EXERCISE 15.2 PAGE NO: 15.12

1. Two angles of a triangle are of measures 105^o and 30^o. Find the measure of the third angle.

Solution:

Given two angles of a triangle are of measures 105^o and 30^o
Let the required third angle be x
We know that sum of all the angles of a triangle = 180^o
$105^o + 30^o + x = 180^o$
$135^o + x = 180^o$
$x = 180^o - 135^o$
$x = 45^o$
Therefore the third angle is 45^o

2. One of the angles of a triangle is 130^o, and the other two angles are equal. What is the measure of each of these equal angles?

Solution:

Given one of the angles of a triangle is 130^o
Also given that remaining two angles are equal
So let the second and third angle be x
We know that sum of all the angles of a triangle = 180^o
$130^o + x + x = 180^o$
$130^o + 2x = 180^o$
$2x = 180^o - 130^o$
$2x = 50^o$
$x = 50/2$
$x = 25^o$
Therefore the two other angles are 25^o each

3. The three angles of a triangle are equal to one another. What is the measure of each of the angles?

Solution:

Given that three angles of a triangle are equal to one another

So let the each angle be x
We know that sum of all the angles of a triangle = 180^o
$x + x + x = 180^o$
$3x = 180^o$
$x = 180/3$
$x = 60^o$
Therefore angle is 60^o each

4. If the angles of a triangle are in the ratio 1: 2: 3, determine three angles.

Solution:
Given angles of the triangle are in the ratio 1: 2: 3
So take first angle as x, second angle as 2x and third angle as 3x
We know that sum of all the angles of a triangle = 180^o
$x + 2x + 3x = 180^o$
$6x = 180^o$
$x = 180/6$
$x = 30^o$
$2x = 30^o \times 2 = 60^o$
$3x = 30^o \times 3 = 90^o$
Therefore the first angle is 30^o, second angle is 60^o and third angle is 90^o.

5. The angles of a triangle are $(x - 40)^o$, $(x - 20)^o$ and $(1/2 - 10)^o$. Find the value of x.

Solution:
Given the angles of a triangle are $(x - 40)^o$, $(x - 20)^o$ and $(1/2 - 10)^o$.
We know that sum of all the angles of a triangle = 180^o
$(x - 40)^o + (x - 20)^o + (1/2 - 10)^o = 180^o$
$x + x + (1/2) - 40^o - 20^o - 10^o = 180^o$
$x + x + (1/2) - 70^o = 180^o$
$(5x/2) = 180^o + 70^o$
$(5x/2) = 250^o$
$x = (2/5) \times 250^o$
$x = 100^o$
Hence the value of x is 100^o

6. The angles of a triangle are arranged in ascending order of magnitude. If the

difference between two consecutive angles is 10^o. Find the three angles.

Solution:

Given that angles of a triangle are arranged in ascending order of magnitude
Also given that difference between two consecutive angles is 10^o
Let the first angle be x
Second angle be $x + 10^o$
Third angle be $x + 10^o + 10^o$
We know that sum of all the angles of a triangle = 180^o
$x + x + 10^o + x + 10^o + 10^o = 180^o$
$3x + 30 = 180$
$3x = 180 - 30$
$3x = 150$
$x = 150/3$
$x = 50^o$
First angle is 50^o
Second angle $x + 10^o = 50 + 10 = 60^o$
Third angle $x + 10^o + 10^o = 50 + 10 + 10 = 70^o$

7. Two angles of a triangle are equal and the third angle is greater than each of those angles by 30^o. Determine all the angles of the triangle

Solution:

Given that two angles of a triangle are equal
Let the first and second angle be x
Also given that third angle is greater than each of those angles by 30^o
Therefore the third angle is greater than the first and second by $30^o = x + 30^o$
The first and the second angles are equal
We know that sum of all the angles of a triangle = 180^o
$x + x + x + 30^o = 180^o$
$3x + 30 = 180$
$3x = 180 - 30$
$3x = 150$
$x = 150/3$
$x = 50^o$
Third angle = $x + 30^o = 50^o + 30^o = 80^o$
The first and the second angle is 50^o and the third angle is 80^o.

8. If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right triangle.

Solution:

Given that one angle of a triangle is equal to the sum of the other two
Let the measure of angles be x, y, z
Therefore we can write above statement as $x = y + z$
$x + y + z = 180^{o}$
Substituting the above value we get
$x + x = 180^{o}$
$2x = 180^{o}$
$x = 180/2$
$x = 90^{o}$
If one angle is 90^{o} then the given triangle is a right angled triangle

9. If each angle of a triangle is less than the sum of the other two, show that the triangle is acute angled.

Solution:

Given that each angle of a triangle is less than the sum of the other two
Let the measure of angles be x, y and z
From the above statement we can write as
$x > y + z$
$y < x + z$
$z < x + y$
Therefore triangle is an acute triangle

10. In each of the following, the measures of three angles are given. State in which cases the angles can possibly be those of a triangle:
(i) $63^{o}, 37^{o}, 80^{o}$
(ii) $45^{o}, 61^{o}, 73^{o}$
(iii) $59^{o}, 72^{o}, 61^{o}$
(iv) $45^{o}, 45^{o}, 90^{o}$
(v) $30^{o}, 20^{o}, 125^{o}$

Solution:

(i) $63^o + 37^o + 80^o = 180^o$
Angles form a triangle

(ii) $45^o, 61^o, 73^o$ is not equal to 180^o
Therefore not a triangle

(iii) $59^o, 72^o, 61^o$ is not equal to 180^0
Therefore not a triangle

(iv) $45^o + 45^o + 90^o = 180^o$
Angles form a triangle

(v) $30^o, 20^o, 125^o$ is not equal to 180^o
Therefore not a triangle

11. The angles of a triangle are in the ratio 3: 4: 5. Find the smallest angle

Solution:
Given that angles of a triangle are in the ratio: 3: 4: 5
Therefore let the measure of the angles be 3x, 4x, 5x
We know that sum of the angles of a triangle $=180^o$
$3x + 4x + 5x = 180^o$
$12x = 180^o$
$x = 180/12$
$x = 15^o$
Smallest angle = 3x
$= 3 \times 15^o$
$= 45^o$
Therefore smallest angle $= 45^o$

12. Two acute angles of a right triangle are equal. Find the two angles.

Solution:
Given that acute angles of a right angled triangle are equal
We know that Right triangle: whose one of the angle is a right angle
Let the measure of angle be x, x, 90^o
$x + x + 90^o = 180^o$

$2x = 90^o$
$x = 90/2$
$x = 45^o$
The two angles are 45^o and 45^o

13. One angle of a triangle is greater than the sum of the other two. What can you say about the measure of this angle? What type of a triangle is this?

Solution:
Given one angle of a triangle is greater than the sum of the other two
Let the measure of the angles be x, y, z
From the question we can write as
$x > y + z$ or
$y > x + z$ or
$z > x + y$
x or y or z > 90^o which is obtuse
Therefore triangle is an obtuse angle

14. In the six cornered figure, (fig. 20), AC, AD and AE are joined. Find ∠FAB + ∠ABC + ∠BCD + ∠CDE + ∠DEF + ∠EFA.

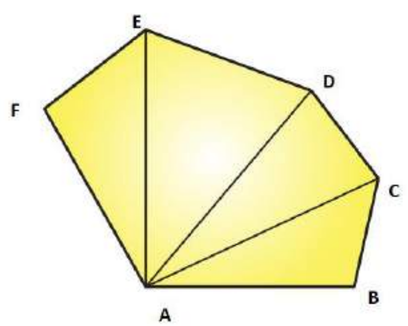


Fig. 20

Solution:
We know that sum of the angles of a triangle is 180^o
Therefore in ΔABC, we have
$\angle CAB + \angle ABC + \angle BCA = 180^o$ …….. (i)
In Δ ACD, we have
$\angle DAC + \angle ACD + \angle CDA = 180^o$ …….. (ii)
In ΔADE, we have
$\angle EAD + \angle ADE + \angle DEA = 180^o$ ………. (iii)

In △AEF, we have

∠FAE + ∠AEF + ∠EFA = 180° .... (iv)

Adding (i), (ii), (iii), (iv) we get

∠CAB + ∠ABC + ∠BCA + ∠DAC + ∠ACD + ∠CDA + ∠EAD + ∠ADE + ∠DEA + ∠FAE + ∠AEF +∠EFA = 720°

Therefore ∠FAB + ∠ABC + ∠BCD + ∠CDE + ∠DEF + ∠EFA = 720°

15. Find x, y, z (whichever is required) from the figures (Fig. 21) given below:

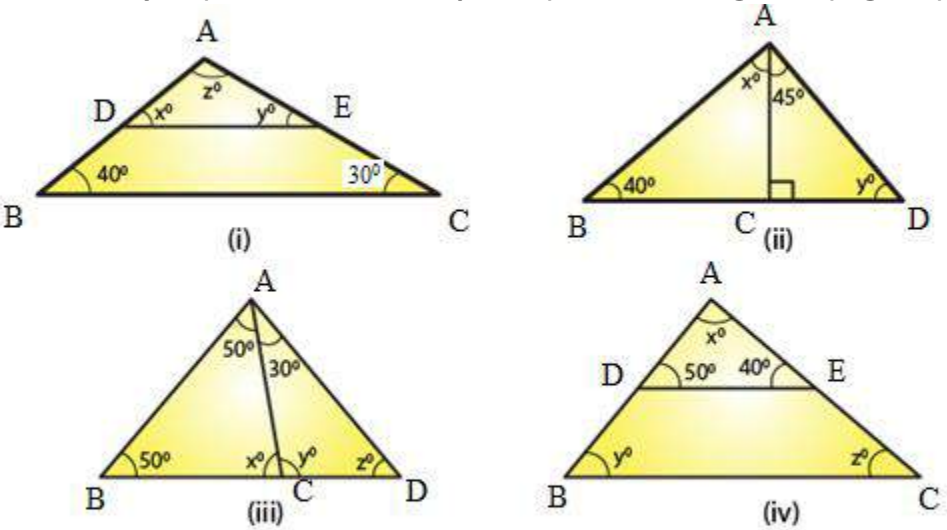


Fig.21

Solution:

(i) In △ABC and △ADE we have,

∠ ADE = ∠ ABC [corresponding angles]

x = 40°

∠AED = ∠ACB (corresponding angles)

y = 30°

We know that the sum of all the three angles of a triangle is equal to 180°

x + y + z = 180° (Angles of △ADE)

Which means: 40° + 30° + z = 180°

z = 180° - 70°

z = 110°

Therefore, we can conclude that the three angles of the given triangle are 40°, 30° and 110°

(ii) We can see that in △ADC, ∠ADC is equal to 90°.

(△ADC is a right triangle)
We also know that the sum of all the angles of a triangle is equal to 180^o.
Which means: $45^o + 90^o + y = 180^o$ (Sum of the angles of △ADC)
$135^o + y = 180^o$
$y = 180^o – 135^o$.
$y = 45^o$.
We can also say that in △ ABC, ∠ABC + ∠ACB + ∠BAC is equal to 180^o.
(Sum of the angles of △ABC)
$40^o + y + (x + 45^o) = 180^o$
$40^o + 45^o + x + 45^o = 180^o$ $(y = 45^o)$
$x = 180^o –130^o$
$x = 50^o$
Therefore, we can say that the required angles are 45^o and 50^o.

(iii) We know that the sum of all the angles of a triangle is equal to 180^o.
Therefore, for △ABD:
∠ABD +∠ADB + ∠BAD = 180° (Sum of the angles of △ABD)
$50^o + x + 50^o = 180^o$
$100^o + x = 180^o$
$x = 180^o – 100^o$
$x = 80^o$
For △ ABC:
∠ABC + ∠ACB + ∠BAC = 180^o (Sum of the angles of △ABC)
$50^o + z + (50^o + 30^o) = 180^o$
$50^o + z + 50^o + 30^o = 180^o$
$z = 180^o – 130^o$
$z = 50^o$
Using the same argument for △ADC:
∠ADC + ∠ACD + ∠DAC = 180^o (Sum of the angles of △ADC)
$y + z + 30^o = 180^o$
$y + 50^o + 30^o = 180^o$ $(z = 50^o)$
$y = 180^o – 80^o$
$y = 100^0$
Therefore, we can conclude that the required angles are 80^o, 50^o and 100^o.

(iv) In △ABC and △ADE we have:
∠ADE = ∠ABC (Corresponding angles)

$y = 50^{\circ}$
Also, $\angle AED = \angle ACB$ (Corresponding angles)
$z = 40^{\circ}$
We know that the sum of all the three angles of a triangle is equal to 180°.
We can write as $x + 50^{\circ} + 40^{\circ} = 180^{\circ}$ (Angles of $\triangle ADE$)
$x = 180^{\circ} - 90^{\circ}$
$x = 90^{\circ}$
Therefore, we can conclude that the required angles are 50°, 40° and 90°.

16. If one angle of a triangle is 60° and the other two angles are in the ratio 1: 2, find the angles.

Solution:
Given that one of the angles of the given triangle is 60°.
Also given that the other two angles of the triangle are in the ratio 1: 2.
Let one of the other two angles be x.
Therefore, the second one will be 2x.
We know that the sum of all the three angles of a triangle is equal to 180°.
$60^{\circ} + x + 2x = 180^{\circ}$
$3x = 180^{\circ} - 60^{\circ}$
$3x = 120^{\circ}$
$x = 120^{\circ}/3$
$x = 40^{\circ}$
$2x = 2 \times 40^{\circ}$
$2x = 80^{\circ}$
Hence, we can conclude that the required angles are 40° and 80°.

17. If one angle of a triangle is 100° and the other two angles are in the ratio 2: 3. Find the angles.

Solution:
Given that one of the angles of the given triangle is 100°.
Also given that the other two angles are in the ratio 2: 3.
Let one of the other two angle be 2x.
Therefore, the second angle will be 3x.
We know that the sum of all three angles of a triangle is 180°.
$100^{\circ} + 2x + 3x = 180^{\circ}$

$5x = 180^o – 100^o$
$5x = 80^o$
$x = 80/5$
$x = 16$
$2x = 2 \times 16$
$2x = 32^o$
$3x = 3\times16$
$3x = 48^o$
Thus, the required angles are 32^o and 48^o.

18. In △ABC, if 3∠A = 4∠B = 6∠C, calculate the angles.

Solution:
We know that for the given triangle, $3\angle A = 6\angle C$
$\angle A = 2\angle C$ ……. (i)
We also know that for the same triangle, $4\angle B = 6\angle C$
$\angle B = (6/4) \angle C$ …… (ii)
We know that the sum of all three angles of a triangle is 180^o.
Therefore, we can say that:
$\angle A + \angle B + \angle C = 180^o$ (Angles of △ABC)…… (iii)
On putting the values of ∠A and ∠B in equation (iii), we get:
$2\angle C + (6/4) \angle C + \angle C = 180^o$
$(18/4) \angle C = 180^o$
$\angle C = 40^o$
From equation (i), we have:
$\angle A = 2\angle C = 2 \times 40$
$\angle A = 80^o$
From equation (ii), we have:
$\angle B = (6/4) \angle C = (6/4) \times 40^o$
$\angle B = 60^o$
$\angle A = 80^o$, $\angle B = 60^o$, $\angle C = 40^o$
Therefore, the three angles of the given triangle are 80^o, 60^o, and 40^o.

19. Is it possible to have a triangle, in which
(i) Two of the angles are right?
(ii) Two of the angles are obtuse?
(iii) Two of the angles are acute?

(iv) Each angle is less than 60^o?
(v) Each angle is greater than 60^o?
(vi) Each angle is equal to 60^o?

Solution:
(i) No, because if there are two right angles in a triangle, then the third angle of the triangle must be zero, which is not possible.

(ii) No, because as we know that the sum of all three angles of a triangle is always 180^o. If there are two obtuse angles, then their sum will be more than 180^o, which is not possible in case of a triangle.

(iii) Yes, in right triangles and acute triangles, it is possible to have two acute angles.

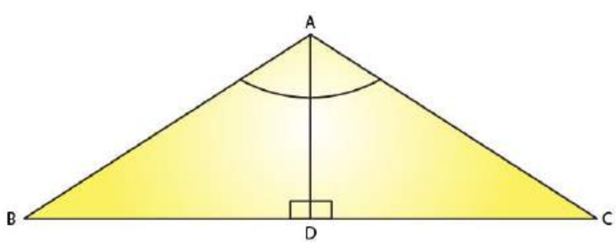
(iv) No, because if each angle is less than 60^o, then the sum of all three angles will be less than 180^o, which is not possible in case of a triangle.

(v) No, because if each angle is greater than 60^0, then the sum of all three angles will be greater than 180^0, which is not possible.

(vi) Yes, if each angle of the triangle is equal to 60^0, then the sum of all three angles will be 180^0, which is possible in case of a triangle.

20. In$\triangle$ ABC, $\angle$A = 100^o, AD bisects $\angle$A and AD $\perp$ BC. Find $\angle$B

Solution:

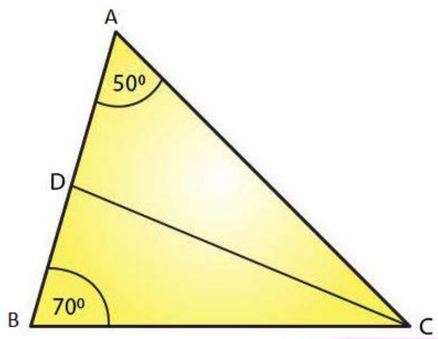


Given that in $\triangle$ABC, $\angle$A = 100^o
Also given that AD $\perp$ BC
Consider $\triangle$ABD
$\angle$BAD = 100/2   (AD bisects $\angle$A)
$\angle$BAD = 50^o

$\angle ADB = 90^o$ (AD perpendicular to BC)
We know that the sum of all three angles of a triangle is 180^o.
Thus,
$\angle ABD + \angle BAD + \angle ADB = 180^o$ (Sum of angles of $\triangle ABD$)
Or,
$\angle ABD + 50^o + 90^o = 180^o$
$\angle ABD = 180^o – 140^o$
$\angle ABD = 40^o$

21. In $\triangle ABC$, $\angle A = 50^o$, $\angle B = 70^o$ and bisector of $\angle C$ meets AB in D. Find the angles of the triangles ADC and BDC

Solution:

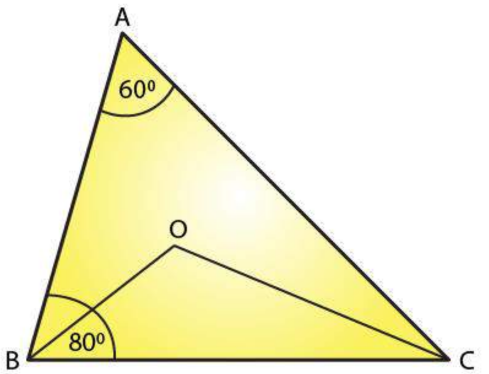


We know that the sum of all three angles of a triangle is equal to 180^o.
Therefore, for the given $\triangle ABC$, we can say that:
$\angle A + \angle B + \angle C = 180^o$ (Sum of angles of $\triangle ABC$)
$50^o + 70^o + \angle C = 180^o$
$\angle C = 180^o – 120^o$
$\angle C = 60^o$
$\angle ACD = \angle BCD = 2\angle C$ (CD bisects $\angle C$ and meets AB in D.)
$\angle ACD = \angle BCD = 60/2 = 30^o$
Using the same logic for the given $\triangle ACD$, we can say that:
$\angle DAC + \angle ACD + \angle ADC = 180^o$
$50^o + 30^o + \angle ADC = 180^o$
$\angle ADC = 180^o – 80^o$
$\angle ADC = 100^o$
If we use the same logic for the given $\triangle BCD$, we can say that
$\angle DBC + \angle BCD + \angle BDC = 180^o$
$70^o + 30^o + \angle BDC = 180^o$

$\angle BDC = 180^{o} - 100^{o}$
$\angle BDC = 80^{o}$
Thus,
For $\triangle ADC$: $\angle A = 50^{o}$, $\angle D = 100^{o}$ $\angle C = 30^{o}$
$\triangle BDC$: $\angle B = 70^{o}$, $\angle D = 80^{o}$ $\angle C = 30^{o}$

22. In ΔABC, $\angle A = 60^{o}$, $\angle B = 80^{o}$, and the bisectors of $\angle B$ and $\angle C$, meet at O. Find
(i) $\angle C$
(ii) $\angle BOC$

Solution:

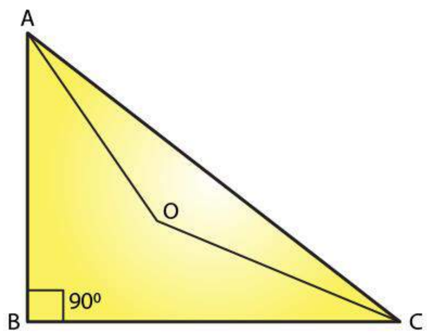


(i) We know that the sum of all three angles of a triangle is 180^{o}.
Hence, for $\triangle ABC$, we can say that:
$\angle A + \angle B + \angle C = 180^{o}$ (Sum of angles of ΔABC)
$60^{o} + 80^{o} + \angle C = 180^{o}$.
$\angle C = 180^{o} - 140^{o}$
$\angle C = 40^{o}$.

(ii)For $\triangle OBC$,
$\angle OBC = \angle B/2 = 80/2$ (OB bisects $\angle B$)
$\angle OBC = 40^{o}$
$\angle OCB = \angle C/2 = 40/2$ (OC bisects $\angle C$)
$\angle OCB = 20^{o}$
If we apply the above logic to this triangle, we can say that:
$\angle OCB + \angle OBC + \angle BOC = 180^{o}$ (Sum of angles of $\triangle OBC$)
$20^{o} + 40^{o} + \angle BOC = 180^{o}$
$\angle BOC = 180^{o} - 60^{o}$
$\angle BOC = 120^{o}$

23. The bisectors of the acute angles of a right triangle meet at O. Find the angle at O between the two bisectors.

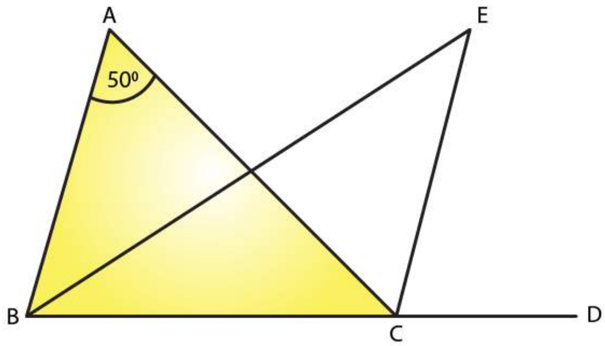
Solution:



Given bisectors of the acute angles of a right triangle meet at O
We know that the sum of all three angles of a triangle is 180^o.
Hence, for △ABC, we can say that:
$\angle A + \angle B + \angle C = 180^o$
$\angle A + 90^o + \angle C = 180^o$
$\angle A + \angle C = 180^o - 90^o$
$\angle A + \angle C = 90^o$
For △OAC:
$\angle OAC = \angle A/2$ (OA bisects ∠A)
$\angle OCA = \angle C/2$ (OC bisects ∠C)
On applying the above logic to △OAC, we get
$\angle AOC + \angle OAC + \angle OCA = 180^o$ (Sum of angles of △AOC)
$\angle AOC + \angle A/2 + \angle C/2 = 180^o$
$\angle AOC + (\angle A + \angle C)/2 = 180^o$
$\angle AOC + 90/2 = 180^o$
$\angle AOC = 180^o - 45^o$
$\angle AOC = 135^o$

24. In △ABC, ∠A = 50° and BC is produced to a point D. The bisectors of ∠ABC and ∠ACD meet at E. Find ∠E.

Solution:



In the given triangle,

∠ACD = ∠A + ∠B. (Exterior angle is equal to the sum of two opposite interior angles.)

We know that the sum of all three angles of a triangle is 180^o.

Therefore, for the given triangle, we know that the sum of the angles = 180^o

∠ABC + ∠BCA + ∠CAB = 180^o

∠A + ∠B + ∠BCA = 180^o

∠BCA = 180^o – (∠A + ∠B)

But we know that EC bisects ∠ACD

Therefore ∠ECA = ∠ACD/2

∠ECA = (∠A + ∠B)/2 [∠ACD = (∠A + ∠B)]

But EB bisects ∠ABC

∠EBC = ∠ABC/2 = ∠B/2

∠EBC = ∠ECA + ∠BCA

∠EBC = (∠A + ∠B)/2 + 180^o - (∠A + ∠B)

If we use same steps for △EBC, then we get,

∠B/2 + (∠A + ∠B)/2 + 180^o - (∠A + ∠B) + ∠BEC = 180^o

∠BEC = ∠A + ∠B - (∠A + ∠B)/2 - ∠B/2

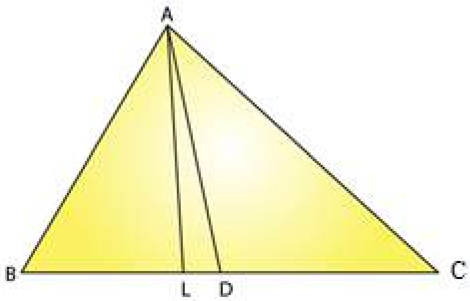
∠BEC = ∠A/2

∠BEC = 50^o/2

= 25^o

25. In ΔABC, ∠B = 60°, ∠C = 40°, AL ⊥ BC and AD bisects ∠A such that L and D lie on side BC. Find ∠LAD

Solution:



We know that the sum of all angles of a triangle is 180°

Consider △ABC, we can write as

$\angle A + \angle B + \angle C = 180^o$

$\angle A + 60^o + 40^o = 180^o$

$\angle A = 80^0$

But we know that ∠DAC bisects ∠A

$\angle DAC = \angle A/2$

$\angle DAC = 80^o/2$

If we apply same steps for the △ADC, we get

We know that the sum of all angles of a triangle is 180°

$\angle ADC + \angle DCA + \angle DAC = 180^o$

$\angle ADC + 40^o + 40^o = 180^o$

$\angle ADC = 180^o - 80^o = 100^0$

We know that exterior angle is equal to the sum of two interior opposite angles

Therefore we have

$\angle ADC = \angle ALD + \angle LAD$

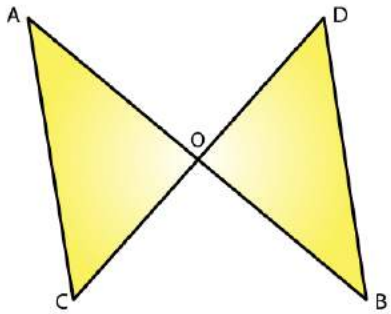
But here AL perpendicular to BC

$100^o = 90^o + \angle LAD$

$\angle LAD = 10^o$

26. Line segments AB and CD intersect at O such that AC ∥ DB. It ∠CAB = 35^o and ∠CDB = 55^o. Find ∠BOD.

Solution:



We know that AC parallel to BD and AB cuts AC and BD at A and B, respectively.
∠CAB = ∠DBA (Alternate interior angles)
∠DBA = 35^o
We also know that the sum of all three angles of a triangle is 180^o.
Hence, for △OBD, we can say that:
∠DBO + ∠ODB + ∠BOD = 180°
$35^o + 55^o$ + ∠BOD = 180^o (∠DBO = ∠DBA and ∠ODB = ∠CDB)
∠BOD = $180^o - 90^o$
∠BOD = 90^o

27. In Fig. 22, ΔABC is right angled at A, Q and R are points on line BC and P is a point such that QP ∥ AC and RP ∥ AB. Find ∠P

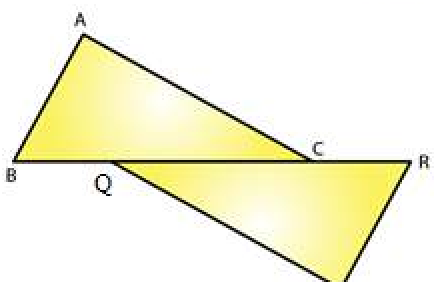


Fig. 22

Solution:
In the given triangle, AC parallel to QP and BR cuts AC and QP at C and Q, respectively.
∠QCA = ∠CQP (Alternate interior angles)
Because RP parallel to AB and BR cuts AB and RP at B and R, respectively,
∠ABC = ∠PRQ (alternate interior angles).
We know that the sum of all three angles of a triangle is 180^o.
Hence, for ΔABC, we can say that:
∠ABC + ∠ACB + ∠BAC = 180^o
∠ABC + ∠ACB + 90^o = 180^o (Right angled at A)
∠ABC + ∠ACB = 90^o

Using the same logic for $\triangle PQR$, we can say that:
$\angle PQR + \angle PRQ + \angle QPR = 180^o$
$\angle ABC + \angle ACB + \angle QPR = 180^o$ ($\angle ACB = \angle PQR$ and $\angle ABC = \angle PRQ$)
Or,
$90^o + \angle QPR = 180^o$ ($\angle ABC + \angle ACB = 90^o$)
$\angle QPR = 90^o$