Exercise 12.1

Page No: 12.8

Question 1: Find the area of a triangle whose sides are respectively 150 cm, 120 cm and 200 cm.

Solution:

We know, Heron's Formula

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Semi Perimeter, s = $\frac{(a+b+c)}{2}$

Where, a, b and c are sides of a triangle

Here, a = 150 cm

b = 120 cm

c = 200 cm

Step 1: Find s

s = (a+b+c)/2

s = (150+200+120)/2

s = 235 cm

Step 2: Find Area of a triangle

Area = $\sqrt{235 \times (235-150) \times (235-120) \times (235-200)}$

= $\sqrt{235 \times (85) \times (115) \times (35)}$

= $\sqrt{80399375}$

= 8966.56

Area of triangle is 8966.56 sq.cm.

Question 2: Find the area of a triangle whose sides are respectively 9 cm, 12 cm and 15 cm.

Solution:

We know, Heron's Formula

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Semi Perimeter, s = $\frac{(a+b+c)}{2}$

Where, a, b and c are sides of a triangle

Here, a = 9 cm

b = 12 cm

c = 15 cm

Step 1: Find s
s = (a+b+c)/2

s = (9 + 12 + 15)/2

s = 18 cm

Step 2: Find Area of a triangle

Area = $\sqrt{(18(18-9)(18-12)(18-15)}$

= $\sqrt{18 \times 9 \times 6 \times 3}$

= $\sqrt{2916}$

= 54

Area of triangle is 54 sq.cm.

Question 3: Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Solution:

Given:

a = 18 cm, b = 10 cm, and perimeter = 42 cm

Let c be the third side of the triangle.

Step 1: Find third side of the triangle, that is c

We know, perimeter = 2s,

2s = 42

s = 21

Again, s = (a+b+c)/2

Put the value of s, we get

21 = (18+10+c)/2

42 = 28 + c

c = 14 cm

Step 2: Find area of triangle

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-18)(21-10)(21-14)}$

$= \sqrt{21 \times 3 \times 11 \times 7}$

$= \sqrt{4851}$

$= 21\sqrt{11}$

Area = $21\sqrt{11}$ square cm.

Question 4: In a triangle ABC, AB = 15cm, BC = 13cm and AC = 14cm. Find the area of triangle ABC and hence its altitude on AC.

Solution:

Let the sides of the given triangle be AB = a, BC = b, AC = c respectively.

Here, a = 15 cm

b = 13 cm

c = 14 cm

From Heron's Formula;

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Semi Perimeter, s = $\frac{(a+b+c)}{2}$

Where, a, b and c are sides of a triangle

s = (15+13+14)/2 = 21

Area = $\sqrt{21(21-13)(21-14)(21-15)}$

= $\sqrt{21 \times 8 \times 7 \times 6}$

= $\sqrt{7056}$

= 84
Area = 84 cm^2
Let, BE is a perpendicular on AC
Now, area of triangle = ½ x Base x Height
½ × BE × AC = 84
BE = 12cm
Hence, altitude is 12 cm.

Question 5: The perimeter of a triangular field is 540 m and its sides are in the ratio 25:17:12. Find the area of the triangle.

Solution:

Let the sides of a given triangle be a = 25x, b = 17x, c = 12x respectively,

Given, Perimeter of triangle = 540 cm

2s = a + b + c

a + b + c = 540 cm

25x + 17x + 12x = 540 cm

54x = 540 cm

x = 10 cm

So, the sides of a triangle are

a = 250 cm

b = 170 cm

c = 120 cm

Semi perimeter, s = (a+b+c)/2

= 540/2

= 270

s = 270 cm

From Heron's Formula;

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{270(270-250)(270-170)(270-120)}$

$= \sqrt{270 \times 20 \times 100 \times 150}$

$= \sqrt{81000000}$

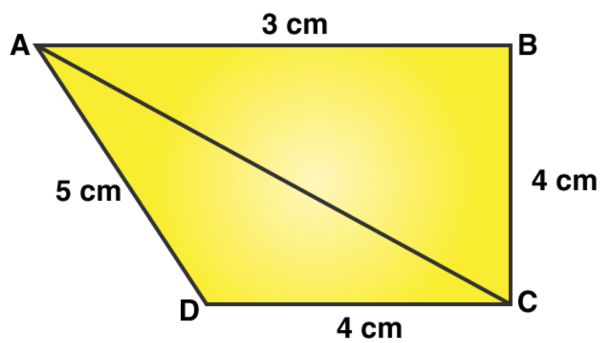
= 9000

Hence, the area of the triangle is 9000 cm^2 .

Exercise 12.2 Page No: 12.19

Question 1: Find the area of the quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

Solution:



Area of the quadrilateral ABCD = Area of △ABC + Area of △ADC ....(1)

△ABC is a right-angled triangle which B.

Area of △ABC = 1/2 x Base x Height

= 1/2×AB×BC

= 1/2×3×4

= 6

Area of △ABC = 6 cm^2 ......(2)

Now, In △CAD,

Sides are given, apply Heron's Formula.

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Semi Perimeter, s = $\frac{(a+b+c)}{2}$

Where, a, b and c are sides of a triangle

Perimeter = 2s = AC + CD + DA

2s = 5 cm + 4 cm + 5 cm

2s = 14 cm

s = 7 cm

Area of the $\triangle$CAD = $\sqrt{7 \times (7-5) \times (7-4) \times (7-5)}$

= $\sqrt{7 \times 2 \times 3 \times 2}$

= 2 $\sqrt{21}$

= 9.16

Area of the $\triangle$CAD = 9.16 cm^2 ...(3)

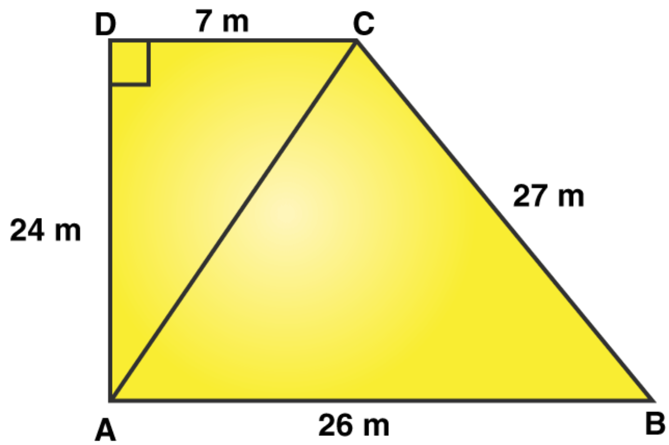
Using equation (2) and (3) in (1), we get

Area of quadrilateral ABCD = (6 + 9.16) cm^2

= 15.16 cm^2.

Question 2: The sides of a quadrilateral field, taken in order are 26 m, 27 m, 7 m, 24 m respectively. The angle contained by the last two sides is a right angle. Find its area.

Solution:



Here,
AB = 26 m, BC = 27 m, CD = 7 m, DA = 24 m

AC is the diagonal joined at A to C point.

Now, in △ADC,

From Pythagoras theorem;

$AC^2 = AD^2 + CD^2$

$AC^2 = 14^2 + 7^2$

AC = 25

Now, area of △ABC
All the sides are known, Apply Heron's Formula.

Area of triangle = $\sqrt{s(s-a)(s-b)\,(s-c)}$

Semi Perimeter, s = $\frac{(a+b+c)}{2}$

Where, a, b and c are sides of a triangle

Perimeter of △ABC= 2s = AB + BC + CA

2s = 26 m + 27 m + 25 m

s = 39 m

Area of a triangle = $\sqrt{39 \times (39-25) \times (39-26) \times (39-27)}$

= $\sqrt{39 \times 14 \times 13 \times 12}$
= $\sqrt{85176}$

= 291.84
Area of a triangle ABC = 291.84 m^2

Now, for area of △ADC, (Right angle triangle)

Area = 1/2 x Base X Height
= 1/2 x 7 x 24
= 84
Thus, the area of a △ADC is 84 m^2

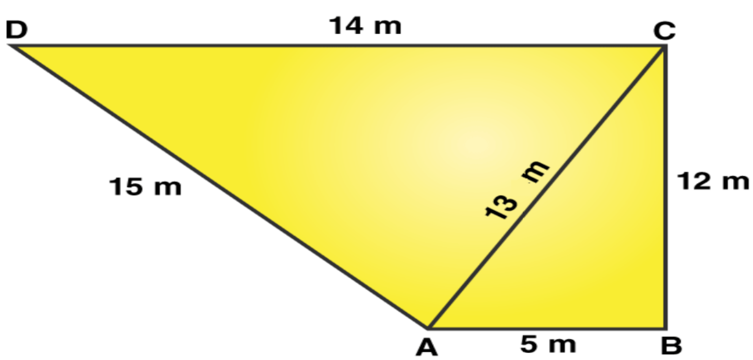
Therefore, Area of rectangular field ABCD = Area of △ABC + Area of △ADC

= 291.84 m^2 + 84 m^2

= 375.8 m^2

Question 3: The sides of a quadrilateral, taken in order as 5, 12, 14, 15 meters respectively, and the angle contained by first two sides is a right angle. Find its area.

Solution:



Here, AB = 5 m, BC = 12 m, CD =14 m and DA = 15 m

Join the diagonal AC.

Now, area of △ABC = 1/2 ×AB×BC

= 1/2×5×12 = 30

Area of △ABC is 30 m^2

In △ABC, (right triangle).
From Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = 5^2 + 12^2$

$AC^2 = 25 + 144 = 169$

or AC = 13

Now in △ADC,
All sides are know, Apply Heron's Formula:

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Semi Perimeter, s = $\frac{(a+b+c)}{2}$

Where, a, b and c are sides of a triangle

Perimeter of △ADC = 2s = AD + DC + AC

2s = 15 m +14 m +13 m

s = 21 m

Area of △ADC = $\sqrt{21 \times (21-13) \times (21-14) \times (21-15)}$

= $\sqrt{21 \times 8 \times 7 \times 6}$
= 84

Area of △ADC = 84 m^2

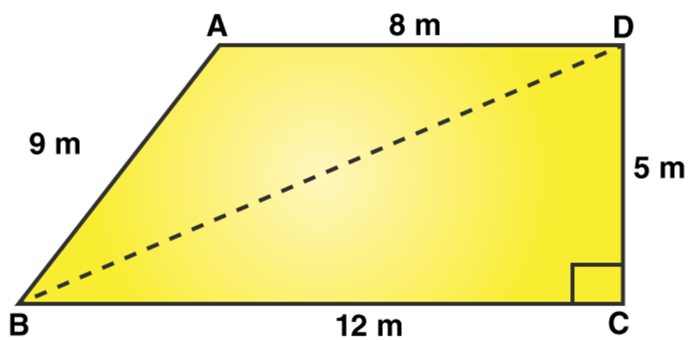
Area of quadrilateral ABCD = Area of △ABC + Area of △ADC

= (30 + 84) m^2

= 114 m^2

Question 4: A park in the shape of a quadrilateral ABCD, has ∠ C = 90^0, AB = 9 m, BC = 12 m, CD = 5 m, AD = 8 m. How much area does it occupy?

Solution:



Here, AB = 9 m, BC = 12 m, CD = 5 m, DA = 8 m.

And BD is a diagonal of ABCD.

In right △BCD,
From Pythagoras theorem;

$BD^2 = BC^2 + CD^2$

$BD^2 = 12^2 + 5^2 = 144 + 25 = 169$

BD = 13 m

Area of △BCD = 1/2×BC×CD

= 1/2×12×5

= 30

Area of △BCD = 30 m^2

Now, In △ABD,

All sides are known, Apply Heron's Formula:

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Semi Perimeter, s = $\frac{(a+b+c)}{2}$

Where, a, b and c are sides of a triangle

Perimeter of △ABD = 2s = 9 m + 8m + 13m

s = 15 m

Area of the △ABD = $\sqrt{15 \times (15-9) \times (15-8) \times (15-13)}$

$= \sqrt{15 \times 6 \times 7 \times 2}$

$= 6\sqrt{35}$

= 35.49

Area of the △ABD = 35.49 m^2

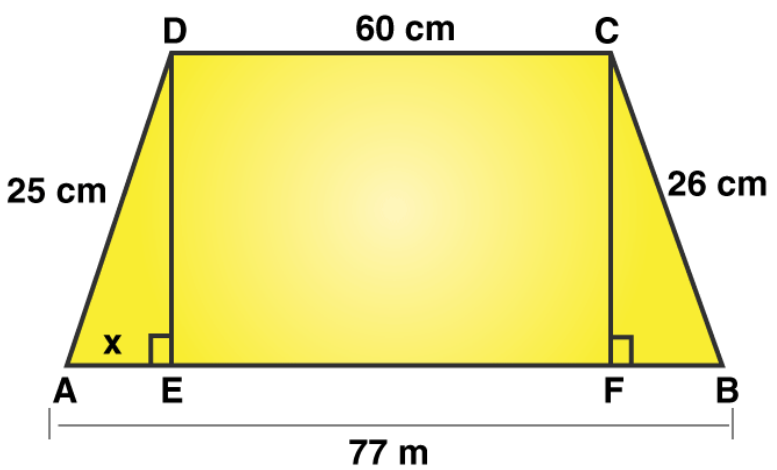
Area of quadrilateral ABCD = Area of △ABD + Area of △BCD

= (35.496 + 30) m^2

= 65.5m^2.

Question 5: Two parallel sides of a trapezium are 60 m and 77 m and the other sides are 25 m and 26 m. Find the area of the trapezium.

Solution:



Given: AB = 77 m , CD = 60 m, BC = 26 m and AD = 25m

AE and CF are diagonals.

DE and CF are two perpendiculars on AB.

Therefore, we get, DC = EF = 60 m

Let's say, AE = x

Then BF = 77 – (60 + x)

BF = 17 – x ...(1)

In right △ADE,
From Pythagoras theorem,

$DE^2 = AD^2 - AE^2$

$DE^2 = 25^2 - x^2$ ....(2)

In right △BCF

From Pythagoras theorem,

$CF^2 = BC^2 - BF^2$

$CF^2 = 26^2 - (17-x)^2$
[Uisng (1)]

Here, DE = CF

So, $DE^2 = CF^2$

(2) => $25^2 - x^2 = 26^2 - (17-x)^2$

$625 - x^2 = 676 - (289 - 34x + x^2)$
$625 - x^2 = 676 - 289 + 34x - x^2$
238 = 34x
x =7

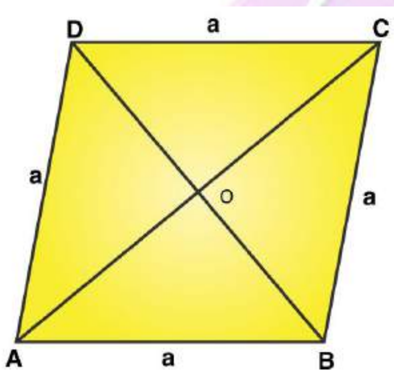
(2) => $DE^2 = 25^2 - (7)^2$
$DE^2 = 625-49$
DE = 24

Area of trapezium = 1/2×(60+77)×24 = 1644

Area of trapezium is 1644 m^2 (Answer)

Question 6: Find the area of a rhombus whose perimeter is 80 m and one of whose diagonal is 24 m.

Solution:



Perimeter of a rhombus = 80 m (given)
We know, Perimeter of a rhombus = 4×side
Let a be the side of a rhombus.

4×a = 80
or a = 20

One of the diagonal, AC = 24 m (given)

Therefore OA = 1/2×AC

OA = 12

In △AOB,
Using Pythagoras theorem:

$OB^2 = AB^2 - OA^2 = 20^2 - 12^2 = 400 - 144 = 256$

or OB = 16

Since diagonal of rhombus bisect each other at 90 degrees.
And OB = OD

Therefore, BD = 2 OB = 2 x 16 = 32 m

Area of rhombus = 1/2×BD×AC = 1/2×32×24 = 384

Area of rhombus = 384 m^2.

Question 7: A rhombus sheet, whose perimeter is 32 m and whose diagonal is 10 m long, is painted on both the sides at the rate of Rs 5 per m^2. Find the cost of painting.

Solution:

Perimeter of a rhombus = 32 m

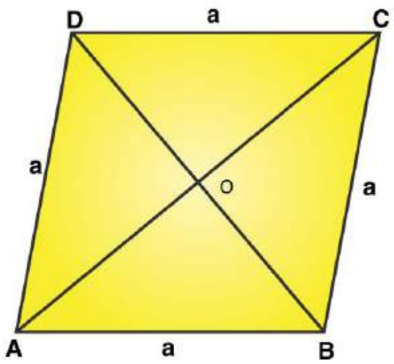
We know, Perimeter of a rhombus = 4×side

=> 4×side = 32

side = a = 8 m

Each side of rhombus is 8 m

AC = 10 m (Given)



Then, OA = 1/2×AC

OA = 1/2×10

OA = 5 m

In right triangle AOB,
From Pythagoras theorem;

$OB^2 = AB^2 – OA^2 = 8^2 – 5^2 = 64 – 25 = 39$

OB = √39 m

And, BD = 2 x OB

BD = 2√39 m

Area of the sheet = 1/2×BD×AC = 1/2 x (2√39 × 10) = 10√39
Area of the sheet is 10√39 m^2

Therefore, cost of printing on both sides of the sheet, at the rate of Rs. 5 per m^2

= Rs. 2 x (10√39 x 5) = Rs. 625.

Exercise VSAQs Page No: 12.23

Question 1: Find the area of a triangle whose base and altitude are 5 cm and 4 cm respectively.
Solution:
Given: Base of a triangle = 5 cm and altitude = 4 cm

Area of triangle = 1/2 x base x altitude

= 1/2 x 5 x 4
= 10
Area of triangle is 10 cm^2.

Question 2: Find the area of a triangle whose sides are 3 cm, 4 cm and 5 cm respectively.

Solution:
Given: Sides of a triangle are 3 cm, 4 cm and 5 cm respectively
Apply Heron's Formula:

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Semi Perimeter, s = $\frac{(a+b+c)}{2}$

Where, a, b and c are sides of a triangle

S = (3+4+5)/2 = 6

Semi perimeter is 6 cm

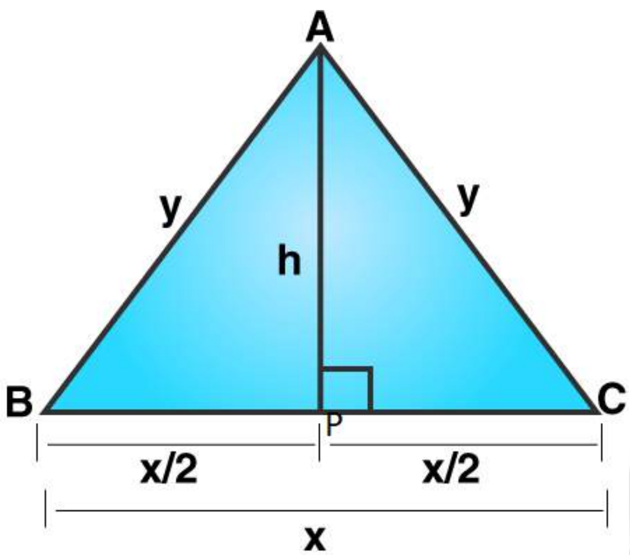
Now,
Area = $\sqrt{6 \times (6-3) \times (6-4) \times (6-5)}$

= $\sqrt{6 \times 3 \times 2 \times 1}$
= $\sqrt{36}$
= 6

Area of given triangle is 6 cm^2.

Question 3: Find the area of an isosceles triangle having the base x cm and one side y cm.

Solution:



In right triangle APC,

Using Pythagoras theorem,

AC^2 = AP^2 + PC^2

y^2 = h^2 + (x/2)^2

or h^2 = y^2 - (x/2)^2

or h = $\sqrt{y^2 - x^2/4}$

Now, Area = 1/2 x Base x Height

= 1/2 x ($\sqrt{y^2 - x^2/4}$)

= x/4 $\sqrt{4y^2 - x^2}$

Question 4: Find the area of an equilateral triangle having each side 4 cm.

Solution: Each side of an equilateral triangle = a = 4 cm

Formula for Area of an equilateral triangle = (√3/4) × a^2

= (√3/4) × 4^2

= 4√3

Area of an equilateral triangle is 4√3 cm^2.

Question 5: Find the area of an equilateral triangle having each side x cm.
Solution:

Each side of an equilateral triangle = a = x cm

Formula for Area of an equilateral triangle = (√3/4) × a^2

= (√3/4) × x^2

= x^2 √3/4

Area of an equilateral triangle is √3x^2/4 cm^2.