EXERCISE 14.2 PAGE NO: 14.20

1. In Fig. 58, line n is a transversal to line l and m. Identify the following:
(i) Alternate and corresponding angles in Fig. 58 (i)
(ii) Angles alternate to ∠d and ∠g and angles corresponding to ∠f and ∠h in Fig. 58 (ii)
(iii) Angle alternate to ∠PQR, angle corresponding to ∠RQF and angle alternate to ∠PQE in Fig. 58 (iii)
(iv) Pairs of interior and exterior angles on the same side of the transversal in Fig. 58 (ii)

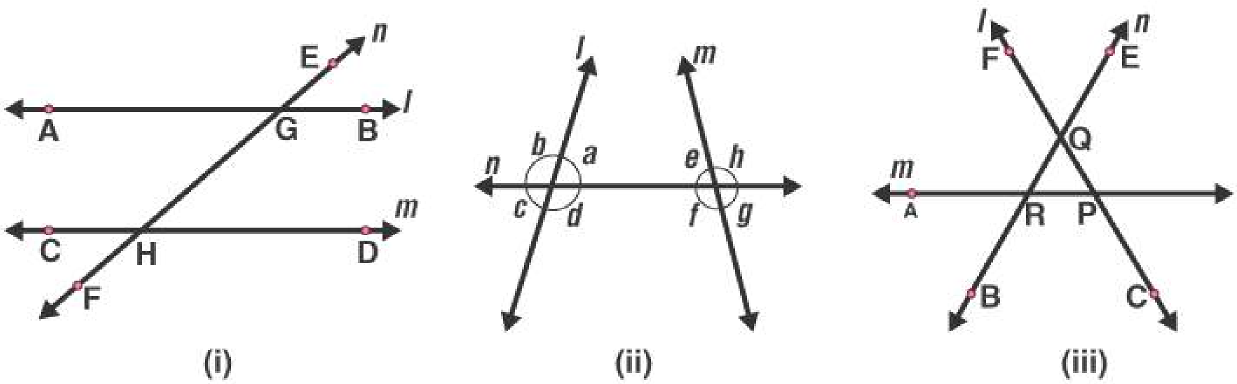

Fig.58

Solution:
(i) A pair of angles in which one arm of both the angles is on the same side of the transversal and their other arms are directed in the same sense is called a pair of corresponding angles.
In Figure (i) Corresponding angles are
∠EGB and ∠GHD
∠HGB and ∠FHD
∠EGA and ∠GHC
∠AGH and ∠CHF
A pair of angles in which one arm of each of the angle is on opposite sides of the transversal and whose other arms include the one segment is called a pair of alternate angles.
The alternate angles are:
∠EGB and ∠CHF

∠HGB and ∠CHG
∠EGA and ∠FHD
∠AGH and ∠GHD

(ii) In Figure (ii)
The alternate angle to ∠d is ∠e.
The alternate angle to ∠g is ∠b.
The corresponding angle to ∠f is ∠c.
The corresponding angle to ∠h is ∠a.

(iii) In Figure (iii)
Angle alternate to ∠PQR is ∠QRA.
Angle corresponding to ∠RQF is ∠ARB.
Angle alternate to ∠POE is ∠ARB.

(iv) In Figure (ii)
Pair of interior angles are
∠a is ∠e.
∠d is ∠f.
Pair of exterior angles are
∠b is ∠h.
∠c is ∠g.

2. In Fig. 59, AB and CD are parallel lines intersected by a transversal PQ at L and M respectively, If ∠CMQ = 60°, find all other angles in the figure.

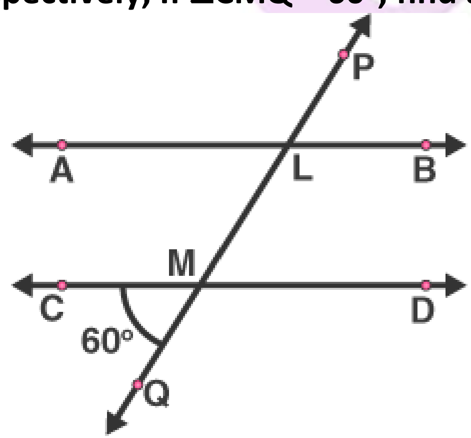

Fig. 59

Solution:

A pair of angles in which one arm of both the angles is on the same side of the transversal and their other arms are directed in the same sense is called a pair of corresponding angles.
Therefore corresponding angles are
$\angle ALM = \angle CMQ = 60^o$ [given]
Vertically opposite angles are
$\angle LMD = \angle CMQ = 60^o$ [given]
Vertically opposite angles are
$\angle ALM = \angle PLB = 60^o$
Here, $\angle CMQ + \angle QMD = 180^o$ are the linear pair
On rearranging we get
$\angle QMD = 180^o – 60^o$
$= 120^o$
Corresponding angles are
$\angle QMD = \angle MLB = 120^o$
Vertically opposite angles
$\angle QMD = \angle CML = 120^o$
Vertically opposite angles
$\angle MLB = \angle ALP = 120^o$

3. In Fig. 60, AB and CD are parallel lines intersected by a transversal by a transversal PQ at L and M respectively. If $\angle LMD = 35^o$ find $\angle ALM$ and $\angle PLA$.

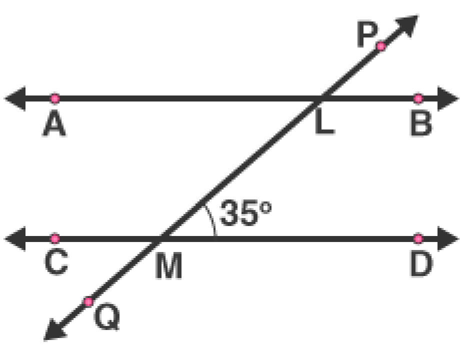

Fig. 60

Solution:
Given that, $\angle LMD = 35^o$
From the figure we can write
$\angle LMD$ and $\angle LMC$ is a linear pair
$\angle LMD + \angle LMC = 180^o$ [sum of angles in linear pair = 180^o]

On rearranging, we get
∠LMC = $180^o - 35^o$
= 145^o
So, ∠LMC = ∠PLA = 145^o
And, ∠LMC = ∠MLB = 145^o
∠MLB and ∠ALM is a linear pair
∠MLB + ∠ALM = 180^o [sum of angles in linear pair = 180^o]
∠ALM = $180^o - 145^o$
∠ALM = 35^0
Therefore, ∠ALM = 35^o, ∠PLA = 145^o.

4. The line n is transversal to line l and m in Fig. 61. Identify the angle alternate to ∠13, angle corresponding to ∠15, and angle alternate to ∠15.

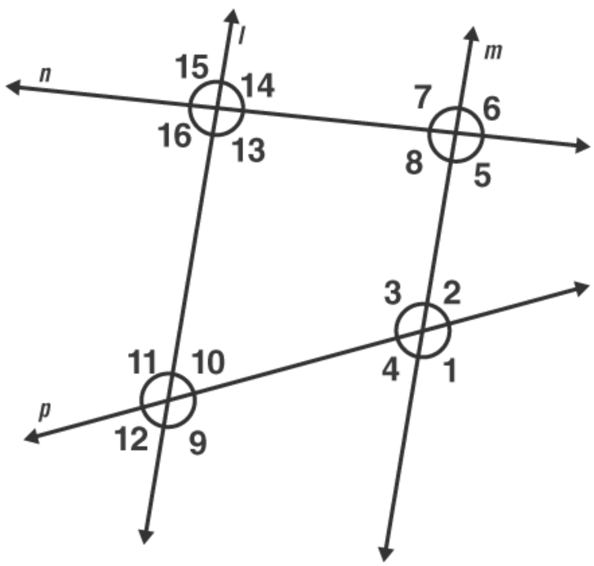

Fig.61

Solution:
Given that, l ∥ m
From the figure the angle alternate to ∠13 is ∠7
From the figure the angle corresponding to ∠15 is ∠7 [A pair of angles in which one arm of both the angles is on the same side of the transversal and their other arms are directed in the same sense is called a pair of corresponding angles.]
Again from the figure angle alternate to ∠15 is ∠5

5. In Fig. 62, line l ∥ m and n is transversal. If ∠1 = 40°, find all the angles and check

that all corresponding angles and alternate angles are equal.

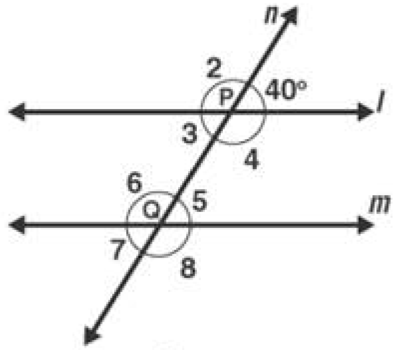

Fig. 62

Solution:

Given that, $\angle 1 = 40^\circ$

$\angle 1$ and $\angle 2$ is a linear pair [from the figure]

$\angle 1 + \angle 2 = 180^\circ$

$\angle 2 = 180^\circ - 40^\circ$

$\angle 2 = 140^\circ$

Again from the figure we can say that

$\angle 2$ and $\angle 6$ is a corresponding angle pair

So, $\angle 6 = 140^\circ$

$\angle 6$ and $\angle 5$ is a linear pair [from the figure]

$\angle 6 + \angle 5 = 180^\circ$

$\angle 5 = 180^\circ - 140^\circ$

$\angle 5 = 40^\circ$

From the figure we can write as

$\angle 3$ and $\angle 5$ are alternate interior angles

So, $\angle 5 = \angle 3 = 40^\circ$

$\angle 3$ and $\angle 4$ is a linear pair

$\angle 3 + \angle 4 = 180^\circ$

$\angle 4 = 180^\circ - 40^\circ$

$\angle 4 = 140^\circ$

Now, $\angle 4$ and $\angle 6$ are a pair of interior angles

So, $\angle 4 = \angle 6 = 140^\circ$

$\angle 3$ and $\angle 7$ are a pair of corresponding angles

So, $\angle 3 = \angle 7 = 40^\circ$

Therefore, $\angle 7 = 40^\circ$

$\angle 4$ and $\angle 8$ are a pair of corresponding angles

So, $\angle 4 = \angle 8 = 140^\circ$

Therefore, $\angle 8 = 140^o$
Therefore, $\angle 1 = 40^o$, $\angle 2 = 140^o$, $\angle 3 = 40^o$, $\angle 4 = 140^o$, $\angle 5 = 40^o$, $\angle 6 = 140^o$, $\angle 7 = 40^o$ and $\angle 8 = 140^o$

6. In Fig.63, line l ∥ m and a transversal n cuts them P and Q respectively. If ∠1 = 75°, find all other angles.

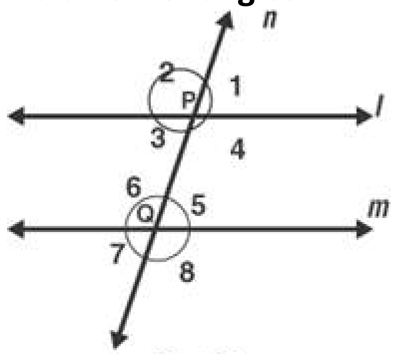

Fig. 63

Solution:
Given that, l ∥ m and $\angle 1 = 75^o$
$\angle 1 = \angle 3$ are vertically opposite angles
We know that, from the figure
$\angle 1 + \angle 2 = 180^o$ is a linear pair
$\angle 2 = 180^o - 75^o$
$\angle 2 = 105^o$
Here, $\angle 1 = \angle 5 = 75^o$ are corresponding angles
$\angle 5 = \angle 7 = 75^o$ are vertically opposite angles.
$\angle 2 = \angle 6 = 105^o$ are corresponding angles
$\angle 6 = \angle 8 = 105^o$ are vertically opposite angles
$\angle 2 = \angle 4 = 105^o$ are vertically opposite angles
So, $\angle 1 = 75^o$, $\angle 2 = 105^o$, $\angle 3 = 75^o$, $\angle 4 = 105^o$, $\angle 5 = 75^o$, $\angle 6 = 105^o$, $\angle 7 = 75^o$ and $\angle 8 = 105^o$

7. In Fig. 64, AB ∥ CD and a transversal PQ cuts at L and M respectively. If ∠QMD = 100^o, find all the other angles.

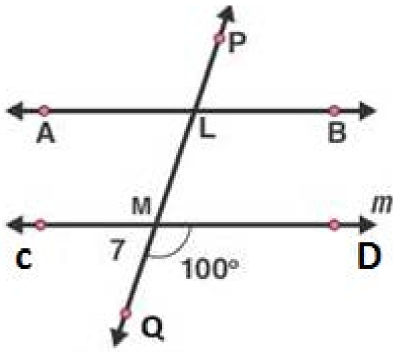

Fig. 64

Solution:

Given that, AB ∥ CD and ∠QMD = 100^{o}

We know that, from the figure ∠QMD + ∠QMC = 180^{o} is a linear pair,

∠QMC = 180^{o} – ∠QMD

∠QMC = 180^{o} – 100°

∠QMC = 80^{o}

Corresponding angles are

∠DMQ = ∠BLM = 100^{o}

∠CMQ = ∠ALM = 80^{o}

Vertically Opposite angles are

∠DMQ = ∠CML = 100^{o}

∠BLM = ∠PLA = 100^{o}

∠CMQ = ∠DML = 80^{o}

∠ALM = ∠PLB = 80^{o}

8. In Fig. 65, l ∥ m and p ∥ q. Find the values of x, y, z, t.

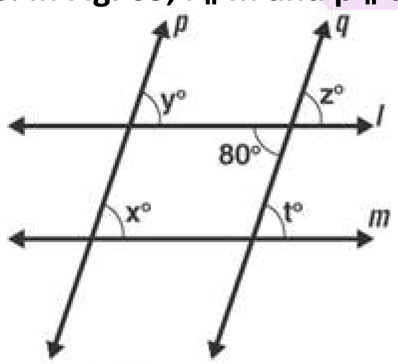

Fig. 65

Solution:

Given that one of the angle is 80^{o}

$\angle z$ and 80^o are vertically opposite angles
Therefore $\angle z = 80^o$
$\angle z$ and $\angle t$ are corresponding angles
$\angle z = \angle t$
Therefore, $\angle t = 80^o$
$\angle z$ and $\angle y$ are corresponding angles
$\angle z = \angle y$
Therefore, $\angle y = 80^o$
$\angle x$ and $\angle y$ are corresponding angles
$\angle y = \angle x$
Therefore, $\angle x = 80^o$

9. In Fig. 66, line l ∥ m, $\angle 1 = 120^o$ and $\angle 2 = 100^o$, find out $\angle 3$ and $\angle 4$.

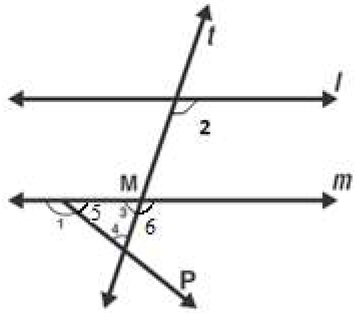

Fig. 66

Solution:
Given that, $\angle 1 = 120^o$ and $\angle 2 = 100^o$
From the figure $\angle 1$ and $\angle 5$ is a linear pair
$\angle 1 + \angle 5 = 180^o$
$\angle 5 = 180^o - 120^o$
$\angle 5 = 60^o$
Therefore, $\angle 5 = 60^o$
$\angle 2$ and $\angle 6$ are corresponding angles
$\angle 2 = \angle 6 = 100^o$
Therefore, $\angle 6 = 100^o$
$\angle 6$ and $\angle 3$ is a linear pair
$\angle 6 + \angle 3 = 180^o$
$\angle 3 = 180^o - 100^o$
$\angle 3 = 80^o$
Therefore, $\angle 3 = 80^o$

By, angles of sum property
$\angle 3 + \angle 5 + \angle 4 = 180^\circ$
$\angle 4 = 180^\circ – 80^\circ – 60^\circ$
$\angle 4 = 40^\circ$
Therefore, $\angle 4 = 40^\circ$

10. In Fig. 67, l ∥ m. Find the values of a, b, c, d. Give reasons.

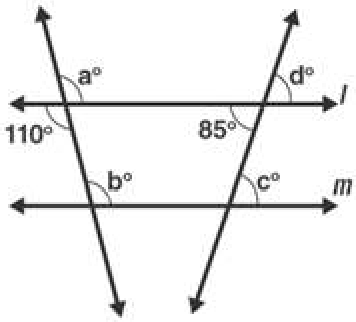

Fig. 67

Solution:
Given l ∥ m
From the figure vertically opposite angles,
$\angle a = 110^\circ$
Corresponding angles, $\angle a = \angle b$
Therefore, $\angle b = 110^\circ$
Vertically opposite angle,
$\angle d = 85^\circ$
Corresponding angles, $\angle d = \angle c$
Therefore, $\angle c = 85^\circ$
Hence, $\angle a = 110^\circ$, $\angle b = 110^\circ$, $\angle c = 85^\circ$, $\angle d = 85^\circ$

11. In Fig. 68, AB ∥ CD and ∠1 and ∠2 are in the ratio of 3: 2. Determine all angles from 1 to 8.

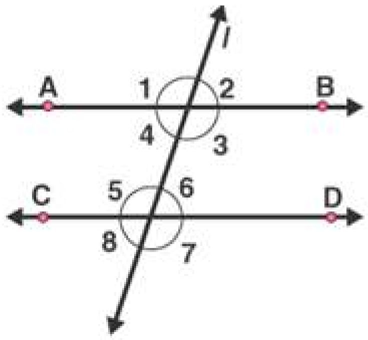

Fig. 68

Solution:

Given ∠1 and ∠2 are in the ratio 3: 2
Let us take the angles as 3x, 2x
∠1 and ∠2 are linear pair [from the figure]
$3x + 2x = 180^o$
$5x = 180^o$
$x = 180^o/5$
$x = 36^o$
Therefore, ∠1 = 3x = 3(36) = 108^o
∠2 = 2x = 2(36) = 72^o
∠1 and ∠5 are corresponding angles
Therefore ∠1 = ∠5
Hence, ∠5 = 108^o
∠2 and ∠6 are corresponding angles
So ∠2 = ∠6
Therefore, ∠6 = 72^o
∠4 and ∠6 are alternate pair of angles
∠4 = ∠6 = 72^o
Therefore, ∠4 = 72^o
∠3 and ∠5 are alternate pair of angles
∠3 = ∠5 = 108^o
Therefore, ∠3 = 108^o
∠2 and ∠8 are alternate exterior of angles
∠2 = ∠8 = 72^o
Therefore, ∠8 = 72^o
∠1 and ∠7 are alternate exterior of angles

$\angle 1 = \angle 7 = 108^o$
Therefore, $\angle 7 = 108^o$
Hence, $\angle 1 = 108^o$, $\angle 2 = 72^o$, $\angle 3 = 108^o$, $\angle 4 = 72^o$, $\angle 5 = 108^o$, $\angle 6 = 72^o$, $\angle 7 = 108^o$, $\angle 8 = 72^o$

12. In Fig. 69, l, m and n are parallel lines intersected by transversal p at X, Y and Z respectively. Find ∠1, ∠2 and ∠3.

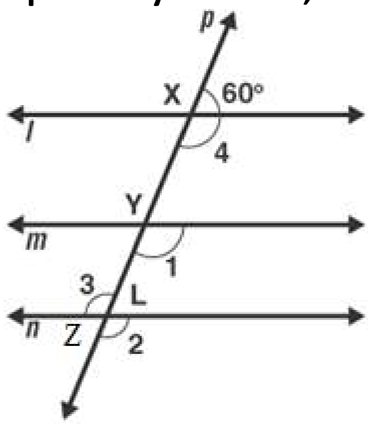

Fig. 69

Solution:
Given l, m and n are parallel lines intersected by transversal p at X, Y and Z
Therefore linear pair,
$\angle 4 + 60^o = 180^o$
$\angle 4 = 180^o - 60^o$
$\angle 4 = 120^o$
From the figure,
$\angle 4$ and $\angle 1$ are corresponding angles
$\angle 4 = \angle 1$
Therefore, $\angle 1 = 120^o$
$\angle 1$ and $\angle 2$ are corresponding angles
$\angle 2 = \angle 1$
Therefore, $\angle 2 = 120^o$
$\angle 2$ and $\angle 3$ are vertically opposite angles
$\angle 2 = \angle 3$
Therefore, $\angle 3 = 120^0$

13. In Fig. 70, if l ∥ m ∥ n and $\angle 1 = 60^o$, find ∠2

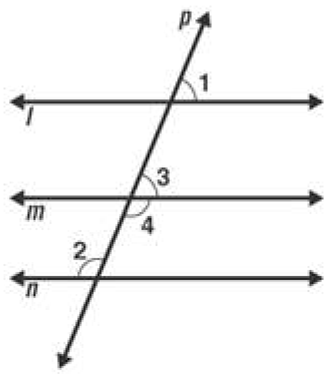

Fig. 70

Solution:

Given that l ‖ m ‖ n

From the figure Corresponding angles are

$\angle 1 = \angle 3$

$\angle 1 = 60^{o}$

Therefore, $\angle 3 = 60^{o}$

$\angle 3$ and $\angle 4$ are linear pair

$\angle 3 + \angle 4 = 180^{o}$

$\angle 4 = 180^{o} - 60^{o}$

$\angle 4 = 120^{o}$

$\angle 2$ and $\angle 4$ are alternate interior angles

$\angle 4 = \angle 2$

Therefore, $\angle 2 = 120^{o}$

14. In Fig. 71, if AB ‖ CD and CD ‖ EF, find ∠ACE

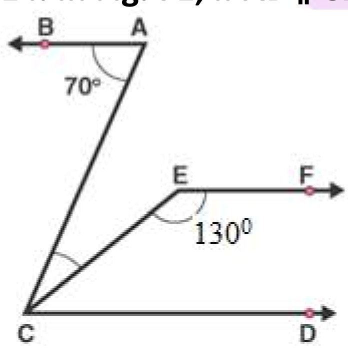

Fig. 71

Solution:

Given that, AB ‖ CD and CD ‖ EF

Sum of the interior angles,

$\angle CEF + \angle ECD = 180^\circ$
$130^\circ + \angle ECD = 180^\circ$
$\angle ECD = 180^\circ - 130^\circ$
$\angle ECD = 50^\circ$
We know that alternate angles are equal
$\angle BAC = \angle ACD$
$\angle BAC = \angle ECD + \angle ACE$
$\angle ACE = 70^\circ - 50^\circ$
$\angle ACE = 20^\circ$
Therefore, $\angle ACE = 20^\circ$

15. In Fig. 72, if l ∥ m, n ∥ p and ∠1 = 85°, find ∠2.

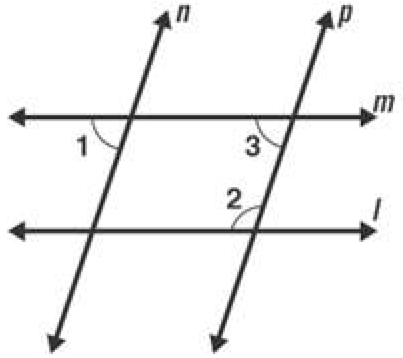

Fig. 72

Solution:
Given that, $\angle 1 = 85^\circ$
$\angle 1$ and $\angle 3$ are corresponding angles
So, $\angle 1 = \angle 3$
$\angle 3 = 85^\circ$
Sum of the interior angles is 180°
$\angle 3 + \angle 2 = 180^\circ$
$\angle 2 = 180^\circ - 85^\circ$
$\angle 2 = 95^\circ$

16. In Fig. 73, a transversal n cuts two lines l and m. If ∠1 = 70° and ∠7 = 80°, is l ∥ m?

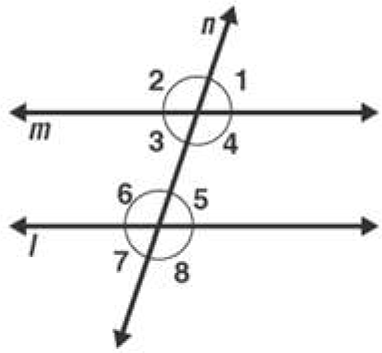

Fig. 73

Solution:

Given ∠1 = 70° and ∠7 = 80°

We know that if the alternate exterior angles of the two lines are equal, then the lines are parallel.

Here, ∠1 and ∠7 are alternate exterior angles, but they are not equal

∠1 ≠ ∠7

17. In Fig. 74, a transversal n cuts two lines l and m such that ∠2 = 65° and ∠8 = 65°. Are the lines parallel?

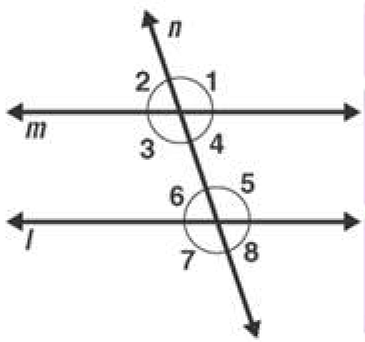

Fig. 74

Solution:

From the figure ∠2 = ∠4 are vertically opposite angles,

∠2 = ∠4 = 65°

∠8 = ∠6 = 65°

Therefore, ∠4 = ∠6

Hence, l ∥ m

18. In Fig. 75, Show that AB ∥ EF.

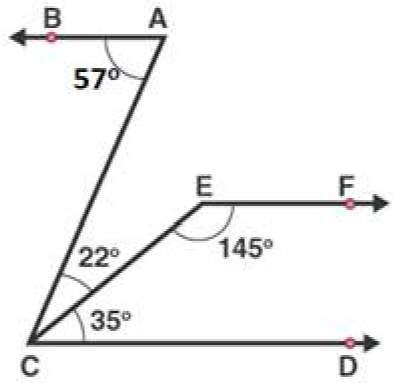

Fig. 75

Solution:

We know that,

∠ACD = ∠ACE + ∠ECD

∠ACD = 22° + 35°

∠ACD = 57° = ∠BAC

Thus, lines BA and CD are intersected by the line AC such that, ∠ACD = ∠BAC

So, the alternate angles are equal

Therefore, AB ∥ CD ……1

Now,

∠ECD + ∠CEF = 35° + 145° = 180°

This, shows that sum of the angles of the interior angles on the same side of the transversal CE is 180°

So, they are supplementary angles

Therefore, EF ∥ CD …….2

From equation 1 and 2

We conclude that, AB ∥ EF

19. In Fig. 76, AB ∥ CD. Find the values of x, y, z.

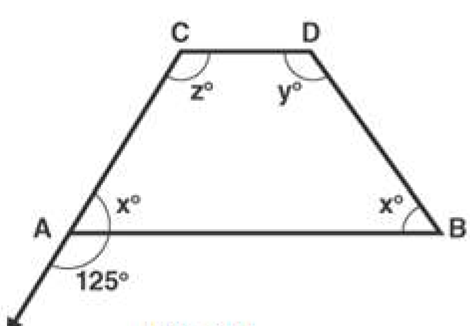

Fig. 76

Solution:

Given that AB ∥ CD

Linear pair,

$\angle x + 125^\circ = 180^\circ$

$\angle x = 180^\circ - 125^\circ$

$\angle x = 55^\circ$

Corresponding angles

$\angle z = 125^\circ$

Adjacent interior angles

$\angle x + \angle z = 180^\circ$

$\angle x + 125^\circ = 180^\circ$

$\angle x = 180^\circ - 125^\circ$

$\angle x = 55^\circ$

Adjacent interior angles

$\angle x + \angle y = 180^\circ$

$\angle y + 55^\circ = 180^\circ$

$\angle y = 180^\circ - 55^\circ$

$\angle y = 125^\circ$

20. In Fig. 77, find out ∠PXR, if PQ ∥ RS.

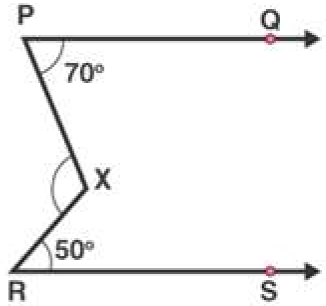

Fig. 77

Solution:

Given PQ ∥ RS

We need to find ∠PXR

$\angle XRS = 50^\circ$

$\angle XPQ = 70^\circ$

Given, that PQ ∥ RS

$\angle PXR = \angle XRS + \angle XPQ$

$\angle PXR = 50^\circ + 70^\circ$

$\angle PXR = 120^\circ$

Therefore, ∠PXR = 120°

21. In Figure, we have
(i) ∠MLY = 2∠LMQ
(ii) ∠XLM = (2x - 10)° and ∠LMQ = (x + 30)°, find x.
(iii) ∠XLM = ∠PML, find ∠ALY
(iv) ∠ALY = (2x - 15)°, ∠LMQ = (x + 40)°, find x.

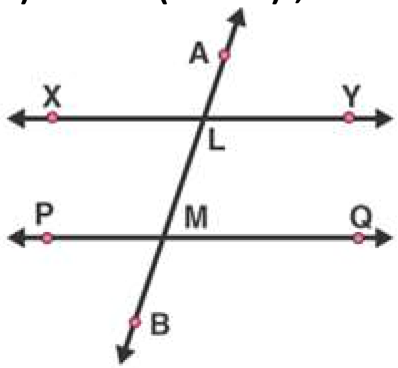

Fig. 78

Solution:

(i) ∠MLY and ∠LMQ are interior angles
∠MLY + ∠LMQ = 180°
2∠LMQ + ∠LMQ = 180°
3∠LMQ = 180°
∠LMQ = 180°/3
∠LMQ = 60°

(ii) ∠XLM = (2x - 10)° and ∠LMQ = (x + 30)°, find x.
∠XLM = (2x - 10)° and ∠LMQ = (x + 30)°
∠XLM and ∠LMQ are alternate interior angles
∠XLM = ∠LMQ
(2x - 10)° = (x + 30)°
2x – x = 30° + 10°
x = 40°
Therefore, x = 40°

(iii) ∠XLM = ∠PML, find ∠ALY
∠XLM = ∠PML

Sum of interior angles is 180 degrees
$\angle XLM + \angle PML = 180^{o}$
$\angle XLM + \angle XLM = 180^{o}$
$2\angle XLM = 180^{o}$
$\angle XLM = 180^{o}/2$
$\angle XLM = 90^{o}$
$\angle XLM$ and $\angle ALY$ are vertically opposite angles
Therefore, $\angle ALY = 90^{o}$

(iv) $\angle ALY = (2x - 15)^{o}$, $\angle LMQ = (x + 40)^{o}$, find x.
$\angle ALY$ and $\angle LMQ$ are corresponding angles
$\angle ALY = \angle LMQ$
$(2x - 15)^{o} = (x + 40)^{o}$
$2x – x = 40^{o} + 15^{o}$
$x = 55^{o}$
Therefore, $x = 55^{o}$

22. In Fig. 79, DE ‖ BC. Find the values of x and y.

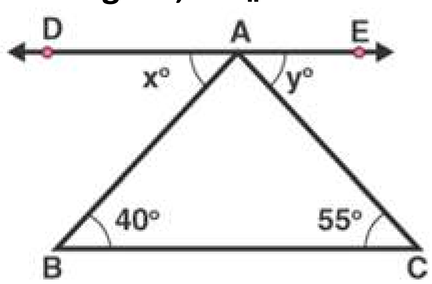

Fig. 79

Solution:
We know that,
ABC, DAB are alternate interior angles
$\angle ABC = \angle DAB$
So, $x = 40^{o}$
And ACB, EAC are alternate interior angles
$\angle ACB = \angle EAC$
So, $y = 55^{o}$

23. In Fig. 80, line AC ‖ line DE and $\angle ABD = 32^{o}$, Find out the angles x and y if $\angle E = 122^{o}$.

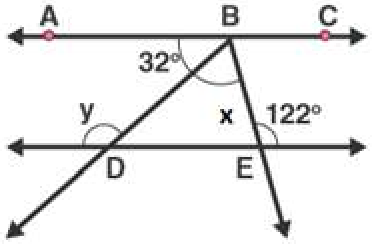

Fig. 80

Solution:

Given line AC ∥ line DE and ∠ABD = 32^o

∠BDE = ∠ABD = 32^o – Alternate interior angles

∠BDE + y = 180^o– linear pair

32^o + y = 180^o

y = 180^o – 32^o

y = 148^o

∠ABE = ∠E = 122^o – Alternate interior angles

∠ABD + ∠DBE = 122^o

32^o + x = 122^o

x = 122^o – 32^o

x = 90^o

24. In Fig. 81, side BC of ΔABC has been produced to D and CE ∥ BA. If ∠ABC = 65^o, ∠BAC = 55^o, find ∠ACE, ∠ECD, ∠ACD.

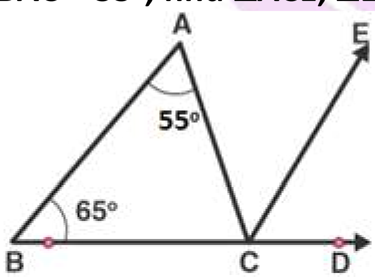

Fig. 81

Solution:

Given ∠ABC = 65^o, ∠BAC = 55^o

Corresponding angles,

∠ABC = ∠ECD = 65^o

Alternate interior angles,

$\angle BAC = \angle ACE = 55^o$
Now, $\angle ACD = \angle ACE + \angle ECD$
$\angle ACD = 55^o + 65^o$
$= 120^o$

25. In Fig. 82, line CA ⊥ AB ‖ line CR and line PR ‖ line BD. Find ∠x, ∠y, ∠z.

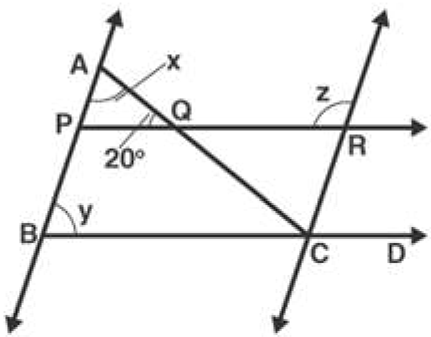

Fig. 82

Solution:
Given that, CA ⊥ AB
$\angle CAB = 90^o$
$\angle AQP = 20^o$
By, angle of sum property
In ΔABC
$\angle CAB + \angle AQP + \angle APQ = 180^o$
$\angle APQ = 180^o - 90^o - 20^o$
$\angle APQ = 70^o$
y and ∠APQ are corresponding angles
$y = \angle APQ = 70^o$
∠APQ and ∠z are interior angles
$\angle APQ + \angle z = 180^o$
$\angle z = 180^o - 70^o$
$\angle z = 110^o$

26. In Fig. 83, PQ ‖ RS. Find the value of x.

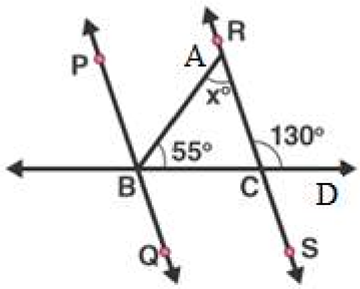

Fig. 83

Solution:

Given, linear pair,

$\angle RCD + \angle RCB = 180^o$

$\angle RCB = 180^o - 130^o$

$= 50^o$

In ΔABC,

$\angle BAC + \angle ABC + \angle BCA = 180^o$

By, angle sum property

$\angle BAC = 180^o - 55^o - 50^o$

$\angle BAC = 75^o$

27. In Fig. 84, AB ‖ CD and AE ‖ CF, ∠FCG = 90^o and ∠BAC = 120^o. Find the value of x, y and z.

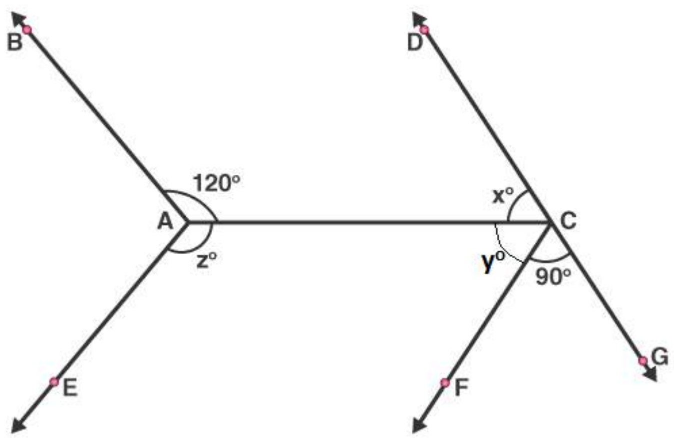

Fig. 84

Solution:

Alternate interior angle

$\angle BAC = \angle ACG = 120^o$

∠ACF + ∠FCG = 120°
So, ∠ACF = 120° – 90°
= 30°
Linear pair,
∠DCA + ∠ACG = 180°
∠x = 180° – 120°
= 60°
∠BAC + ∠BAE + ∠EAC = 360°
∠CAE = 360° – 120° – (60° + 30°)
= 150°

28. In Fig. 85, AB ∥ CD and AC ∥ BD. Find the values of x, y, z.

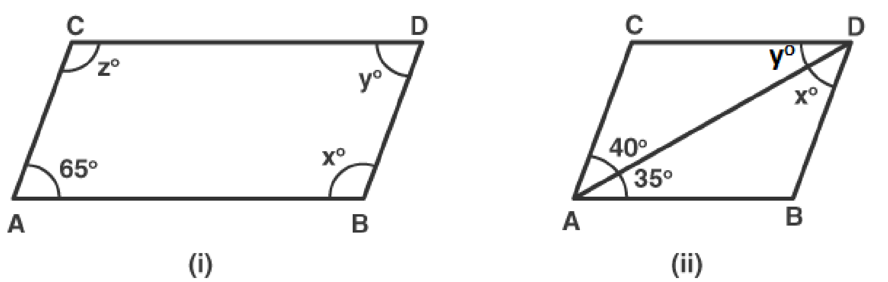

Fig. 85

Solution:
(i) Since, AC ∥ BD and CD ∥ AB, ABCD is a parallelogram
Adjacent angles of parallelogram,
∠CAB + ∠ACD = 180°
∠ACD = 180° – 65°
= 115°
Opposite angles of parallelogram,
∠CAB = ∠CDB = 65°
∠ACD = ∠DBA = 115°

(ii) Here,
AC ∥ BD and CD ∥ AB
Alternate interior angles,
∠CAD = x = 40°
∠DAB = y = 35°

29. In Fig. 86, state which lines are parallel and why?

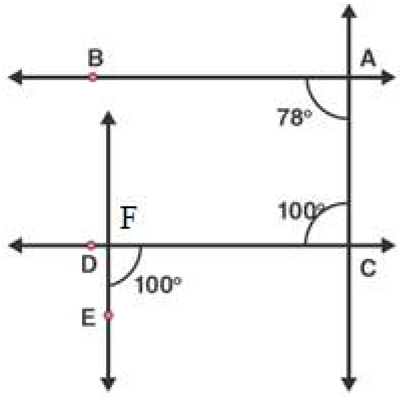

Fig. 86

Solution:

Let, F be the point of intersection of the line CD and the line passing through point E.
Here, ∠ACD and ∠CDE are alternate and equal angles.
So, ∠ACD = ∠CDE = 100°
Therefore, AC ∥ EF

30. In Fig. 87, the corresponding arms of ∠ABC and ∠DEF are parallel. If ∠ABC = 75°, find ∠DEF.

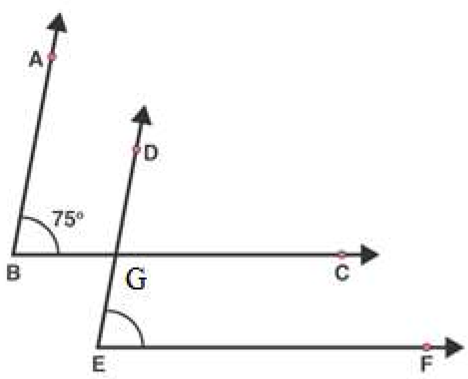

Fig. 87

Solution:

Let, G be the point of intersection of the lines BC and DE
Since, AB ∥ DE and BC ∥ EF
The corresponding angles are,
∠ABC = ∠DGC = ∠DEF = 75°