqrt{x+2}}dx$$

For the first- and second-part using identity $\int x^n dx=\frac{x^{n+1}}{n+1}$

$$\frac{2}{3}(x+2)^{\frac{3}{2}}-2\sqrt{x+2}$$

For the second part

Assume $x+2=t^2$

$dx = 2\,t\,dt$

So,

$$\int \frac{2\,dt}{(t^2-3)}$$

Using identity $\int \frac{dz}{(z)^2-1} = \frac{1}{2}\log\left|\frac{z-1}{z+1}\right| + c$

$$= 2.\frac{1}{2\sqrt{3}}\log\left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + c$$

$$= \frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{(x+2)}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c \quad (\text{Using } t^2 = x+2)$$

Hence integral is

$$I = \frac{2}{3}(x+2)^{\frac{3}{2}} - 2\sqrt{x+2} + \frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{(x+2)}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c$$

5. $\int \frac{x}{(x-3)\sqrt{x+1}}dx$

Solution:

The given equation can be written as

$$\int \frac{(x-3)+3}{(x-3)\sqrt{x+1}}dx$$

$$\int \frac{(x-3)}{(x-3)\sqrt{x+1}}dx + \int \frac{3}{(x-3)\sqrt{x+1}}dx$$

For the first part using identity $\int x^n dx = \frac{x^{n+1}}{n+1}$

$$2\sqrt{x+1} + c$$

For the second part $\int \frac{3}{(x-3)\sqrt{x+1}}dx$,

Assume $x + 1 = t^2$

$dx = 2t\ dt$

$$\int \frac{3.2dt}{(t^2-4)}$$

Using identity $\int \frac{dz}{(z)^2-1} = \frac{1}{2}\log\left|\frac{z-1}{z+1}\right| + c$

$$=3 \times 2 \times \frac{1}{2(2)} \log\left|\frac{t-2}{t+2}\right| + c$$

$$= \frac{3}{2}\log\left|\frac{\sqrt{(x+1)}\ -2}{\sqrt{x+1}\ +2}\right| + c$$

Hence integral is

$$= 2\sqrt{x+1} + \frac{3}{2}\log\left|\frac{\sqrt{(x+1)}\ -2}{\sqrt{x+1}\ +2}\right| + c$$