EXERCISE 6.2

1. Using laws of exponents, simplify and write the answer in exponential form

(i) $2^3 \times 2^4 \times 2^5$

(ii) $5^{12} \div 5^3$

(iii) $(7^2)^3$

(iv) $(3^2)^5 \div 3^4$

(v) $3^7 \times 2^7$

(vi) $(5^{21} \div 5^{13}) \times 5^7$

Solution:

(i) Given $2^3 \times 2^4 \times 2^5$

We know that first law of exponents states that $a^m \times a^n \times a^p = a^{(m+n+p)}$

Therefore above equation can be written as $2^3 \times 2^4 \times 2^5 = 2^{(3+4+5)}$

$= 2^{12}$

(ii) Given $5^{12} \div 5^3$

According to the law of exponents we have $a^m \div a^n = a^{m-n}$

Therefore given question can be written as $5^{12} \div 5^3 = 5^{12-3} = 5^9$

(iii) Given $(7^2)^3$

According to the law of exponents we have $(a^m)^n = a^{mn}$

Therefore given question can be written as $(7^2)^3 = 7^6$

(iv) Given $(3^2)^5 \div 3^4$

According to the law of exponents we have $(a^m)^n = a^{mn}$

Therefore $(3^2)^5 \div 3^4 = 3^{10} \div 3^4$

According to the law of exponents we have $a^m \div a^n = a^{m-n}$

$3^{10} \div 3^4 = 3^{(10-4)} = 3^6$

(v) Given $3^7 \times 2^7$

We know that law of exponents states that $a^m \times b^m = (a \times b)^m$

$3^7 \times 2^7 = (3 \times 2)^7 = 6^7$

(vi) Given $(5^{21} \div 5^{13}) \times 5^7$

According to the law of exponents we have $a^m \div a^n = a^{m-n}$

$= 5^{(21-13)} \times 5^7$

$= 5^8 \times 5^7$

According to the law of exponents we have $a^m \times a^n = a^{(m+n)}$

$= 5^{(8+7)} = 5^{15}$

2. Simplify and express each of the following in exponential form:

(i) $\{(2^3)^4 \times 2^8\} \div 2^{12}$

(ii) $(8^2 \times 8^4) \div 8^3$

(iii) $(5^7/5^2) \times 5^3$

(iv) $(5^4 \times x^{10}y^5)/(5^4 \times x^7y^4)$

Solution:

(i) Given $\{(2^3)^4 \times 2^8\} \div 2^{12}$

$\{(2^3)^4 \times 2^8\} \div 2^{12} = \{2^{12} \times 2^8\} \div 2^{12}$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 2^{(12+8)} \div 2^{12}$[According to the law of exponents we have $a^m \times a^n = a^{(m+n)}$]

$= 2^{20} \div 2^{12}$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 2^{(20-12)}$

$= 2^8$

(ii) Given $(8^2 \times 8^4) \div 8^3$

$(8^2 \times 8^4) \div 8^3$ [According to the law of exponents we have $a^m \times a^n = a^{(m+n)}$]

$= 8^{(2+4)} \div 8^3$

$= 8^6 \div 8^3$[According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 8^{(6-3)} = 8^3 = (2^3)^3 = 2^9$

(iii) Given $(5^7/5^2) \times 5^3$

$= 5^{(7-2)} \times 5^3$[According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 5^5 \times 5^3$[According to the law of exponents we have $a^m \times a^n = a^{(m+n)}$]

$= 5^{(5+3)} = 5^8$

(iv) Given $(5^4 \times x^{10}y^5)/(5^4 \times x^7y^4)$

$= (5^{4-4} \times x^{10-7}y^{5-4})$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

$= 5^0x^3y^1$ [since $5^0 = 1$]

$= 1x^3y$

3. Simplify and express each of the following in exponential form:

(i) $\{(3^2)^3 \times 2^6\} \times 5^6$

(ii) $(x/y)^{12} \times y^{24} \times (2^3)^4$

(iii)$(5/2)^6 \times (5/2)^2$
(iv) $(2/3)^5 \times (3/5)^5$

Solution:
(i) Given $\{(3^2)^3 \times 2^6\} \times 5^6$
= $\{3^6 \times 2^6\} \times 5^6$[According to the law of exponents we have $(a^m)^n = a^{mn}$]
= $6^6 \times 5^6$ [since law of exponents states that $a^m \times b^m = (a \times b)^m$]
= 30^6

(ii) Given $(x/y)^{12} \times y^{24} \times (2^3)^4$
= $(x^{12}/y^{12}) \times y^{24} \times 2^{12}$
= $x^{12} \times y^{24-12} \times 2^{12}$[According to the law of exponents we have $a^m \div a^n = a^{m-n}$]
= $x^{12} \times y^{12} \times 2^{12}$
= $(2xy)^{12}$

(iii) Given $(5/2)^6 \times (5/2)^2$
= $(5/2)^{6+2}$[According to the law of exponents we have $a^m \times a^n = a^{(m+n)}$]
= $(5/2)^8$

(iv) Given $(2/3)^5 \times (3/5)^5$
= $(2/5)^5$[since law of exponents states that $a^m \times b^m = (a \times b)^m$]

4. Write 9 × 9 × 9 × 9 × 9 in exponential form with base 3.

Solution:
Given $9 \times 9 \times 9 \times 9 \times 9 = (9)^5 = (3^2)^5$
= 3^{10}

5. Simplify and write each of the following in exponential form:
(i) $(25)^3 \div 5^3$
(ii) $(81)^5 \div (3^2)^5$
(iii) $9^8 \times (x^2)^5 / (27)^4 \times (x^3)^2$
(iv) $3^2 \times 7^8 \times 13^6 / 21^2 \times 91^3$

Solution:
(i) Given $(25)^3 \div 5^3$
= $(5^2)^3 \div 5^3$[According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 5^6 \div 5^3$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]
$= 5^{6-3}$
$= 5^3$

(ii) Given $(81)^5 \div (3^2)^5$[According to the law of exponents we have $(a^m)^n = a^{mn}$]
$= (81)^5 \div 3^{10}$[$81 = 3^4$]
$= (3^4)^5 \div 3^{10}$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]
$= 3^{20} \div 3^{10}$
$= 3^{20-10}$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]
$= 3^{10}$

(iii) Given $9^8 \times (x^2)^5 / (27)^4 \times (x^3)^2$
$= (3^2)^8 \times (x^2)^5 / (3^3)^4 \times (x^3)^2$[According to the law of exponents we have $(a^m)^n = a^{mn}$]
$= 3^{16} \times x^{10}/3^{12} \times x^6$
$= 3^{16-12} \times x^{10-6}$[According to the law of exponents we have $a^m \div a^n = a^{m-n}$]
$= 3^4 \times x^4$
$= (3x)^4$

(iv) Given $(3^2 \times 7^8 \times 13^6)/(21^2 \times 91^3)$
$= (3^2 \times 7^2 7^6 \times 13^6)/(21^2 \times 13^3 \times 7^3)$[According to the law of exponents we have $(a^m)^n = a^{mn}$]
$= (21^2 \times 7^6 \times 13^6)/(21^2 \times 13^3 \times 7^3)$
$= (7^6 \times 13^6)/(13^3 \times 7^3)$
$= 91^6/91^3$[According to the law of exponents we have $a^m \div a^n = a^{m-n}$]
$= 91^{6-3}$
$= 91^3$

6. Simplify:
(i) $(3^5)^{11} \times (3^{15})^4 - (3^5)^{18} \times (3^5)^5$
(ii) $(16 \times 2^{n+1} - 4 \times 2^n)/(16 \times 2^{n+2} - 2 \times 2^{n+2})$
(iii) $(10 \times 5^{n+1} + 25 \times 5^n)/(3 \times 5^{n+2} + 10 \times 5^{n+1})$
(iv) $(16)^7 \times (25)^5 \times (81)^3/(15)^7 \times (24)^5 \times (80)^3$

Solution:
(i) Given $(3^5)^{11} \times (3^{15})^4 - (3^5)^{18} \times (3^5)^5$
$= (3)^{55} \times (3)^{60} - (3)^{90} \times (3)^{25}$[According to the law of exponents we have $(a^m)^n = a^{mn}$]
$= 3^{55+60} - 3^{90+25}$
$= 3^{115} - 3^{115}$

= 0

(ii) Given $(16 \times 2^{n+1} – 4 \times 2^n)/(16 \times 2^{n+2} – 2 \times 2^{n+2})$

$= (2^4 \times 2^{(n+1)} -2^2 \times 2^n)/(2^4 \times 2^{(n+2)} -2^{2+1} \times 2^2)$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 2^2 \times 2^{(n+3-2n)}/)2^2\times 2^{(n+4-2n+1)}$

$= 2^n \times 2^3 – 2^n/ 2^n \times 2^4 – 2^n \times 2$

$= 2^n(2^3 – 1)/ 2^n(2^4 – 1)$ [According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

= 8 -1 /16 -2

= 7/14

= (1/2)

(iii) Given $(10 \times 5^{n+1} + 25 \times 5^n)/(3 \times 5^{n+2} + 10 \times 5^{n+1})$

$= (10 \times 5^{n+1} + 5^2 \times 5^n)/(3 \times 5^{n+2} + (2 \times 5) \times 5^{n+1})$

$= (10 \times 5^{n+1} + 5 \times 5^{n+1})/(3 \times 5^{n+2} + (2 \times 5) \times 5^{n+1})$ [According to the law of exponents we have $(a^m)^n = a^{mn}$]

$= 5^{n+1} (10+5)/ 5^{n+1} (10+15)$[According to the law of exponents we have $a^m \div a^n = a^{m-n}$]

= 15/25

= (3/5)

(iv) Given $(16)^7 \times(25)^5\times (81)^3/(15)^7 \times(24)^5\times (80)^3$

$= (16)^7 \times(5^2)^5\times (3^4)^3/(3 \times 5)^7 \times(3 \times 8)^5\times (16 \times 5)^3$

$= (16)^7 \times(5^2)^5\times (3^4)^3/3^7 \times 5^7 \times 3^5 \times 8^5\times 16^3\times 5^3$

$= (16)^7/ 8^5 \times 16^{3}$

$= (16)^4/8^5$

$= (2 \times 8)^4/8^5$

$= 2^4/8$

= (16/8)

= 2

7. Find the values of n in each of the following:

(i) $5^{2n} \times 5^3 = 5^{11}$

(ii) $9 \times 3^n = 3^7$

(iii) $8 \times 2^{n+2} = 32$

(iv) $7^{2n+1} \div 49 = 7^3$

(v) $(3/2)^4 \times (3/2)^5 = (3/2)^{2n+1}$

(vi) $(2/3)^{10}\times \{(3/2)^2\}^5 = (2/3)^{2n – 2}$

Solution:

(i) Given $5^{2n} \times 5^3 = 5^{11}$

$= 5^{2n+3} = 5^{11}$

On equating the coefficients, we get

$2n + 3 = 11$

$\Rightarrow 2n = 11 - 3$

$\Rightarrow 2n = 8$

$\Rightarrow n = (8/2)$

$\Rightarrow n = 4$

(ii) Given $9 \times 3^n = 3^7$

$= (3)^2 \times 3^n = 3^7$

$= (3)^{2+n} = 3^7$

On equating the coefficients, we get

$2 + n = 7$

$\Rightarrow n = 7 - 2 = 5$

(iii) Given $8 \times 2^{n+2} = 32$

$= (2)^3 \times 2^{n+2} = (2)^5$ [since $2^3 = 8$ and $2^5 = 32$]

$= (2)^{3+n+2} = (2)^5$

On equating the coefficients, we get

$3 + n + 2 = 5$

$\Rightarrow n + 5 = 5$

$\Rightarrow n = 5 - 5$

$\Rightarrow n = 0$

(iv) Given $7^{2n+1} \div 49 = 7^3$

$= 7^{2n+1} \div 7^2 = 7^3$ [since $49 = 7^2$]

$= 7^{2n+1-2} = 7^3$

$= 7^{2n-1} = 7^3$

On equating the coefficients, we get

$2n - 1 = 3$

$\Rightarrow 2n = 3 + 1$

$\Rightarrow 2n = 4$

$\Rightarrow n = 4/2 = 2$

(v) Given $(3/2)^4 \times (3/2)^5 = (3/2)^{2n+1}$

$= (3/2)^{4+5} = (3/2)^{2n+1}$
$= (3/2)^{9} = (3/2)^{2n+1}$
On equating the coefficients, we get
$2n + 1 = 9$
$\Rightarrow 2n = 9 - 1$
$\Rightarrow 2n = 8$
$\Rightarrow n = 8/2 = 4$

(vi) Given $(2/3)^{10} \times \{(3/2)^2\}^5 = (2/3)^{2n-2}$
$= (2/3)^{10} \times (3/2)^{10} = (2/3)^{2n-2}$
$= 2^{10} \times 3^{10}/3^{10} \times 2^{10} = (2/3)^{2n-2}$
$= 1 = (2/3)^{2n-2}$
$= (2/3)^0 = (2/3)^{2n-2}$
On equating the coefficients, we get
$0 = 2n - 2$
$2n - 2 = 0$
$2n = 2$
$n = 1$

8. If $(9^n \times 3^2 \times 3^n - (27)^n)/ (3^3)^5 \times 2^3 = (1/27)$, find the value of n.

Solution:
Given $(9^n \times 3^2 \times 3^n - (27)^n)/ (3^3)^5 \times 2^3 = (1/27)$
$= (3^2)^n \times 3^3 \times 3^n - (3^3)^n/ (3^{15} \times 2^3) = (1/27)$
$= 3^{(2n+2+n)} - (3^3)^n/ (3^{15} \times 2^3) = (1/27)$
$= 3^{(3n+2)} - (3^3)^n/ (3^{15} \times 2^3) = (1/27)$
$= 3^{3n} \times 3^2 - 3^{3n}/ (3^{15} \times 2^3) = (1/27)$
$= 3^{3n} \times (3^2 - 1)/ (3^{15} \times 2^3) = (1/27)$
$= 3^{3n} \times (9 - 1)/ (3^{15} \times 2^3) = (1/27)$
$= 3^{3n} \times (8)/ (3^{15} \times 2^3) = (1/27)$
$= 3^{3n} \times 2^3/ (3^{15} \times 2^3) = (1/27)$
$= 3^{3n}/3^{15} = (1/27)$
$= 3^{3n-15} = (1/27)$
$= 3^{3n-15} = (1/3^3)$
$= 3^{3n-15} = 3^{-3}$
On equating the coefficients, we get
$3n - 15 = -3$

$\Rightarrow 3n = -3 + 15$

$\Rightarrow 3n = 12$

$\Rightarrow n = 12/3 = 4$