Exercise 10(A)

1. Which of the following sequences are in arithmetic progression?
(i) 2, 6, 10, 14, ….
(ii) 15, 12, 9, 6, ….
(iii) 5, 9, 12, 18, ….
(iv) 1/2, 1/3, 1/4, 1/5, ….
Solution:

(i) 2, 6, 10, 14, ….
Finding the difference between the terms,
$d_1 = 6 – 2 = 4$
$d_2 = 10 - 6 = 4$
$d_3 = 14 – 10 = 4$
As $d_1 = d_2 = d_3$, the given sequence is in arithmetic progression.

(ii) 15, 12, 9, 6, ….
Finding the difference between the terms,
$d_1 = 12 – 15 = -3$
$d_2 = 9 - 12 = -3$
$d_3 = 6 – 9 = -3$
As $d_1 = d_2 = d_3$, the given sequence is in arithmetic progression.

(iii) 5, 9, 12, 18, ….
Finding the difference between the terms,
$d_1 = 9 – 5 = 4$
$d_2 = 12 - 9 = 3$
$d_3 = 18 – 12 = 6$
As $d_1 \neq d_2 \neq d_3$, the given sequence is not in arithmetic progression.

(iv) 1/2, 1/3, 1/4, 1/5, ….
Finding the difference between the terms,
$d_1 = 1/3 – 1/2 = -1/6$
$d_2 = 1/4 – 1/3 = -1/12$
$d_3 = 1/5 – 1/4 = -1/20$
As $d_1 \neq d_2 \neq d_3$, the given sequence is not in arithmetic progression.

2. The n^{th} term of sequence is (2n - 3), find its fifteenth term.
Solution:

Given, n^{th} term of sequence is (2n - 3)
So, the 15^{th} term is when n = 15
$t_{15} = 2(15) – 3 = 30 – 3 = 27$
Thus, the 15^{th} term of the sequence is 27.

3. If the p^{th} term of an A.P. is (2p + 3); find the A.P.

Solution:

Given, p^{th} term of an A.P. = (2p + 3)
So, on putting p = 1, 2, 3, …, we have
$t_1 = 2(1) + 3 = 5$
$t_2 = 2(2) + 3 = 7$
$t_3 = 2(3) + 3 = 9$
…..
Hence, the sequence A.P. is 5, 7, 9, …

4. Find the 24^{th} term of the sequence:
12, 10, 8, 6,……
Solution:

Given sequence,
12, 10, 8, 6,……
The common difference:
10 – 12 = -2
8 – 10 = -2
6 – 8 = -2 ….
So, the common difference(d) of the sequence is -2 and a = 12.
Now, the general term of this A.P. is given by
$t_n = a + (n - 1)d = 12 + (n - 1)(-2) = 12 – 2n + 2 = 14 – 2n$
For 24^{th} term, n = 24
$t_n = 14 – 2(24) = 14 – 48 = -34$
Therefore, the 24^{th} term is -34

5. Find the 30^{th} term of the sequence:
1/2, 1, 3/2, …….
Solution:

Given sequence,
1/2, 1, 3/2, …….
So,
a = ½
d = 1 – ½ = ½
We know that,
$t_n = a + (n - 1)d$
Hence, the 30^{th} term will be
$t_{30} = ½ + (30 - 1)(1/2) = ½ + 29/2 = 30/2 = 15$
Therefore, the 30^{th} term is 15.

6. Find the 100^{th} term of the sequence:
√3, 2√3, 3√3, ….
Solution:

Given A.P. is √3, 2√3, 3√3, ….
So,
a = √3
d = 2√3 - √3 = √3
The general term is given by,
$t_n = a + (n - 1)d$
For 100th term
$t_{100} = \sqrt{3} + (100 - 1)\sqrt{3} = \sqrt{3} + 99\sqrt{3} = 100\sqrt{3}$
Therefore, the 100th term of the given A.P. is 100√3.

7. Find the 50th term of the sequence:
1/n, (n+1)/n, (2n+1)/n, ……
Solution:

Given sequence,
1/n, (n+1)/n, (2n+1)/n, ……
So,
a = 1/n
d = (n+1)/n – 1/n = (n+1-1)/n = 1
Then, the general term is given by
$t_n = a + (n - 1)d$
For 50th term, n = 50
$t_{50} = 1/n + (50 - 1)1 = 1/n + 49 = (49n + 1)/n$
Hence, the 50th term of the given sequence is (49n + 1)/n.

8. Is 402 a term of the sequence: 8, 13, 18, 23,…………?
Solution:

Give sequence, 8, 13, 18, 23,…………
d = 13 – 8 = 5 and a = 8
So, the general term is given by
$t_n = a + (n - 1)d$
$t_n = 8 + (n - 1)5 = 8 + 5n – 5 = 3 + 5n$
Now,
If 402 is a term of the sequence whose nth is given by (3 + 5n) then n must be a non-negative integer.
3 + 5n = 402
5n = 399
n = 399/5
So, clearly n is a fraction.
Thus, we can conclude that 402 is not a term of the given sequence.

9. Find the common difference and 99th term of the arithmetic progression:
$7\frac{3}{4}, 9\frac{1}{2}, 11\frac{1}{4}, \ldots\ldots\ldots\ldots$
Solution:

Given, A.P, $7\frac{3}{4}, 9\frac{1}{2}, 11\frac{1}{4}, \ldots\ldots\ldots\ldots$
i.e., 31/4, 19/2, 45/4, …….
So,
a = 31/4
Common difference, d = 19/2 – 31/4 = (38 - 31)/ 4 = 7/4
Then the general term of the A.P
$t_n = a + (n - 1)d$
t_{99} = (31/4) + (99 - 1) x (7/4)
= (31/4) + 93 x (7/4)
= (31/4) + (686 / 4)
= (31 + 686)/ 4
= (717)/ 4
= 179 ¼
Hence, the 99th term of A.P. is $179\frac{1}{4}$

10. How many terms are there in the series :
(i) 4, 7, 10, 13, …………, 148?
(ii) 0.5, 0.53, 0.56, ……………, 1.1?
(iii) 3/4, 1, 1 ¼, ……., 3?
Solution:

(i) Given series, 4, 7, 10, 13, …………, 148
Here,
a = 4 and d = 7 – 4 = 3
So, the given term is given by $t_n = 4 + (n - 1)3 = 4 + 3n – 3$
$t_n = 1 + 3n$
Now,
148 = 1 + 3n
147 = 3n
n = 147/3 = 49
Thus, there are 49 terms in the series.

(ii) Given series, 0.5, 0.53, 0.56, ……………, 1.1
Here,
a = 0.5 and d = 0.53 – 0.5 = 0.03
So, the given term is given by $t_n = 0.5 + (n - 1)0.03 = 0.5 + 0.03n – 0.03$
$t_n = 0.47 + 0.03n$
Now,
1.1 = 0.47 + 0.03n
1.1 – 0.47 = 0.03n
n = 0.63/0.03 = 21
Thus, there are 21 terms in the series.

(iii) Given series, 3/4, 1, 1 ¼, ……., 3
Here,
a = 3/4 and d = 1 – 3/4 = 1/4

So, the given term is given by $t_n = 3/4 + (n - 1)1/4 = (3 + n – 1)/4$
$t_n = (2 + n)/ 4$
Now,
$3 = (2 + n)/ 4$
$12 = 2 + n$
$n = 10$
Thus, there are 10 terms in the series.

Exercise 10(B)

1. In an A.P., ten times of its tenth term is equal to thirty times of its 30^{th} term. Find its 40^{th} term.
Solution:

Given condition,
$10\ t_{10} = 30\ t_{30}$ in an A.P.
To find: $t_{40} = ?$
We know that,
$t_n = a + (n - 1)d$
So,
$10\ t_{10} = 30\ t_{30}$
$10(a + (10 - 1)d) = 30(a + (30 - 1)d)$
$10(a + 9d) = 30(a + 29d)$
$a + 9d = 3(a + 29d)$
$a + 9d = 3a + 87d$
$2a + 78d = 0$
$2(a + 39d) = 0$
$a + 39d = a + (40 - 1)d = t_{40} = 0$
Therefore, the 40^{th} term of the A.P. is 0

2. How many two-digit numbers are divisible by 3?
Solution:

The 2-digit numbers divisible by 3 are as follows:
12, 15, 18, 21, ………, 99
It's seen that the above forms an A.P. with
a = 12, d = 3 and last term(n^{th} term) = 99
We know that,
$t_n = a + (n - 1)d$
So,
$99 = 12 + (n - 1)3$
$99 = 12 + 3n – 3$
$99 = 9 + 3n$
$3n = 90$
$n = 90/3 = 30$
Hence, the number of 2-digit numbers divisible by 3 is 30

3. Which term of A.P. 5, 15, 25 ………… will be 130 more than its 31^{st} term?
Solution:

Given A.P. is 5, 15, 25, ……
a = 5, d = 10
From the question, we have
$t_n = t_{31} + 130$
We know that,

$t_n = a + (n - 1)d$
So,
$5 + (n - 1)10 = 5 + (31 - 1)10 + 130$
$10n – 10 = 300 + 130$
$10n = 430 + 10 = 440$
$n = 440/10 = 44$
Thus, the 44^{th} term of the given A.P. is 130 more than its 31^{st} term.

4. Find the value of p, if x, 2x + p and 3x + 6 are in A.P
Solution:

Given that,
x, 2x + p and 3x + 6 are in A.P
So, the common difference between the terms must be the same.
Hence,
$2x + p – x = 3x + 6 – (2x + p)$
$x + p = x + 6 – p$
$2p = 6$
$p = 3$

5. If the 3^{rd} and the 9^{th} terms of an arithmetic progression are 4 and -8 respectively, which term of it is zero?
Solution:

Given in an A.P.
$t_3 = 4$ and $t_9 = -8$
We know that,
$t_n = a + (n - 1)d$
So,
$a + (3 - 1)d = 4$ and $a + (9 - 1)d = -8$
$a + 2d = 4$ and $a + 8d = -8$
Subtracting both the equations we get,
$6d = -12$
$d = -2$
Using the value of d,
$a + 2(-2) = 4$
$a – 4 = 4$
$a = 8$
Now,
$t_n = 0$
$8 + (n - 1)(-2) = 0$
$8 – 2n + 2 = 0$
$10 – 2n = 0$
$10 = 2n$
$n = 5$
Thus, the 5^{th} term of the A.P. is zero.

6. How many three-digit numbers are divisible by 87?
Solution:

The 3-digit numbers divisible by 87 are starting from:
174, 261, ……. , 957
This forms an A.P. with a = 174 and d = 87
And we know that,
$t_n = a + (n - 1)d$
So,
957 = 174 + (n - 1)87
783 = (n - 1)87
(n - 1) = 9
n = 10
Thus, 10 three-digit numbers are divisible by 87.

7. For what value of n, the n^{th} term of A.P 63, 65, 67, …….. and n^{th} term of A.P. 3, 10, 17,…….. are equal to each other?
Solution:

Given,
$A.P._1$ = 63, 65, 67, ……..
a = 63, d = 2 and $t_n = 63 + (n - 1)2$
$A.P._2$ = 3, 10, 17, ……..
a = 3, d = 7 and $t_n = 3 + (n - 1)7$
Then according to the question,
n^{th} of $A.P._1$ = n^{th} of $A.P._2$
63 + (n - 1)2 = 3 + (n - 1)7
60 + 2n – 2 = 7n – 7
58 + 7 = 5n
n = 65/5 = 13
Therefore, the 13^{th} term of both the A.P.s is equal to each other.

8. Determine the A.P. whose 3^{rd} term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.
Solution:

Given,
t_3 of an A.P. = 16 and
$t_7 = t_5 + 12$
We know that,
$t_n = a + (n - 1)d$
So,
$t_3 = a + (3 - 1)d = 16$
a + 2d = 16 …… (i)
And,
a + (7 - 1)d = a + (5 - 1)d + 12
6d = 4d + 12

$2d = 12$
$d = 6$
Using ‘d’ in (i) we get,
$a + 2(6) = 16$
$a + 12 = 16$
$a = 4$
Hence, after finding the first term = 4 and common difference = 6 the A.P. can be formed.
i.e. 4, 10, 16, 22, 28, …….

Exercise 10(C)

1. Find the sum of the first 22 terms of the A.P.: 8, 3, -2, ………..
Solution:

Given, A.P. is 8, 3, -2, ………..
Here,
$a = 8$
$d = 3 – 8 = -5$
And we know that,
$S_n = n/2[2a + (n - 1)d]$
So,
$S_{22} = 22/2[2(8) + (22 - 1)(-5)]$
$= 11[16 + (21 x -5)]$
$= 11[16 - 105]$
$= 11[-89]$
$= - 979$

2. How many terms of the A.P. :
24, 21, 18, ……… must be taken so that their sum is 78?
Solution:

Given, A.P. is 24, 21, 18, ………
Here,
$a = 24$
$d = 21 – 24 = -3$ and
$S_n = 78$
We know that,
$S_n = n/2[2a + (n - 1)d]$
So,
$78 = n/2[2(24) + (n - 1)(-3)]$
$78 = n/2[48 – 3n + 3]$
$156 = n[51 – 3n]$
$3n^2 – 51n + 156 = 0$
$n^2 – 17n + 52 = 0$
$n^2 – 13n – 4n + 52 = 0$
$n(n - 13) – 4(n - 13) = 0$
$(n - 4) (n – 13) = 0$
So, $n - 4 = 0$ or $n – 13 = 0$
Thus, $n = 4$ or 13
Hence, the required number of terms to be taken can be first 4 or first 13.

3. Find the sum of 28 terms of an A.P. whose n^{th} term is 8n - 5.
Solution:

Given,

n^{th} term of an A.P. = 8n - 5
So,
a = 8(1) – 5 = 8 – 5 = 3
Here, the last term is the 28^{th} term
l = 8(28) - 5 = 224 – 5 = 219
We know that,
$S_n = n/2(a + l)$
Thus,
$S_{28} = 28/2(3 + 219) = 14(222) = 3108$

4. (i) Find the sum of all odd natural numbers less than 50
Solution:

Odd natural numbers less than 50 are:
1, 3, 5, 7, …….. ,49
This forms an A.P. with a = 1, d = 2 and l = 49
Now,
l = a + (n – 1)d
49 = 1 + (n - 1)2
49 = 1 + 2n – 2
50 = 2n
n = 25
We know that,
$S_{25} = n/2 [a + l]$
$= 25/2 [1 + 49]$
$= 25/2 [50] = 25 \times 25$
Thus, $S_{25} = 625$

(ii) Find the sum of first 12 natural numbers each of which is a multiple of 7.
Solution:

The first 12 natural numbers which are multiples of 7 are:
7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77 and 84
This forms an A.P where,
a = 7, d = 7, l = 84 and n = 12
So,
$S_n = n/2 (a + l)$
$S_{12} = 12/2 [7 + 84]$
$= 6 [91]$
$= 546$

5. Find the sum of first 51 terms of an A.P. whose 2^{nd} and 3^{rd} terms are 14 and 18 respectively.
Solution:

Given,

Number of terms of an A.P. (n) = 51
$t_2 = 14$ and $t_3 = 18$
So, the common difference (d) = $t_3 - t_2 = 18 - 14 = 4$
And,
$t_2 = a + d$
$14 = a + 4$
$a = 10$
Now, we know that
$S_n = n/2[2a + (n - 1)d]$
Thus,
$S_{51} = 51/2[2(10) + (51 - 1)(4)]$
$= 51/2[20 + 50x4]$
$= 51/2[20 + 200]$
$= 51/2[220]$
$= 51 \times 110$
$= 5610$

6. The sum of first 7 terms of an A.P is 49 and that of first 17 terms of it is 289. Find the sum of first n terms.
Solution:

Given,
$S_7 = 49$
$S_{17} = 289$
We know that,
$S_n = n/2[2a + (n - 1)d]$
So,
$S_7 = 7/2[2a + (7 - 1)d] = 49$
$7[2a + 6d] = 98$
$2a + 6d = 14$
$a + 3d = 7$ …..(1)
And,
$S_{17} = 17/2[2a + (17 - 1)d] = 289$
$17/2[2a + 16d] = 289$
$17[a + 8d] = 289$
$a + 8d = 289/17$
$a + 8d = 17$ …… (2)
Subtracting (1) from (2), we have
$5d = 10$
$d = 2$
Using value of d in (1), we get
$a + 3(2) = 7$
$a = 7 - 6 = 1$
Therefore,
$S_n = n/2[2(1) + (n - 1)(2)]$
$= n/2[2 + 2n - 2]$

$= n/2[2n]$
$S_n = n^2$

7. The first term of an A.P is 5, the last term is 45 and the sum of its terms is 1000. Find the number of terms and the common difference of the A.P.
Solution:

Given,
First term of an A.P. = 5 = a
Last term of the A.P = 45 = l
$S_n = 1000$
We know that,
$S_n = n/2[a + l]$
$1000 = n/2[5 + 45]$
$1000 = n/2[50]$
$20 = n/2$
$n = 40$
Now,
$l = a + (n - 1)d$
$45 = 5 + (40 - 1)d$
$40 = 39d$
$d = 40/39$
Therefore, the number of terms are 40 and the common difference is 40/39.

Exercise 10(D) Page No: 146

1. Find three numbers in A.P. whose sum is 24 and whose product is 440.
Solution:

Let's assume the terms in the A.P to be (a - d), a, (a + d) with common difference as d.
Given conditions,
$S_n = 24$
$(a - d) + a + (a + d) = 3a = 24$
$a = 24/3 = 8$
And,
Product of terms = 440
$(a - d) \times a \times (a + d) = 440$
$a(a^2 – d^2) = 440$
$8(64 – d^2) = 440$
$(64 – d^2) = 55$
$d^2 = 9$
$d = \pm 3$
Hence,
When a = 8 and d = 3, we have
A.P. = 5, 8, 11
And, when a = 8 and d = -3 we have
A.P. = 11, 8, 5

2. The sum of three consecutive terms of an A.P. is 21 and the sum of their squares is 165. Find these terms.
Solution:

Let the three consecutive terms of an A.P. be (a - d), a, (a + d)
Given conditions,
Sum of the three consecutive terms = 21
$(a - d) + a + (a + d) = 21$
$3a = 21$
$a = 7$
And,
Sum of squares = $(a - d)^2 + a^2 + (a + d)^2 = 165$
$a^2 – 2ad + d^2 + a^2 + a^2 + 2ad + d^2 = 165$
$3a^2 + 2d^2 = 165$
$3(49) + 2d^2 = 165$
$2d^2 = 165 – 147 = 18$
$d^2 = 9$
$d = \pm 3$
Hence,
When a = 7 and d = 3, we have
A.P. = 4, 7, 10
And, when a = 7 and d = -3 we have

A.P. = 10, 7, 4

3. The angles of a quadrilateral are in A.P. with common difference 20º. Find its angles.
Solution:

Given, the angles of a quadrilateral are in A.P. with common difference 20^o.
So, let the angles be taken as x, $x + 20^o$, $x + 40^o$ and $x + 60^o$.
We know that,
Sum of all the interior angles of a quadrilateral is 360^o
$x + x + 20^o + x + 40^o + x + 60^o = 360^o$
$4x + 120^o = 360^o$
$4x = 360^o - 120^o = 240^o$
$x = 240^o / 4 = 60^o$
Hence, the angles are
60^o, $(60^o + 20^o)$, $(60^o + 40^o)$ and $(60^o + 60^o)$
i.e. 60^o, 80^o, 100^o and 120^o

4. Divide 96 into four parts which are in A.P and the ratio between product of their means to product of their extremes is 15: 7.
Solution:

Let 96 be divided into 4 parts as $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$ which are in A.P with common difference of $2d$.
Given,
$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 96$
$4a = 96$
So, $a = 24$
And, given that,
$(a - d)(a + d)/ (a - 3d) (a + 3d) = 15/7$
$[a^2 - d^2]/ [a^2 - 9d^2] = 15/7$
Substituting the value of a, we get,
$(576 - d^2) / (576 - 9d^2) = 15 / 7$
On cross multiplication, we get,
$4032 - 7d^2 = 8640 - 135d^2$
$128d^2 = 4608$
$d^2 = 36$
$d = +6$ or -6

Hence,
When $a = 24$ and $d = 6$
The parts are 6, 18, 30 and 42
And, when $a = 24$ and $d = -6$
The parts are 42, 30, 18 and 6

5. Find five numbers in A.P. whose sum is 12.5 and the ratio of the first to the last terms is 2: 3.
Solution:

Let the five numbers in A.P be taken as $(a - 2d)$, $(a - d)$, a, $(a + d)$ and $(a + 2d)$.
Given conditions,
$(a - 2d) + (a - d) + a + (a + d) + (a + 2d) = 12.5$
$5a = 12.5$
$a = 12.5/5 = 2.5$
And,
$(a - 2d)/ (a + 2d) = 2/3$
$3(a - 2d) = 2(a + 2d)$
$3a - 6d = 2a + 4d$
$a = 10d$
$2.5 = 10d$
$d = 0.25$
Therefore, the five numbers in A.P are $(2.5 - 2(0.25))$, $(2.5 - 0.25)$, 2.5, $(2.5 + 0.25)$ and $(2.5 + 2(0.25))$
i.e. 2, 2.25, 2.5, 2.75 and 3.

Exercise 10(E)

1. Two cars start together in the same direction from the same place. The first car goes at uniform speed of 10 km h^{-1}. The second car goes at a speed of 8 km h^{-1} in the first hour and thereafter increasing the speed by 0.5 km h^{-1} each succeeding hour. After how many hours will the two cars meet?

Let's assume the two cars meet after n hours.
Then, this means that two cars travel the same distance in n hours.
So,
Distance travelled by the 1st car in n hours = 10 x n km
Distance travelled by the 2nd car in n hours = n/2[2x8 + (n – 1) x 0.5] km
10n = n/2[2x8 + (n – 1) x 0.5]
20 = [16 + 0.5n – 0.5]
20 = 15.5 + 0.5n
4.5 = 0.5n
n = 9
Hence, the two cars will meet after 9 hours.

2. A sum of Rs. 700 is to be paid to give seven cash prizes to the students of a school for their overall academic performance. If the cost of each prize is Rs. 20 less than its preceding prize; find the value of each of the prizes.

From the question, it's understood that
n = 7
d = -20
S_7 = 700
We know that,
S_n = n/2[2a + (n - 1)d]
700 = 7/2[2a + (7 -1)(-20)]
200 = [2a + (7 -1)(-20)]
200 = 2a – 120
2a = 320
a = 160
Hence, the value of each prize will be
1st prize – Rs 160, 2nd prize – Rs 140, 3rd prize – Rs 120, 4th prize – Rs 100, 5th prize – Rs 80, 6th prize – Rs 60 and 7th prize – Rs 40

Exercise 10(F)

1. The 6th term of an A.P. is 16 and the 14th term is 32. Determine the 36th term.
Solution:

Given,
$t_6 = 16$ and $t_{14} = 32$
Let's take 'a' to be the first term and 'd' to be the common difference of the given A.P.
We know that,
$t_n = a + (n - 1)d$
$\Rightarrow a + 5d = 16$ ….(1)
And,
$\Rightarrow a + 13d = 32$ ….(2)
Now, subtracting (1) from (2), we get
$8d = 16$
$d = 2$
Using d in (1) we get,
$a + 5(2) = 16$
$\Rightarrow a = 6$
Therefore, the 36th term = $t_{36} = a + 35d = 6 + 35(2) = 76$

2. If the third and the 9th term of an A.P. be 4 and -8 respectively, find which term is zero?
Solution:

Given,
$t_3 = 4$ and $t_9 = -8$
We know that,
$t_n = a + (n - 1)d$
So,
$4 = a + (3 - 1)d$
$a + 2d = 4$ ….. (1)
And,
$-8 = a + (9 - 1)d$
$a + 8d = -8$ ….. (2)
Subtracting (1) from (2), we have
$6d = -8 – 4$
$6d = -12$
$d = -2$
Using d in (1), we get
$a + 2(-2) = 4$
$a = 4 + 4 = 8$
Now,
$t_n = 8 + (n - 1)(-2)$
Let nth term of this A.P. be 0
$8 + (n – 1) (-2) = 0$
$8 – 2n + 2 = 0$

$10 - 2n = 0$
$2n = 10$
$n = 5$
Therefore, the 5th term of an A.P. is zero.

3. An A.P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term of the A.P.
Solution:

Given,
Number of terms in an A.P, $n = 50$
And, $t_3 = 12$
We know that,
$t_n = a + (n - 1)d$
$\Rightarrow a + 2d = 12$ ….(1)
Last term, $l = 106$
$t_{50} = 106$
$a + 49d = 106$ ….(2)
Subtracting (1) from (2), we get
$47d = 94$
$d = 2$
Substituting the value of d in equation (1), we get
$a + 2(2) = 12$
$a = 8$
Therefore, the 29th term is
$t_{29} = a + 28d = 8 + 28(2) = 8 + 56 = 64$

4. Find the arithmetic mean of:
(i) -5 and 41
(ii) 3x - 2y and 3x + 2y
(iii) $(m + n)^2$ and $(m - n)^2$
Solution:

(i) Arithmetic mean of -5 and 41 = (-5 + 41)/ 2 = 36/2 = 18
(ii) Arithmetic mean of (3x – 2y) and (3x + 2y) = [(3x – 2y) + (3x + 2y)]/ 2 = 6x/ 2 = 3x

(iii) Arithmetic mean of $(m + n)^2$ and $(m - n)^2$

$$= \frac{(m+n)^2 + (m-n)^2}{2}$$
$$= \frac{m^2 + n^2 + 2mn + m^2 + n^2 - 2mn}{2}$$
$$= \frac{2(m^2 + n^2)}{2}$$
$$= m^2 + n^2$$

5. Find the sum of first 10 terms of the A.P.
4 + 6 + 8 + ……
Solution:

Given A.P. is 4 + 6 + 8 + ……
Here,
a = 4 and d = 6 - 4 = 2
And, n = 10
$S_n = n/2\ [2a + (n – 1)d]$
$S_{10} = 10/2\ [2a + (10 - 1)d]$
$= 5\ [2(4) + 9(2)]$
$= 5\ [8 + 18]$
$= 5 \times 26$
$= 130$

6. Find the sum of first 20 terms of an A.P. whose first term is 3 and the last term is 57.
Solution:

Given,
First term, a = 3 and last term, l = 57
And, n = 20
$S = n/2\ (a + l)$
$= 20/2\ (3 + 57)$
$= 10\ (60)$
$= 600$

7. How many terms of the series 18 + 15 + 12 +…….. when added together will give 45?
Solution:

Given series, 18 + 15 + 12 +……..
Here,
a = 18 and d = 15 – 18 = -3
Let's consider the number of terms to be added as 'n'.
So, we have
$S_n = n/2\ [2a + (n - 1)d]$
$45 = n/2\ [2(18) + (n - 1)(-3)]$
$90 = n[36 - 3n + 3]$
$90 = n[39 - 3n]$
$90 = 3n[13 - n]$
$30 = 13n - n^2$
$n^2 - 13n + 30 = 0$
$n^2 - 10n - 3n + 30 = 0$
$n(n - 10) - 3(n - 10) = 0$
$(n - 10)(n - 3) = 0$
$n - 10 = 0$ or $n - 3 = 0$
$n = 10$ or $n = 3$

Therefore, the required number of terms to be added is 3 or 10.

8. The n^{th} term of a sequence is 8 - 5n. Show that the sequence is an A.P.
Solution:

Given, $t_n = 8 - 5n$
Now, replacing n by (n + 1), we get
$t_{n+1} = 8 - 5(n + 1) = 8 - 5n - 5 = 3 - 5n$
Now,
$t_{n+1} - t_n = (3 - 5n) - (8 - 5n) = -5$
As, $(t_{n+1} - t_n)$ is independent of n and is thus a constant.
Therefore, the given sequence having n^{th} term (8 - 5n) is an A.P.

9. Find the general term (n^{th} term) and 23^{rd} term of the sequence 3, 1, -1, -3, ….. .
Solution:

Given sequence is 3, 1, -1, -3, …..
Now,
1 - 3 = -1 - 1 = -3 - (-1) = -2
Thus, the given sequence is an A.P. where a = 3 and d = -2.
So, the general term of an A.P is given by
$t_n = a + (n - 1)d$
$= 3 + (n - 1)(-2)$
$= 3 - 2n + 2$
$= 5 - 2n$
Therefore, the 23^{rd} term $= t_{23} = 5 - 2(23) = 5 - 46 = -41$

10. Which term of the sequence 3, 8, 13, ... is 78?
Solution:

The given sequence is 3, 8, 13, …..
Now,
8 - 3 = 13 - 8 = 5
Hence, the given sequence is an A.P. with first term a = 3 and common difference d = 5.
Let the n^{th} term of the given A.P. be 78.
78 = 3 + (n - 1)(5)
75 = 5n - 5
5n = 80
n = 16
Therefore, the 16^{th} term of the given sequence is 78.

11. Is -150 a term of 11, 8, 5, 2, ... ?
Solution:

Given sequence is 11, 8, 5, 2, …..
It’s seen that,

8 - 11 = 5 - 8 = 2 - 5 = -3
Thus, the given sequence is an A.P. with a = 11 and d = -3.
So, the general term of an A.P. is given by
$t_n = a + (n - 1)d$
-150 = 11 + (n - 1)(-3)
-161 = -3n + 3
3n = 164
n = 164/3 (which is a fraction)
As the number of terms cannot be a fraction.
Hence, clearly -150 is not a term of the given sequence.

12. How many two digit numbers are divisible by 3?
Solution:

The two-digit numbers divisible by 3 are given below:
12, 15, 18, 21, ….., 99
It's clear that the above sequence forms an A.P
Where,
a = 12, d = 3 and last term (l) = 99
And, the general term is given by
$t_n = a + (n - 1)d$
So,
99 = 12 + (n - 1)3
99 = 12 + 3n – 3
99 = 9 + 3n
90 = 3n
n = 90/3 = 30
Therefore, there are 30 two-digit numbers that are divisible by 3.