MODEL QUESTION PAPER-1

For Reduced Syllabus 2020-21
MATHEMATICS :SECOND PUC
Subject code: 35

Time: 3 hours 15 minute **Max. Marks: 100**

Instructions:

i. The question paper has five parts namely A, B, C, D and E. Answer all the parts.
ii. Use the graph sheet for the question on linear programming in **PART – E**.

PART A

Answer ALL the questions **$10\times1=10$**

1. Define an empty relation.

2. Write the domain of the function $y=\sec^{-1}x$.

3. If a matrix has 5 elements, what are the possible orders it can have?

4. Find the values of x for which $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix}=\begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$.

5. If $y=\tan\sqrt{x}$, find $\dfrac{dy}{dx}$.

6. Find $\int(2x^2+e^x)\,dx$.

7. Define a negative vector.

8. If a line makes angles 90°, 135° and 45° with the *x*, *y* and *z*-axis respectively, find its direction cosines.

9. Define Optimal solution in a linear programming problem.

10. If $P(A)=\dfrac{7}{13}$, $P(B)=\dfrac{9}{13}$ and $P(A\cap B)=\dfrac{4}{13}$, evaluate $P(A|B)$.

PART B

Answer any TEN questions: **$10\times2=20$**

11. Let $*$ be a binary operation on Q defined by $a*b=\dfrac{ab}{2},\forall a,b\in Q$. Show that $*$ is associative.

12. Find the principal value of $\cot^{-1}\left(\dfrac{-1}{\sqrt{3}}\right)$.

13. Find the area of the triangle whose vertices are $(-2,-3),(3,2)$ and $(-1,-8)$ using determinants.

14. Find $\dfrac{dy}{dx}$, if $y=\cos(\log x+e^x),\ x>0$

15. Find $\dfrac{dy}{dx}$, if $\sin^2 x+\cos^2 y=1$.

16. If $y=x^3+\tan x, then\ find\ \frac{d^2y}{dx^2}$.

17. Find the slope of the tangent to the curve $y=x^3-x$ at $x=2$.

18. Find $\int_0^{\pi}(sin^2\frac{x}{2}-cos^2\frac{x}{2})dx.$

19. Find $\int x\sec^2 x\,dx$.

20. Find the order and degree of the differential equation, $y'''+2y''+y'=0$.

21. Find the projection of the vector $\vec{a}=\hat{i}+3j+7k$ on the vector $\vec{b}=7\hat{i}-j+8k$.

22. Find the area of the parallelogram whose adjacent sides are determined by vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

23. Find the equation of the plane with intercepts 2, 3 and 4 on *X*, *Y* and *Z* axes respectively.

24. Assume that each child born in a family is equally likely to be a boy or girl. If a family has two children, what is the conditional probability that both are girls, given that the youngest is a girl.

PART C

Answer any TEN questions: **$10 \times 3 = 30$**

25. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2): T_1 \text{ is similar to } T_2\}$, is equivalence relation.

26. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $A + A'$ is a symmetric matrix (ii) $A - A'$ is a skew- symmetric matrix.

27. If $x = 2at^2$, $y = at^4$, then find $\frac{dy}{dx}$.

28. Find $\frac{dy}{dx}$, if $x^y = y^x$.

29. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is (a) strictly increasing (b) strictly decreasing.

30. Evaluate: $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cdot \cos^2 x} dx$.

31. Find $\int \frac{(x-3)e^x}{(x-1)^3} dx$.

32. Evaluate: $\int \frac{dx}{(x+1)(x+2)}$.

33. Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the *y*-axis in the first quadrant.

34. Solve, $\frac{dy}{dx} = e^{x+y}$.

35. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$.

36. Show that the points $A(-2\hat{i} + 3\hat{j} + 5\hat{k}), B(\hat{i} + 2\hat{j} + 3\hat{k})$ and $C(7\hat{i} - \hat{k})$ are collinear.

37. Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$, $x + y + z - 2 = 0$ and the point (2, 2, 1).

38. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

PART D

Answer any SIX questions: **$6 \times 5 = 30$**

39. Check the injectivity and surjectivity of the function $f: R \to R$ defined by $f(x) = 3 - 4x$. Is it a bijective function?

40. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ then compute $A + B$ and $B - C$

ɔ, verify that $A+(B-C)=(A+B)-C$.

41. Solve the system of equations by matrix method:
$2x+3y+3z=5,\ x-2y+z=-4,\ 3x-y-2z=3.$

42. If $y=\left(\tan^{-1}x\right)^2$, show that $(1+x^2)^2y_2+2x(1+x^2)y_1=2$.

43. The length x of a rectangle is decreasing at the rate of $3\ cm/minute$ and the width y is Increasing at the rate of $2\ cm/minute$. When $x=10\ cm$ and $y=6\ cm$, find the rates of change of (i) the perimeter (ii) the area of the rectangle.

44. Find the integral of $\frac{1}{\sqrt{a^2-x^2}}$ with respect to x and hence evaluate $\frac{1}{\sqrt{9-25x^2}}$.

45. Using the method of integration, find the area enclosed by the circle $x^2+y^2=a^2$.

46. Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2\ (x\neq 0)$.

47. Derive the equation of the line in space passing through a given point and parallel to a given vector both in vector and Cartesian form.

48. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem.

PART E

Answer any ONE question: **1 × 10=10**

49. (a) Prove that $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$ and hence evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx$. **6**

(b) Find the value of k if $f(x)=\begin{cases} kx+1, & if\ \ x\le\pi \\ \cos x, & if\ \ x>\pi \end{cases}$ is continuous at $x=\pi$. **4**

50.(a) Miximise $z=4x+y$ subject to constraints $x+y\le 50,\ 3x+y\le 90,\ x\ge 0,\ y\ge 0$ by graphical method. **6**

(b) If $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, satisfies the equation $A^2-5A+7I=O$, then find the inverse of A using this equation, where I is the identity matrix of order 2. **4**

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