EXERCISE 4.1   PAGE NO: 4.6

1. Find the principal value of the following:

(i) $\sin^{-1}(-\sqrt{\dfrac{3}{2}})$

(ii) $\sin^{-1}(\cos\dfrac{2\pi}{3})$

(iii) $\sin^{-1}(\dfrac{\sqrt{3}-1}{2\sqrt{2}})$

(iv) $\sin^{-1}(\dfrac{\sqrt{3}+1}{2\sqrt{2}})$

(v) $\sin^{-1}(\cos\dfrac{3\pi}{4})$

(vi) $\sin^{-1}(\tan\dfrac{5\pi}{4})$

Solution:

$(i) Let \sin^{-1}(\dfrac{-\sqrt{3}}{2}) = y$

$Then\ siny = (\dfrac{-\sqrt{3}}{2})$

$= -\sin(\dfrac{\pi}{3})$

$= \sin(-\dfrac{\pi}{3})$

$We\ know\ that\ the\ principal\ value\ of\ \sin^{-1}\ is\ \left[\dfrac{-\pi}{2},\dfrac{\pi}{2}\right]$

$And - \sin\dfrac{\pi}{3} = \sin(\dfrac{-\pi}{3})$

$Therefore\ principal\ value\ of \sin^{-1}(\dfrac{-\sqrt{3}}{2}) = \dfrac{-\pi}{3}$

$(ii) Let \sin^{-1}(\cos\dfrac{2\pi}{3}) = y$

$Then \ siny = \cos(\dfrac{2\pi}{3})$

$= -\sin(\dfrac{\pi}{2} + \dfrac{\pi}{6})$

$= -sin(\dfrac{\pi}{6})$

$We \ know \ that \ the \ principal \ value \ of \ \sin^{-1} \ is \ \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$

$And \ -sin(\dfrac{\pi}{6}) = cos(\dfrac{2\pi}{3})$

$Therefore \ principal \ value \ of \ \sin^{-1}(\cos\dfrac{2\pi}{3}) \ is\dfrac{-\pi}{6}$

(iii) Given functions can be written as

$$\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)$$

Taking 1/√2 as common from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$$

Taking √3/2 as common, and 1/√2 from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2} - \frac{1}{\sqrt{2}} \times \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

On simplifying, we get

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

By substituting the values,

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

Taking LCM and cross multiplying we get,

$$= \frac{\pi}{12}$$

(iv) The given question can be written as

$$\sin^{-1}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)$$

Taking 1/√2 as common from the above equation we get

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$$

Taking √3/2 as common, and 1/√2 from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} + \frac{1}{\sqrt{2}} \times \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

On simplifying we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

By substituting the corresponding values we get

$$= \frac{\pi}{3} + \frac{\pi}{4}$$

$$= \frac{7\pi}{12}$$

(v) Let

$$\sin^{-1}\left(\cos\frac{3\pi}{4}\right) = y$$

Then above equation can be written as

$$\sin y = \cos\frac{3\pi}{4} = -\sin\left(\pi - \frac{3\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right)$$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore above equation becomes,

$$-\sin\left(\frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$$

Therefore the principal value of $\sin^{-1}\left(\cos\frac{3\pi}{4}\right)$ is $-\frac{\pi}{4}$

(vi) Let

$$y = \sin^{-1}\left(\tan\frac{5\pi}{4}\right)$$

Therefore above equation can be written as

$$\text{Sin } y = \left(\tan\frac{5\pi}{4}\right) = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1 = \sin\left(\frac{\pi}{2}\right)$$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\sin\left(\frac{\pi}{2}\right) = \tan\frac{5\pi}{4}$$

Therefore the principal value of $\sin^{-1}\left(\tan\frac{5\pi}{4}\right)$ is $\frac{\pi}{2}$.

2. Find the value of each of the following:

(i) $\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{\sqrt{2}}$

(ii) $\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right\}$

Solution:

(i) The given question can be written as,

$$\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{\sqrt{2}} = \sin^{-1}\frac{1}{2} - \sin^{-1}\left(2 \times \frac{1}{\sqrt{2}}\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}\right)$$

On simplifying, we get

$$= \sin^{-1}\frac{1}{2} - \sin^{-1}(1)$$

By substituting the corresponding values, we get

$$= \frac{\pi}{6} - \frac{\pi}{2}$$

$$= -\frac{\pi}{3}$$

(ii) Given question can be written as

We know that $\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) = \pi/3$

$$= \sin^{-1}\left\{\cos\left(\frac{\pi}{3}\right)\right\}$$

Now substituting the values we get,

$$= \sin^{-1}\left\{\frac{1}{2}\right\}$$

$$= \frac{\pi}{6}$$