MODEL QUESTION PAPER-2

For Reduced Syllabus 2020-21
MATHEMATICS :SECOND PUC
Subject code: 35

Time: 3 hours 15 minute **Max. Marks: 100**

Instructions:

i. The question paper has five parts namely A, B, C, D and E. Answer all the parts.
ii. Use the graph sheet for the question on linear programming in **PART – E**.

PART – A

I. Answer all the questions **10 x 1 = 10**

1. Examine whether the operation $*: Z^+ \to Z^+$ defined by $a * b = |a - b|$, where Z^+ is the set of all positive integers, is a binary operation or not.
2. Find the domain of $\sin^{-1} x$.
3. Construct a 2×2 matrix whose elements are given by $a_{ij} = \frac{(i+j)^2}{2}$.
4. If A is a square matrix and $\text{adj}(A) = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$, then find $|A|$.
5. Differentiate $\cos\sqrt{x}$ with respect to x.
6. Evaluate: $\int \sqrt{ax+b}\, dx$.
7. Find the vector components of the vector with initial point (2,1) and terminal point (-5,7).
8. Find the distance of the plane $3x - 4y + 12z - 3 = 0$ from the origin.
9. Define the objective function in a linear programming problem.
10. If F is an event of a sample space S of an experiment then find $P(S | F)$.

PART – B

II. Answer any Ten questions **10 x 2 = 20**

11. On R $*$ is defined by $a * b = \frac{a+b}{2}$, verify whether $*$ is associative.
12. Evaluate: $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$.
13. Find the equation of the line passing through (1, 2) and (3, 6) using determinants.
14. Find $\frac{dy}{dx}$, if $ax + by^2 = \cos y$.
15. Differentiate $\cos^{-1}(\sin x)$ with respect to x.
16. If $y = x^{\sin x}$, x>0. Find $\frac{dy}{dx}$.
17. Find the local maximum value of the function $g(x) = x^2 - 3x$.
18. Evaluate: $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}$ dx.
19. Evaluate: $\int \log_e x \; dx$.
20. Find order and degree of the differential equation $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$.
21. Find a vector in the direction of vector $\vec{a} = \hat{i} - 2\hat{j}$ that has magnitude 7 units.
22. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the positive direction of the axes.
23. Find the Cartesian equation of the line that passes through the points

-5) and (3, -2, 6).

24. Two cards drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black

PART – C

III. Answer any TEN questions **10 x 3 = 30**

25. Show that the relation R in the set of all integers Z defined by $R=\{(a,b): 2\, divides\, a-b\}$ is an equivalence relation.
26. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute that is AB=BA.
27. Find $\frac{dy}{dx}$, if $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$.
28. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, for $-1<x<1$ and $x\neq y$, prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$.
29. Find the point at which the tangent to the curve $y=\sqrt{4x-3}-1$ has its slope $\frac{2}{3}$
30. Evaluate: $\int \tan^4 x\, dx.$
31. Evaluate: $\int \frac{x}{(x+1)(x+2)}dx$.
32. Evaluate: $\int_0^{\frac{\pi}{4}} \sin 2x\, dx.$
33. Find the area of the region bounded by the curve $y=x^2$ and the line $y=2$.
34. Solve: $\mathrm{y\log y dx - x dy = 0}$.
35. Show that the position vectors of the point P which divides the line joining the points A and B having position vectors $\vec{a}$ and $\vec{b}$ internally in the ration m : n is $\frac{m\vec{b}+n\vec{a}}{m+n}$
36. Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ Where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$.
37. Find the distance between the parallel lines $\vec{r}=\hat{i}+2j-4k+\lambda(2\hat{i}+3j+6k)$ and $\vec{r}=3\hat{i}+3j-5k+\mu(2\hat{i}+3j+6k)$.
38. Bag I contains 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag-II

PART – D

Answer any six following questions **6 x 5 = 30**

39. Verify whether the function $f: N\to N$ defined by $f(x)=x^2$ is one-one, onto and bijective.
40. If $A=\begin{bmatrix}0\\1\\2\end{bmatrix}$, $B=\begin{bmatrix}1 & 5 & 7\end{bmatrix}$ verity that $(AB)'=B'A'$.
41. Solve $4x+3y+2z=60,\ 2x+4y+6z=90$ and $6x+2y+3z=70$ by a matrix method..
42. If $y=e^{a\cos^{-1}x}$, $-1\le x\le 1$, , then prove that $(1-x^2)y_2-xy_1-a^2y=0$.

...cle moves along the curve $6y = x^3 + 2$, find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

44. Find the integral $\frac{1}{x^2 - a^2}$ with respect to x and hence evaluate $\int \frac{1}{4x^2 - 9}\, dx$.

45. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a > b)$ by method of integration.

46. Solve the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$.

47. Derive the equation of a plane in normal form both in vector and Cartesian form

48. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that
(i) both balls are red
(ii) first ball is black and second is red.
(iii) one of them is black and other is red.

PART – E

Answer any ONE of the following question **10 x 1 = 10**

49. a) Prove that $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ and hence evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$. 6

b) Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax + b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$$ is continuous function. 4

50. a) Solve the following linear programming problem graphically:
Minimize and maximize $z = x + 2y$, subject to constraints
$x + 2y \ge 100,\ 2x - y \le 0,\ 2x + y \le 200\ \ and\ \ x, y \ge 0$. 6

a) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, satisfying the equation $A^2 - 4A + I = O$,
Where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Find A^{-1}. 4

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