EXERCISE 15.3 PAGE NO: 15.19

1. In Fig. 35, ∠CBX is an exterior angle of ΔABC at B. Name
(i) The interior adjacent angle
(ii) The interior opposite angles to exterior ∠CBX
Also, name the interior opposite angles to an exterior angle at A.

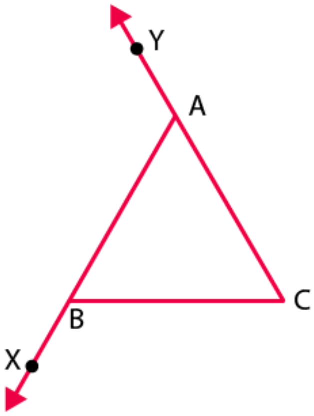


Fig. 35

Solution:

(i) The interior adjacent angle is ∠ABC

(ii) The interior opposite angles to exterior ∠CBX is ∠BAC and ∠ACB

Also the interior angles opposite to exterior ∠BAY are ∠ABC and ∠ACB

2. In the fig. 36, two of the angles are indicated. What are the measures of ∠ACX and ∠ACB?

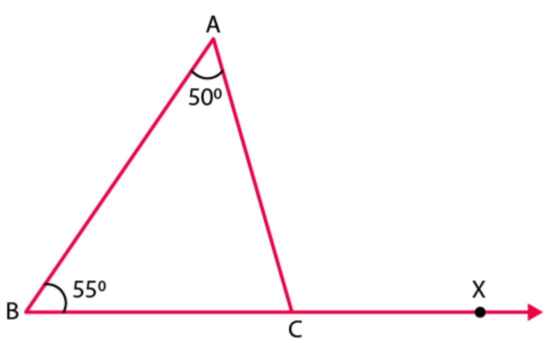


Fig. 36

Solution:

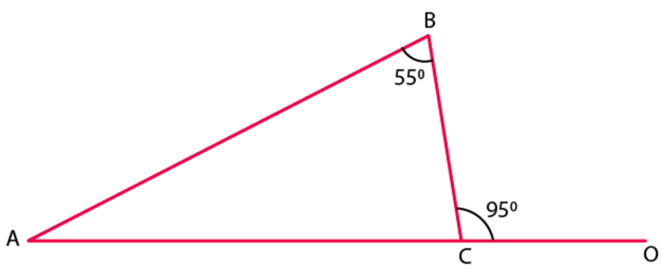
Given that in △ABC, ∠A = 50^{o} and ∠B = 55^{o}

We know that the sum of angles in a triangle is 180^{o}

Therefore we have
$\angle A + \angle B + \angle C = 180^o$
$50^o + 55^o + \angle C = 180^o$
$\angle C = 75^o$
$\angle ACB = 75^o$
$\angle ACX = 180^o - \angle ACB = 180^o - 75^o = 105^o$

3. In a triangle, an exterior angle at a vertex is 95^o and its one of the interior opposite angles is 55^o. Find all the angles of the triangle.

Solution:



We know that the sum of interior opposite angles is equal to the exterior angle.
Hence, for the given triangle, we can say that:
$\angle ABC + \angle BAC = \angle BCO$
$55^o + \angle BAC = 95^o$
$\angle BAC = 95^o - 55^o$
$\angle BAC = 40^o$
We also know that the sum of all angles of a triangle is 180^o.
Hence, for the given $\triangle ABC$, we can say that:
$\angle ABC + \angle BAC + \angle BCA = 180^o$
$55^o + 40^o + \angle BCA = 180^o$
$\angle BCA = 180^o - 95^o$
$\angle BCA = 85^o$

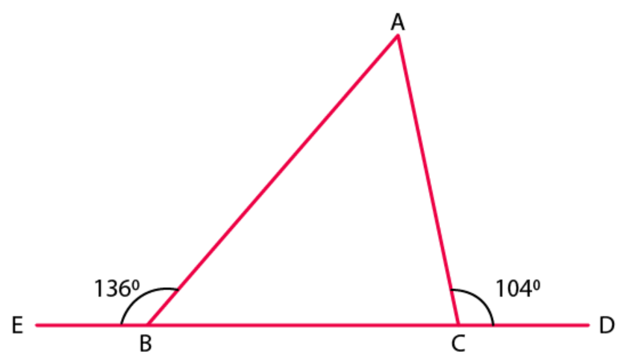
4. One of the exterior angles of a triangle is 80^o, and the interior opposite angles are equal to each other. What is the measure of each of these two angles?

Solution:
Let us assume that A and B are the two interior opposite angles.
We know that $\angle A$ is equal to $\angle B$.

We also know that the sum of interior opposite angles is equal to the exterior angle.
Therefore from the figure we have,
$\angle A + \angle B = 80^o$
$\angle A + \angle A = 80^o$ (because $\angle A = \angle B$)
$2\angle A = 80^o$
$\angle A = 80/2 = 40^o$
$\angle A = \angle B = 40^o$
Thus, each of the required angles is of 40^o.

5. The exterior angles, obtained on producing the base of a triangle both ways are 104^o and 136^o. Find all the angles of the triangle.

Solution:



In the given figure, $\angle ABE$ and $\angle ABC$ form a linear pair.
$\angle ABE + \angle ABC = 180^o$
$\angle ABC = 180^o - 136^o$
$\angle ABC = 44^o$
We can also see that $\angle ACD$ and $\angle ACB$ form a linear pair.
$\angle ACD + \angle ACB = 180^o$
$\angle ACB = 180^o - 104^o$
$\angle ACB = 76^o$
We know that the sum of interior opposite angles is equal to the exterior angle.
Therefore, we can write as
$\angle BAC + \angle ABC = 104^o$
$\angle BAC = 104^o - 44^o = 60^o$
Thus,
$\angle ACE = 76^o$ and $\angle BAC = 60^o$

6. In Fig. 37, the sides BC, CA and BA of a △ABC have been produced to D, E and F respectively. If ∠ACD = 105° and ∠EAF = 45°; find all the angles of the △ABC.

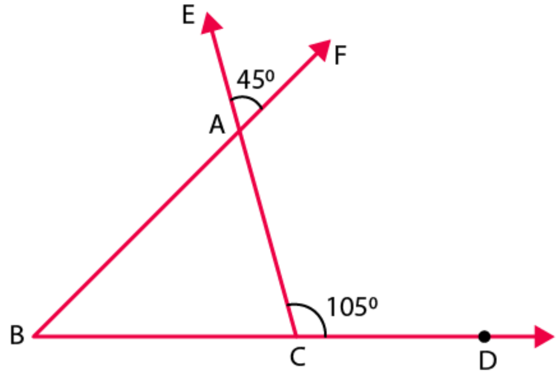


Fig. 37

Solution:

In a △ABC, ∠BAC and ∠EAF are vertically opposite angles.

Hence, we can write as

∠BAC = ∠EAF = 45°

Considering the exterior angle property, we have

∠BAC + ∠ABC = ∠ACD = 105°

On rearranging we get

∠ABC = 105°– 45° = 60°

We know that the sum of angles in a triangle is 180°

∠ABC + ∠ACB +∠BAC = 180°

∠ACB = 75°

Therefore, the angles are 45°, 60° and 75°.

7. In Fig. 38, AC perpendicular to CE and C ∠A: ∠B: ∠C= 3: 2: 1. Find the value of ∠ECD.

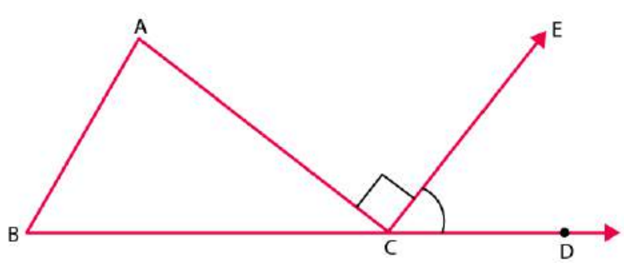


Fig. 38

Solution:

In the given triangle, the angles are in the ratio 3: 2: 1.

Let the angles of the triangle be 3x, 2x and x.

We know that sum of angles in a triangle is 180°

$3x + 2x + x = 180^o$
$6x = 180^o$
$x = 30^o$
Also, $\angle ACB + \angle ACE + \angle ECD = 180^o$
$x + 90^o + \angle ECD = 180^o$ ($\angle ACE = 90^o$)
We know that $x = 30^o$
Therefore
$\angle ECD = 60^o$

8. A student when asked to measure two exterior angles of △ABC observed that the exterior angles at A and B are of 103^o and 74^o respectively. Is this possible? Why or why not?
Solution:
We know that sum of internal and external angle is equal to 180^o
Internal angle at A + External angle at A = 180^o
Internal angle at A + 103^o = 180^o
Internal angle at A = 77^o
Internal angle at B + External angle at B = 180^o
Internal angle at B + 74^o = 180^o
Internal angle at B = 106^o
Sum of internal angles at A and B = $77^o + 106^o = 183^o$
It means that the sum of internal angles at A and B is greater than 180^o, which cannot be possible.

9. In Fig.39, AD and CF are respectively perpendiculars to sides BC and AB of △ABC. If $\angle FCD = 50^o$, find $\angle BAD$

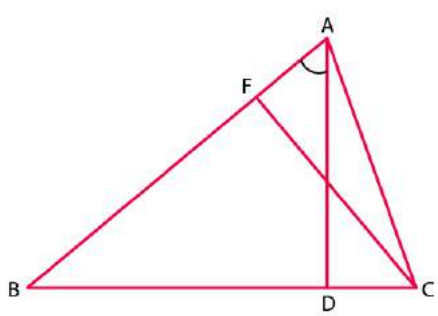


Fig. 39

Solution:
We know that the sum of all angles of a triangle is 180^o
Therefore, for the given △FCB, we have

$\angle FCB + \angle CBF + \angle BFC = 180^o$
$50^o + \angle CBF + 90^o = 180^o$
$\angle CBF = 180^o - 50^o - 90^o = 40^o$
Using the above steps for $\triangle ABD$, we can say that:
$\angle ABD + \angle BDA + \angle BAD = 180^o$
$\angle BAD = 180^o - 90^o - 40^o = 50^o$

10. In Fig.40, measures of some angles are indicated. Find the value of x.

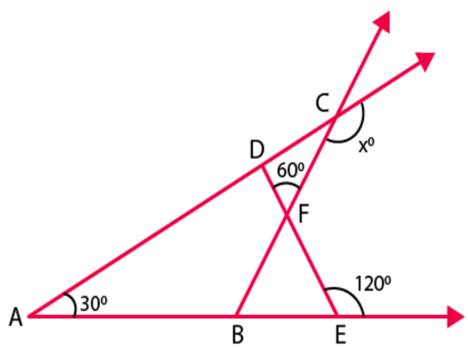


Fig. 40

Solution:
We know that the sum of the angles of a triangle is 180^o
From the figure we have,
$\angle AED + 120^o = 180^o$ (Linear pair)
$\angle AED = 180^o - 120^o = 60^o$
We know that the sum of all angles of a triangle is 180^o.
Therefore, for $\triangle ADE$, we have
$\angle ADE + \angle AED + \angle DAE = 180^o$
$60^o + \angle ADE + 30^o = 180^o$
$\angle ADE = 180^o - 60^o - 30^o = 90^o$
From the given figure, we have
$\angle FDC + 90^o = 180^o$ (Linear pair)
$\angle FDC = 180^o - 90^o = 90^o$
Using the same steps for $\triangle CDF$, we get
$\angle CDF + \angle DCF + \angle DFC = 180^o$
$90^o + \angle DCF + 60^o = 180^o$
$\angle DCF = 180^o - 60^o - 90^o = 30^o$
Again from the figure we have
$\angle DCF + x = 180^o$ (Linear pair)
$30^o + x = 180^o$

$x = 180^o – 30^o = 150^o$

11. In Fig. 41, ABC is a right triangle right angled at A. D lies on BA produced and DE perpendicular to BC intersecting AC at F. If ∠AFE = 130°, find
(i) ∠BDE
(ii) ∠BCA
(iii) ∠ABC

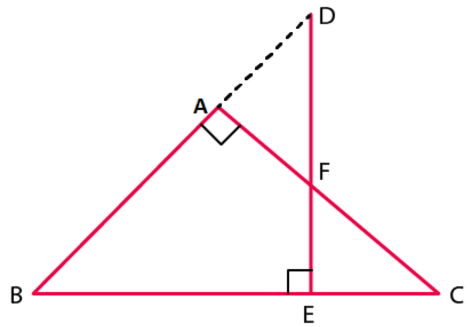


Fig. 41

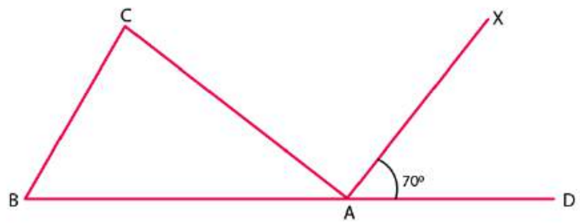
Solution:
(i) Here,
∠BAF + ∠FAD = 180° (Linear pair)
∠FAD = 180° - ∠BAF = 180° – 90° = 90°
Also from the figure,
∠AFE = ∠ADF + ∠FAD (Exterior angle property)
∠ADF + 90° = 130°
∠ADF = 130° – 90° = 40°
∠BDE = 40°

(ii) We know that the sum of all the angles of a triangle is 180°.
Therefore, for △BDE, we have
∠BDE + ∠BED + ∠DBE = 180°
∠DBE = 180° – ∠BDE - ∠BED
∠DBE = 180° – 40° – 90° = 50° …. Equation (i)
Again from the figure we have,
∠FAD = ∠ABC + ∠ACB (Exterior angle property)
90° = 50° + ∠ACB
∠ACB = 90° – 50° = 40°

(iii) From equation we have
∠ABC = ∠DBE = 50°

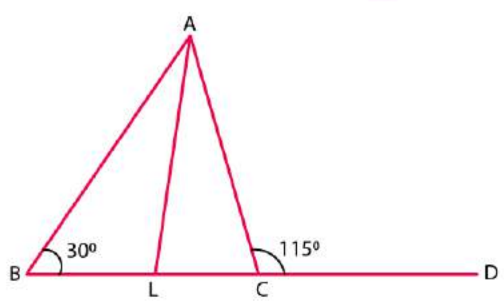
12. ABC is a triangle in which ∠B = ∠C and ray AX bisects the exterior angle DAC. If ∠DAX = 70^o. Find ∠ACB.

Solution:



Given that ABC is a triangle in which ∠B = ∠C
Also given that AX bisects the exterior angle DAC
∠CAX = ∠DAX (AX bisects ∠CAD)
∠CAX =70^o [given]
∠CAX +∠DAX + ∠CAB =180^o
70^o+ 70^o + ∠CAB =180^o
∠CAB =180^o –140^o
∠CAB =40^o
∠ACB + ∠CBA + ∠CAB = 180^o (Sum of the angles of ΔABC)
∠ACB + ∠ACB+ 40^o = 180^o (∠C = ∠B)
2∠ACB = 180^o – 40^o
∠ACB = 140/2
∠ACB = 70^o

13. The side BC of ΔABC is produced to a point D. The bisector of ∠A meets side BC in L. If ∠ABC= 30^o and ∠ACD = 115^o, find ∠ALC

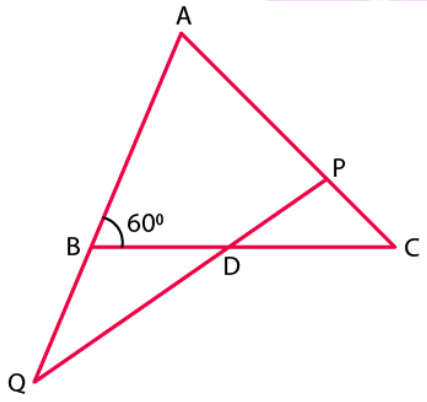


Solution:
Given that ∠ABC= 30^o and ∠ACD = 115^o
From the figure, we have

∠ACD and ∠ACL make a linear pair.
$\angle ACD + \angle ACB = 180^\circ$
$115^\circ + \angle ACB = 180^\circ$
$\angle ACB = 180^\circ - 115^\circ$
$\angle ACB = 65^\circ$
We know that the sum of all angles of a triangle is 180°.
Therefore, for△ ABC, we have
$\angle ABC + \angle BAC + \angle ACB = 180^\circ$
$30^\circ + \angle BAC + 65^\circ = 180^\circ$
$\angle BAC = 85^\circ$
$\angle LAC = \angle BAC/2 = 85/2$
Using the same steps for △ALC, we get
$\angle ALC + \angle LAC + \angle ACL = 180^\circ$
$\angle ALC + 85/2 + 65^\circ = 180^\circ$
We know that ∠ACL = ∠ACB
$\angle ALC = 180^\circ - 85/2 - 65^\circ$
$\angle ALC = 72\frac{1}{2}^\circ$

14. D is a point on the side BC of △ABC. A line PDQ through D, meets side AC in P and AB produced at Q. If ∠A = 80°, ∠ABC = 60° and ∠PDC = 15°, find
(i) ∠AQD
(ii) ∠APD

Solution:



From the figure we have
∠ABD and ∠QBD form a linear pair.
$\angle ABC + \angle QBC = 180^\circ$
$60^\circ + \angle QBC = 180^\circ$

$\angle QBC = 120^o$
$\angle PDC = \angle BDQ$ (Vertically opposite angles)
$\angle BDQ = 15^o$
(i) In $\triangle QBD$:
$\angle QBD + \angle QDB + \angle BQD = 180^o$ (Sum of angles of $\triangle QBD$)
$120^o + 15^o + \angle BQD = 180^o$
$\angle BQD = 180^o - 135^o$
$\angle BQD = 45^o$
$\angle AQD = \angle BQD = 45^o$
(ii) In $\triangle AQP$:
$\angle QAP + \angle AQP + \angle APQ = 180^o$ (Sum of angles of $\triangle AQP$)
$80^o + 45^o + \angle APQ = 180^o$
$\angle APQ = 55^o$
$\angle APD = \angle APQ = 55^o$

15. Explain the concept of interior and exterior angles and in each of the figures given below. Find x and y (Fig. 42)

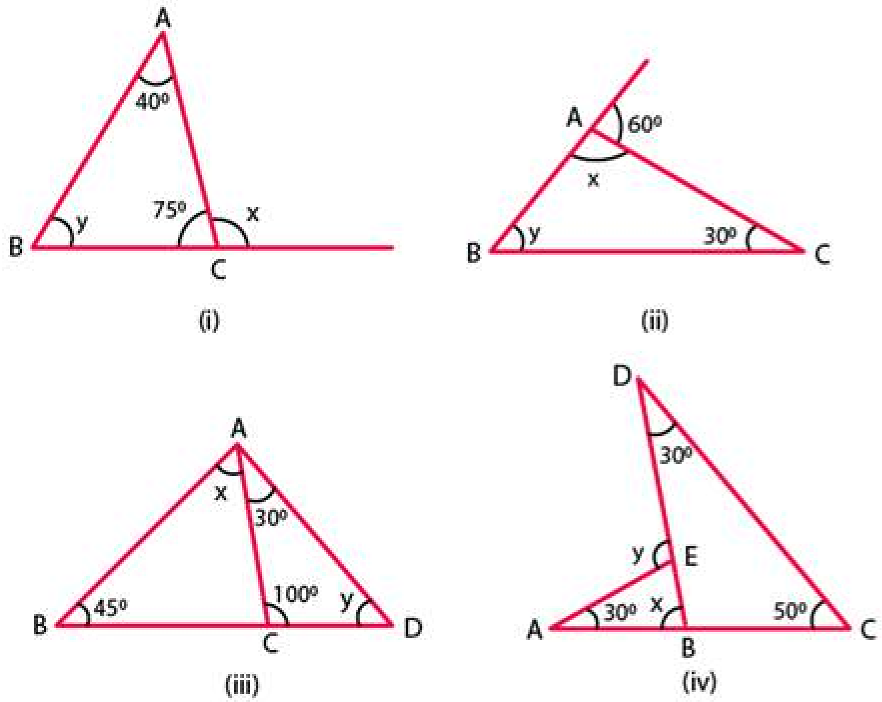


Fig. 42

Solution:
The interior angles of a triangle are the three angle elements inside the triangle.
The exterior angles are formed by extending the sides of a triangle, and if the side of a

triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.
Using these definitions, we will obtain the values of x and y.
(i) From the given figure, we have
$\angle ACB + x = 180^o$ (Linear pair)
$75^o + x = 180^o$
$x = 105^o$
We know that the sum of all angles of a triangle is 180^o
Therefore, for $\triangle ABC$, we can say that:
$\angle BAC + \angle ABC + \angle ACB = 180^o$
$40^o + y + 75^o = 180^o$
$y = 65^o$

(ii) From the figure, we have
$x + 80^o = 180^o$ (Linear pair)
$x = 100^o$
In $\triangle ABC$, we have
We also know that the sum of angles of a triangle is 180^o
$x + y + 30^o = 180^o$
$100^o + 30^o + y = 180^o$
$y = 50^o$

(iii) We know that the sum of all angles of a triangle is 180°.
Therefore, for $\triangle ACD$, we have
$30^o + 100^o + y = 180^o$
$y = 50^o$
Again from the figure we can write as
$\angle ACB + 100° = 180°$
$\angle ACB = 80^o$
Using the above rule for $\triangle ACB$, we can say that:
$x + 45^o + 80^o = 180^o$
$x = 55^o$

(iv) We know that the sum of all angles of a triangle is 180^o.
Therefore, for $\triangle DBC$, we have
$30^o + 50^o + \angle DBC = 180^o$
$\angle DBC = 100^o$

From the figure we can say that
x + ∠DBC = 180° is a Linear pair
x = 80°
From the exterior angle property we have
y = 30° + 80° = 110°

16. Compute the value of x in each of the following figures:

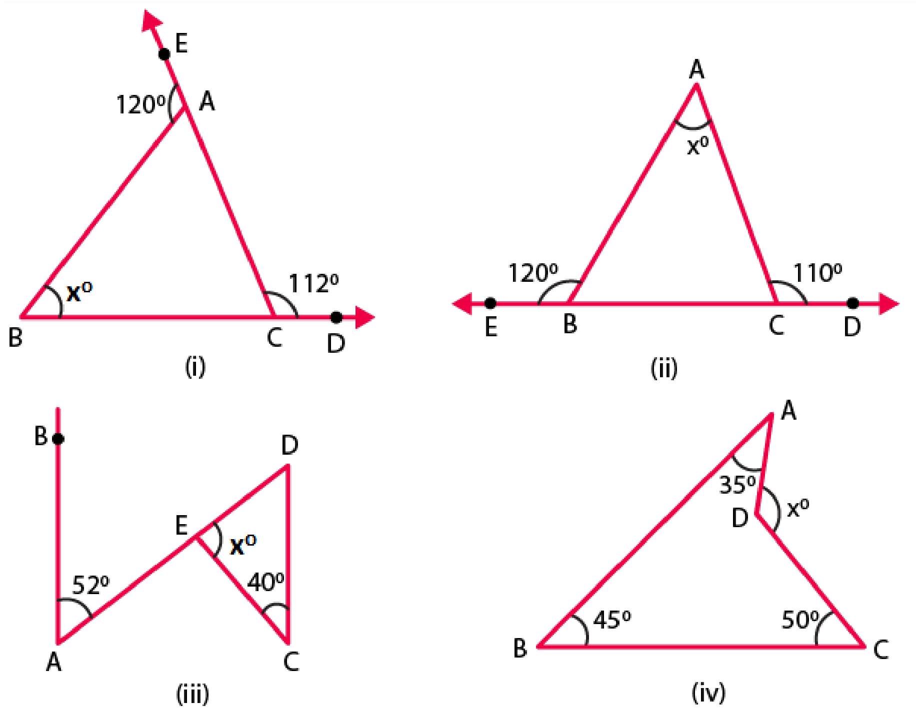


Fig. 43

Solution:
(i) From the given figure, we can write as
∠ACD + ∠ACB = 180° is a linear pair
On rearranging we get
∠ACB = 180° – 112° = 68°
Again from the figure we have,
∠BAE + ∠BAC = 180° is a linear pair
On rearranging we get,
∠BAC = 180°– 120° = 60°
We know that the sum of all angles of a triangle is 180°.
Therefore, for △ABC:
x + ∠BAC + ∠ACB = 180°

$x = 180^\circ - 60^\circ - 68^\circ = 52^\circ$
$x = 52^\circ$

(ii) From the given figure, we can write as
$\angle ABC + 120^\circ = 180^\circ$ is a linear pair
$\angle ABC = 60^\circ$
Again from the figure we can write as
$\angle ACB + 110^\circ = 180^\circ$ is a linear pair
$\angle ACB = 70^\circ$
We know that the sum of all angles of a triangle is 180°.
Therefore, consider $\triangle ABC$, we get
$x + \angle ABC + \angle ACB = 180^\circ$
$x = 50^\circ$

(iii) From the given figure, we can write as
$\angle BAD = \angle ADC = 52^\circ$ are alternate angles
We know that the sum of all the angles of a triangle is 180°.
Therefore, consider $\triangle DEC$, we have
$x + 40^\circ + 52^\circ = 180^\circ$
$x = 88^\circ$

(iv) In the given figure, we have a quadrilateral and also we know that sum of all angles in a quadrilateral is 360°.
Thus,
$35^\circ + 45^\circ + 50^\circ +$ reflex $\angle ADC = 360^\circ$
On rearranging we get,
Reflex $\angle ADC = 230^\circ$
$230^\circ + x = 360^\circ$ (A complete angle)
$x = 130^\circ$