

JEE MAIN – 2021



25th Feb Shift – II Maths Question Paper and Solutions

Section A

1. A plane passes through the points $A(1, 2, 3)$, $B(2, 3, 1)$ and $C(2, 4, 2)$. If O is the origin and P is $(2, -1, 1)$, then the projection of \overrightarrow{OP} on this plane is of length:

(1) $\sqrt{\frac{2}{5}}$

(2) $\sqrt{\frac{2}{3}}$

(3) $\sqrt{\frac{2}{11}}$

(4) $\sqrt{\frac{2}{7}}$

Ans. (3)

Sol. $A(1, 2, 3)$, $B(2, 3, 1)$, $C(2, 4, 2)$, $O(0, 0, 0)$

Equation of plane passing through A, B, C will be

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2-1 & 3-2 & 1-3 \\ 2-1 & 4-2 & 2-3 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$$

$$(x-1)(-1+4) - (y-2)(-1+2) + (z-3)(2-1) = 0$$

$$(x-1)(3) - (y-2)(1) + (z-3)(1) = 0$$

$$3x - 3 - y + 2 + z - 3 = 0$$

$3x - y + z - 4 = 0$, is the required plane equation

Now, given $O(0, 0, 0)$ & $P(2, -1, 1)$

Plane is $3x - y + z - 4 = 0$

Let O' & P' be the foot of perpendiculars.

for O'

$$\frac{x-0}{3} = \frac{y-0}{-1} = \frac{z-0}{1} = \frac{-(0-0+0-4)}{9+1+1}$$

$$\frac{x}{3} = \frac{y}{-1} = \frac{z}{1} = \frac{4}{11}$$

$$\Rightarrow O' = \left(\frac{12}{11}, -\frac{4}{11}, \frac{4}{11}\right)$$

for P'

$$\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-1}{1} = \frac{-(3(2)-(-1)+1-4)}{9+1+1}$$

$$\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-1}{1} = \left(\frac{-4}{11}\right)$$

$$\Rightarrow P' = \left(\frac{10}{11}, -\frac{7}{11}, \frac{7}{11}\right)$$

$$O'P' = \sqrt{\left(\frac{10}{11} - \frac{12}{11}\right)^2 + \left(-\frac{7}{11} + \frac{4}{11}\right)^2 + \left(\frac{7}{11} - \frac{4}{11}\right)^2}$$

$$O'P' = \frac{1}{11} \sqrt{4+9+9}$$

$$O'P' = \frac{\sqrt{22}}{11}$$

$$O'P' = \sqrt{\frac{2}{11}}$$

2. The contrapositive of the statement “If you will work, you will earn money” is:

(1) If you will not earn money, you will not work

(2) You will earn money, if you will not work

(3) If you will earn money, you will work

(4) To earn money, you need to work

Ans. (1)

Sol. Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

p : you will work

q : you will earn money

$\sim q$: you will not earn money

$\sim p$: you will not work

$\sim q \rightarrow \sim p$: if you will not earn money, you will not work

3. If $\alpha, \beta \in \mathbb{R}$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to:

(1) 7

(2) -3

(3) 3

(4) -7

Ans: (4)

Sol. $1 - 2i$ is a root of $z^2 + \alpha z + \beta = 0$. So,

$$(1 - 2i)^2 + \alpha(1 - 2i) + \beta = 0$$

$$\Rightarrow 1 - 4 - 4i + \alpha - 2i\alpha + \beta = 0$$

$$\Rightarrow (\alpha + \beta - 3) - i(4 + 2\alpha) = 0$$

$$\Rightarrow \alpha + \beta - 3 = 0 \text{ \& } 4 + 2\alpha = 0$$

So, $\alpha = -2, \beta = 5$

Therefore, $\alpha - \beta = -7$

4. If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x \, dx$, then

(1) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in G.P.

(2) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

(3) $I_2 + I_4, I_3 + I_5, I_4 + I_6$ are in A.P.

(4) $I_2 + I_4, (I_3 + I_5)^2, I_4 + I_6$ are in G.P.

Ans: (2)

Sol. $I_{n+2} + I_n = \int_{\pi/4}^{\pi/2} \cot^n x \cdot \cos e c^2 x \, dx = \left[\frac{-(\cot x)^{n+1}}{n+1} \right]_{\pi/4}^{\pi/2}$

$$\Rightarrow I_{n+2} + I_n = \frac{1}{n+1}$$

$$\Rightarrow I_2 + I_4 = \frac{1}{3}, I_3 + I_5 = \frac{1}{4}, I_4 + I_6 = \frac{1}{5}$$

So, $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

5. If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is:

(1) 1

(2) 3

(3) 2

(4) 4

Ans: (1)

Sol. Given, $AA^T = I_2$

$$\begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix} \begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1 + \alpha^2 & \alpha - \alpha\beta \\ \alpha - \alpha\beta & \alpha^2 + \beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$1 + \alpha^2 = 1$$

$$\Rightarrow \alpha^2 = 0$$

$$\alpha^2 + \beta^2 = 1$$

$$\Rightarrow \beta^2 = 1$$

$$\therefore \alpha^4 = 0 \text{ \& } \beta^4 = 1$$

$$\beta^4 = 1$$

$$\text{Therefore, } \alpha^4 + \beta^4 = 1$$

6. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set $A \times B$.

Then:

(1) $y = 273x$

(2) $2y = 91x$

(3) $y = 91x$

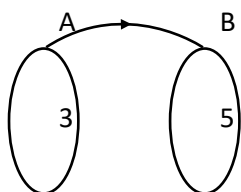
(4) $2y = 273x$

Ans: (2)

Sol. Number of elements in $A = 3$

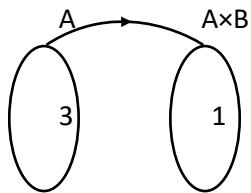
Number of elements in $B = 5$

Number of elements in $A \times B = 15$



Number of one-one function is $x = 5 \times 4 \times 3$

$$\Rightarrow x = 60$$



Number of one-one function is $y = 15 \times 14 \times 13$

$$\Rightarrow y = 15 \times 4 \times \frac{14}{4} \times 13$$

$$\Rightarrow y = 60 \times \frac{7}{2} \times 13$$

$$\Rightarrow 2y = (13)(7x) \Rightarrow 2y = 91x$$

7. If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is:

(1) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

(2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$

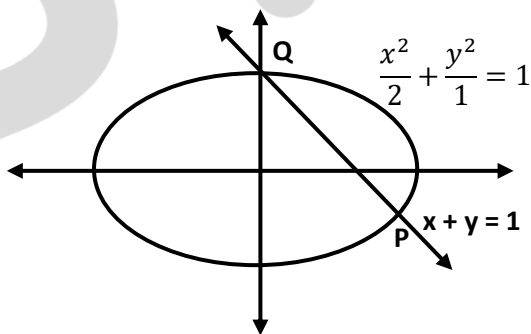
(3) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$

(4) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$

Ans. (1)

Sol. Ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$

Line $x + y = 1$



Using homogenization

$$x^2 + 2y^2 = 2(1)^2$$

$$\Rightarrow x^2 + 2y^2 = 2(x + y)^2$$

$$\Rightarrow x^2 + 2y^2 = 2x^2 + 2y^2 + 4xy$$

$$\Rightarrow x^2 + 4xy = 0$$

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for $ax^2 + 2hxy + by^2 = 0$, obtuse angle between lines θ is

$$\tan \theta = \pm \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\Rightarrow \tan \theta = \pm 4$$

$$\Rightarrow \tan \theta = -4$$

$$\cot \theta = -\frac{1}{4}$$

$$\theta = \cot^{-1}\left(-\frac{1}{4}\right)$$

$$\theta = \pi - \cot^{-1}\left(\frac{1}{4}\right)$$

$$\theta = \pi - \left(\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)\right)$$

$$\theta = \frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$$



8. The integral $\int \frac{e^{3 \log_e 2x} + 5e^{2 \log_e 2x}}{e^{4 \log_e x} + 5e^{3 \log_e x} - 7e^{2 \log_e x}} dx, x > 0$, is equal to:

(where c is a constant of integration)

(1) $\log_e |x^2 + 5x - 7| + c$

(2) $\frac{1}{4} \log_e |x^2 + 5x - 7| + c$

(3) $4 \log_e |x^2 + 5x - 7| + c$

(4) $\log_e \sqrt{x^2 + 5x - 7} + c$

Ans. (3)

Sol.

$$\begin{aligned} & \int \frac{e^{3 \log_e 2x} + 5e^{2 \log_e 2x}}{e^{4 \log_e x} + 5e^{3 \log_e x} - 7e^{2 \log_e x}} dx \\ &= \int \frac{8x^3 + 5(4x^2)}{x^4 + 5x^3 - 7x^2} dx \\ &= \int \frac{8x^3 + 20x^2}{x^4 + 5x^3 - 7x^2} dx \\ &= \int \frac{8x + 20}{x^2 + 5x - 7} dx \\ &= \int \frac{4(2x + 5)}{x^2 + 5x - 7} dx \quad \left\{ \begin{array}{l} \text{Let } x^2 + 5x - 7 = t \\ \Rightarrow (2x + 5)dx = dt \end{array} \right\} \\ &= \int \frac{4dt}{t} \\ &= 4 \log_e |t| + C \\ &= 4 \log_e |(x^2 + 5x - 7)| + c \end{aligned}$$



9. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is:

(1) $\frac{x^2}{9} - \frac{y^2}{4} = 1$

(2) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(3) $x^2 - y^2 = 9$

(4) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

Ans. (2)

Sol. For ellipse $e_1 = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

Foci = $(\pm 3, 0)$

Let equation of hyperbola be $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$, passes through $(\pm 3, 0)$

$A^2 = 9, A = 3, e_2 = \frac{5}{3}$

$e_2^2 = 1 + \frac{B^2}{A^2}$

$\frac{25}{9} = 1 + \frac{B^2}{9} \Rightarrow B^2 = 16$

$\frac{x^2}{9} - \frac{y^2}{16} = 1$

10. $\lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \dots + \frac{n}{(2n-1)^2} \right]$ is equal to:

(1) 1

(2) $\frac{1}{3}$

(3) $\frac{1}{2}$

(4) $\frac{1}{4}$

Ans. (3)

Sol. $\lim_{n \rightarrow \infty} \sum_{r=0}^{n-1} \frac{n}{(n+r)^2} = \lim_{n \rightarrow \infty} \sum_{r=0}^{n-1} \frac{n}{n^2 \left(1 + \frac{r}{n}\right)^2} = \int_0^1 \frac{dx}{(1+x)^2}$

$= - \left[\frac{1}{1+x} \right]_0^1 \Rightarrow - \left[\frac{1}{2} - 1 \right] = \frac{1}{2}$

11. In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is:

(1) $\frac{7}{45}$

(2) $\frac{8}{45}$

(3) $\frac{14}{45}$

(4) $\frac{28}{45}$

Ans. (4)

Sol. Based on Bayes' theorem

$$\begin{aligned} \text{Probability} &= \frac{\left(\frac{160}{400} \times \frac{35}{100}\right)}{\left(\frac{160}{400} \times \frac{35}{100}\right) + \left(\frac{100}{400} \times \frac{20}{100}\right) + \left(\frac{140}{400} \times \frac{10}{100}\right)} \\ &= \frac{5600}{9000} \\ &= \frac{28}{45} \end{aligned}$$

12. The following system of linear equations

$$2x + 3y + 2z = 9$$

$$3x + 2y + 2z = 9$$

$$x - y + 4z = 8$$

- (1) does not have any solution
- (2) has a unique solution
- (3) has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$
- (4) has infinitely many solutions

Ans. (2)

Sol. $\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 3 & 2 & 2 \\ 1 & -1 & 4 \end{vmatrix} = -20 \neq 0$

It has unique solution.

13. The minimum value of $f(x) = a^{ax} + a^{1-ax}$, where $a, x \in \mathbb{R}$ and $a > 0$, is equal to:

(1) $a + \frac{1}{a}$

(2) $a + 1$

(3) $2a$

(4) $2\sqrt{a}$

Ans. (4)

Sol. Using AM \geq GM inequality, we get

$$\frac{a^{ax} + \frac{a}{a^{ax}}}{2} \geq \left(a^{ax} \cdot \frac{a}{a^{ax}} \right)^{1/2} \Rightarrow a^{ax} + a^{1-ax} \geq 2\sqrt{a}$$



14. A function $f(x)$ is given by $f(x) = \frac{5^x}{5^x+5}$, then the sum of the series

$$f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \dots + f\left(\frac{39}{20}\right)$$

is equal to:

(1) $\frac{19}{2}$

(2) $\frac{49}{2}$

(3) $\frac{39}{2}$

(4) $\frac{29}{2}$

Ans. (3)

Sol.

$$f(x) = \frac{5^x}{5^x+5} \dots (i)$$

$$\Rightarrow f(2-x) = \frac{5^{2-x}}{5^{2-x}+5}$$

$$\Rightarrow f(2-x) = \frac{5}{5^x+5} \dots (ii)$$

Adding equation (i) and (ii)

$$f(x) + f(2-x) = 1$$

$$f\left(\frac{1}{20}\right) + f\left(\frac{39}{20}\right) = 1$$

$$f\left(\frac{2}{20}\right) + f\left(\frac{38}{20}\right) = 1$$

:

:

$$f\left(\frac{19}{20}\right) + f\left(\frac{21}{20}\right) = 1$$

$$\text{and } f\left(\frac{20}{20}\right) = f(1) = \frac{1}{2}$$

$$\therefore f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \dots + f\left(\frac{39}{20}\right) = 19 + \frac{1}{2} \Rightarrow \frac{39}{2}$$

15. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is:

(1) 4

(2) 1

(3) 2

(4) 3

Ans. (3)

Sol. α and β be the roots of $x^2 - 6x - 2 = 0$. So,

$$\alpha^2 - 6\alpha - 2 = 0 \Rightarrow \alpha^2 - 2 = 6\alpha$$

$$\beta^2 - 6\beta - 2 = 0 \Rightarrow \beta^2 - 2 = 6\beta$$

Now,

$$\begin{aligned} \frac{a_{10} - 2a_8}{3a_9} &= \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{3(\alpha^9 - \beta^9)} \\ &= \frac{(\alpha^{10} - 2\alpha^8) - (\beta^{10} - 2\beta^8)}{3(\alpha^9 - \beta^9)} \\ &= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{3(\alpha^9 - \beta^9)} \\ &= \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{3(\alpha^9 - \beta^9)} = \frac{6(\alpha^9 - \beta^9)}{3(\alpha^9 - \beta^9)} = \frac{6}{3} = 2 \end{aligned}$$

16. Let A be a 3×3 matrix with $\det(A) = 4$. Let R_i denote the i^{th} row of A . If a matrix B is obtained by performing the operation $R_2 \rightarrow 2R_2 + 5R_3$ on $2A$, then $\det(B)$ is equal to:

(1) 64

(2) 16

(3) 80

(4) 128

Ans. (1)

Sol.

$$A = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

$$2A = \begin{bmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 2R_{21} & 2R_{22} & 2R_{23} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{bmatrix}$$

$$R_2 \rightarrow 2R_2 + 5R_3$$

$$B = \begin{bmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 4R_{21} + 10R_{31} & 4R_{22} + 10R_{32} & 4R_{23} + 10R_{33} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{bmatrix}$$

$$|B| = \begin{vmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 4R_{21} + 10R_{31} & 4R_{22} + 10R_{32} & 4R_{23} + 10R_{33} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{vmatrix}$$

$$R_2 \rightarrow R_2 - 5R_3$$

$$|B| = \begin{vmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 4R_{21} & 4R_{22} & 4R_{23} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{vmatrix}$$

$$|B| = 2 \times 2 \times 4 \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix}$$

$$= 16 \times 4$$

$$= 64$$

17. The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is:

(1) $\frac{1}{2}$

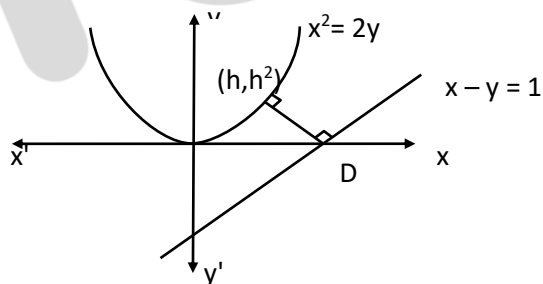
(2) 0

(3) $\frac{1}{2\sqrt{2}}$

(4) $\frac{1}{\sqrt{2}}$

Ans. (3)

Sol. Shortest distance must be along common normal



$$m_1 (\text{slope of line } x - y = 1) = 1 \Rightarrow \text{slope of perpendicular line} = -1$$

$$m_2 = \frac{2x}{2} = x \Rightarrow m_2 = h \Rightarrow \text{slope of normal} = -\frac{1}{h}$$

$$-\frac{1}{h} = -1 \Rightarrow h = 1$$

$$\text{so point is } \left(1, \frac{1}{2}\right)$$

$$D = \frac{\left|1 - \frac{1}{2} - 1\right|}{\sqrt{1+1}} = \frac{1}{2\sqrt{2}}$$



18. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is:

- (1) $\frac{1}{5}$
- (2) $\frac{2}{9}$
- (3) $\frac{97}{297}$
- (4) $\frac{122}{297}$

Ans. (3)

Sol. Total cases

$$= {}^4C_1 \times 9 \times 9 \times 9 - {}^3C_1 \times 9 \times 9$$

(as 4 digit number having 0 at thousands place have to be excluded)

For a number to have remainder 2 when divided by 5 its unit digit should be 2 or 7

Case 1: when unit digit is 2

$$\text{Number of four digit number} = {}^3C_1 \times 9 \times 9 - {}^2C_1 \times 9$$

Case 1: when unit digit is 7

$$\text{Number of four digit number} = 8 \times 9 \times 9$$

$$\text{So total number favorable cases} = 3 \times 9^2 - 2 \times 9 + 8 \times 9^2$$

$$\begin{aligned} \therefore \text{required Probability} &= \frac{(3 \times 9 \times 9) - (2 \times 9) + (8 \times 9 \times 9)}{(4 \times 9^3) - (3 \times 9^2)} \\ &= \frac{97}{297} \end{aligned}$$

19. $\operatorname{cosec}\left[2 \cot^{-1}(5) + \cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to:

- (1) $\frac{75}{56}$
- (2) $\frac{65}{56}$
- (3) $\frac{56}{33}$
- (4) $\frac{65}{33}$

Ans. (2)

Sol. $\operatorname{cosec}\left(2 \cot^{-1}(5) + \cos^{-1}\left(\frac{4}{5}\right)\right)$

$$= \operatorname{cosec} \left(2 \tan^{-1} \left(\frac{1}{5} \right) + \cos^{-1} \left(\frac{4}{5} \right) \right)$$

$$= \operatorname{cosec} \left(\tan^{-1} \left(\frac{2 \left(\frac{1}{5} \right)}{1 - \left(\frac{1}{5} \right)^2} \right) + \cos^{-1} \left(\frac{4}{5} \right) \right)$$

$$= \operatorname{cosec} \left(\tan^{-1} \left(\frac{5}{12} \right) + \cos^{-1} \left(\frac{4}{5} \right) \right)$$

$$\text{Let } \tan^{-1} \left(\frac{5}{12} \right) = \theta \Rightarrow \sin \theta = \frac{5}{13}, \cos \theta = \frac{12}{13}$$

$$\text{and } \cos^{-1} \left(\frac{4}{5} \right) = \phi \Rightarrow \cos \phi = \frac{4}{5} \text{ and } \sin \phi = \frac{3}{5}$$

$$= \operatorname{cosec}(\theta + \phi)$$

$$= \frac{1}{\sin \theta \cos \phi + \cos \theta \sin \phi}$$

$$= \frac{1}{\frac{5}{13} \cdot \frac{4}{5} + \frac{12}{13} \cdot \frac{3}{5}} = \frac{65}{56}$$

20. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to:

(1) $\frac{1+\sqrt{3}}{2}$

(2) $\frac{1-\sqrt{3}}{2}$

(3) $\frac{\sqrt{3}}{2}$

(4) $\frac{1}{2}$

Ans. (1)

Sol.

$$2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right) - \left[2 \cos^2 \left(\frac{x+y}{2} \right) - 1 \right] = \frac{3}{2}$$

$$\Rightarrow 2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right) - 2 \cos^2 \left(\frac{x+y}{2} \right) = \frac{1}{2}$$

$$\Rightarrow 4 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right) - 4 \cos^2 \left(\frac{x+y}{2} \right) = 1 = \cos^2 \left(\frac{x-y}{2} \right) + \sin^2 \left(\frac{x-y}{2} \right)$$

$$\Rightarrow 4 \cos^2 \left(\frac{x+y}{2} \right) + \cos^2 \left(\frac{x-y}{2} \right) - 4 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right) + \sin^2 \left(\frac{x-y}{2} \right) = 0$$



$$\Rightarrow \left(\cos\left(\frac{x-y}{2}\right) - 2 \cos\left(\frac{x+y}{2}\right) \right)^2 + \sin^2\left(\frac{x-y}{2}\right) = 0$$

$$\Rightarrow \sin\frac{x-y}{2} = 0 \Rightarrow x = y$$

$$\text{And } \cos\frac{x-y}{2} = 2 \cos\frac{x+y}{2}$$

$$\Rightarrow \cos x = \frac{1}{2} = \cos y$$

$$\therefore \sin x + \cos y = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$$

1. If $\lim_{x \rightarrow 0} \frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)}$ exists and is equal to b, then the value of $a - 2b$ is _____.

Ans. (5)

Sol. $\lim_{x \rightarrow 0} \frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)}$

Applying L'Hospital Rule

$$\lim_{x \rightarrow 0} \frac{a - 4e^{4x}}{a(e^{4x} - 1) + ax(4e^{4x})}$$

So for limit to exist, $a = 4$

Applying L'Hospital Rule

$$\lim_{x \rightarrow 0} \frac{-16e^{4x}}{a(4e^{4x}) + a(4e^{4x}) + ax(16e^{4x})}$$

$$\frac{-16}{4a + 4a} = \frac{-16}{32} = -\frac{1}{2} = b$$

$$a - 2b = 4 - 2\left(-\frac{1}{2}\right) = 4 + 1 = 5$$

2. A line is a common tangent to the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then $2(a+c)$ is equal to _____.

Ans. (9)

Sol. Circle: $(x - 3)^2 + y^2 = 9$

Parabola: $y^2 = 4x$

Let common tangent equation be $y = mx + \frac{a}{m}$

$$\Rightarrow y = mx + \frac{1}{m}$$

$$\Rightarrow m^2x - my + 1 = 0$$

the above line is also tangent to circle

$$(x - 3)^2 + y^2 = 9$$

$$\therefore \perp \text{ from } (3, 0) \text{ to line} = 3$$

$$\Rightarrow \left| \frac{3m^2 - 0 + 1}{\sqrt{m^2 + m^4}} \right| = 3$$

$$\Rightarrow (3m^2 + 1)^2 = 9(m^2 + m^4)$$

$$\Rightarrow m = \pm \frac{1}{\sqrt{3}}$$

\therefore tangent is

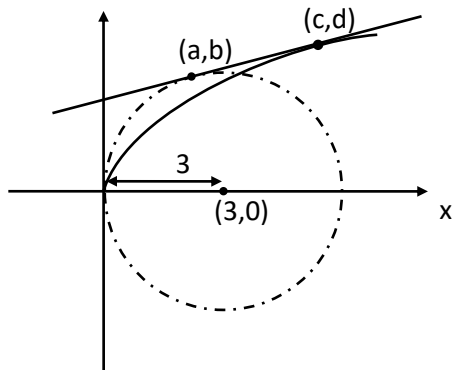
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$$y = \frac{1}{\sqrt{3}}x + \sqrt{3} \quad \text{or} \quad y = -\frac{1}{\sqrt{3}}x - \sqrt{3}$$

(it will be used) (rejected)

$$m = \frac{1}{\sqrt{3}}$$



for Parabola, point of contact is $\left(\frac{a}{m^2}, \frac{2a}{m}\right) \equiv (3, 2\sqrt{3}) \equiv (c, d)$

Solving Circle $(x - 3)^2 + y^2 = 9$ & line equation $y = \frac{1}{\sqrt{3}}x + \sqrt{3}$

$$(x - 3)^2 + \left(\frac{1}{\sqrt{3}}x + \sqrt{3}\right)^2 = 9$$

$$\Rightarrow x^2 + 9 - 6x + \frac{1}{3}x^2 + 3 + 2x = 9$$

$$\Rightarrow \frac{4}{3}x^2 - 4x + 3 = 0$$

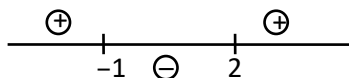
$$\Rightarrow x = \frac{3}{2} = a$$

$$\therefore 2(a + c) = 2\left(\frac{3}{2} + 3\right) = 9$$

3. The value of $\int_{-2}^2 |3x^2 - 3x - 6| dx$ is _____.

Ans. (19)

Sol. $x^2 - x - 2 = (x - 2)(x + 1)$



$$3 \int_{-2}^2 |x^2 - x - 2| dx$$



$$\begin{aligned}
 &= 3 \left\{ \int_{-2}^{-1} (x^2 - x - 2) dx + \int_{-1}^2 (-x^2 + x + 2) dx \right\} \\
 &= 3 \left[\left(\frac{x^3}{3} - \frac{x^2}{2} - 2x \right)_{-2}^{-1} - \left(\frac{x^3}{3} - \frac{x^2}{2} - 2x \right)_{-1}^2 \right] \\
 &= 19
 \end{aligned}$$

4. If the remainder when x is divided by 4 is 3, then the remainder when $(2020+x)^{2022}$ is divided by 8 is _____.

Ans. (1)

Sol. Let $x = 4k + 3$

$$\begin{aligned}
 &(2020 + x)^{2022} \\
 &= (2020 + 4k + 3)^{2022} \\
 &= (2024 + 4k - 1)^{2022} \\
 &= (4A - 1)^{2022} \\
 &= {}^{2022}C_0(4A)^{2022}(-1)^0 + {}^{2022}C_1(4A)^{2021}(-1)^1 + \dots + {}^{2022}C_{2021}(4A)^1(-1)^{2021} + {}^{2022}C_{2022}(4A)^0(-1)^{2022}
 \end{aligned}$$

Which will be of the form $8\lambda + 1$

So, Remainder is 1.

5. A line L passing through origin is perpendicular to the lines

$$L_1: \vec{r} = (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}$$

$$L_2: \vec{r} = (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}$$

If the co-ordinates of the point in the first octant on L_2 at the distance of $\sqrt{17}$ from the point of intersection of L and L_1 are (a, b, c) , then $18(a+b+c)$ is equal to _____.

Ans. (44)

Sol. $L_1: \vec{r} = (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}$

$$L_1: \frac{x-3}{1} = \frac{y+1}{2} = \frac{z-4}{2} \Rightarrow \text{D.R. of } L_1 = 1, 2, 2$$

$$L_2: \vec{r} = (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}$$

$$L_2: \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} \Rightarrow \text{D.R. of } L_2 = 2, 2, 1$$



D.R. of required L is \perp to L_1 & L_2

$$\therefore \text{D.R. of } L \text{ is parallel to } (L_1 \times L_2) \Rightarrow \langle -2, 3, -2 \rangle$$

$$\therefore \text{Equation of } \ell : \frac{x}{2} = \frac{y}{-3} = \frac{z}{2}$$

Solving L & L_1

$$\Rightarrow (2\lambda, -3\lambda, 2\lambda) = (\mu + 3, 2\mu - 1, 2\mu + 4)$$

$$\Rightarrow \mu = -1, \lambda = 1$$

So, intersection point $P(2, -3, 2)$

Let, $Q(2v + 3, 2v + 3, v + 2)$ be required point on L_2

$$\text{Now, } PQ = \sqrt{(2v + 3 - 2)^2 + (2v + 3 + 3)^2 + (v + 2 - 2)^2} = \sqrt{17}$$

$$\Rightarrow (2v + 1)^2 + (2v + 6)^2 + (v)^2 = 17$$

$$\Rightarrow 9v^2 + 28v + 20 = 0$$

$$\Rightarrow v = -2 \text{ (rejected), } \frac{10}{9} \text{ (accepted)}$$

$$Q\left(3 - \frac{20}{9}, 3 - \frac{20}{9}, 2 - \frac{10}{9}\right)$$

$$\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$$

$$\therefore 18(a + b + c)$$

$$= 18\left(\frac{7}{9} + \frac{7}{9} + \frac{8}{9}\right)$$

$$= 44$$

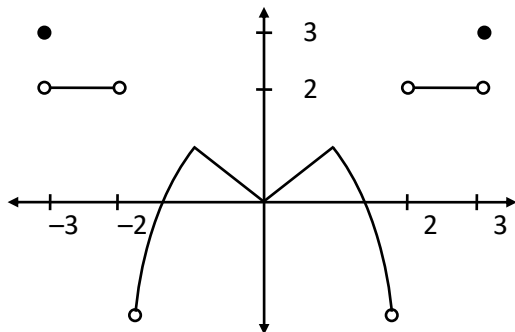
6. A function f is defined on $[-3, 3]$ as

$$f(x) = \begin{cases} \min\{|x|, 2 - x^2\}, & -2 \leq x \leq 2 \\ [|x|], & 2 < |x| \leq 3 \end{cases}$$

where $[x]$ denotes the greatest integer $\leq x$. The number of points, where f is not differentiable in $(-3, 3)$ is _____.

Ans. (5)

Sol.



Points of non-differentiability in $(-3, 3)$ are at $x = -2, -1, 0, 1, 2$.

i.e. 5 points.

7. If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to _____.

Ans. 4

Sol. $4y^3 \frac{dy}{dx} = 1$ and $x \frac{dy}{dx} + y = 0$

$$m_1 = \frac{1}{4y_1^3} = \frac{dy}{dx} = \frac{-y_1}{x_1} = m_2$$

$$m_1 m_2 = -1$$

$$\frac{1}{4y_1^3} \times \frac{-y_1}{x_1} = -1 \because x_1 = y_1^4$$

$$\frac{1}{4y_1^6} = 1 \text{ and } x_1 y_1 = k$$

$$y_1^6 = \frac{1}{4} \Rightarrow k = y_1^5$$

$$\Rightarrow k^6 = y_1^{30}$$

$$\Rightarrow k^6 = \left(\frac{1}{4}\right)^5$$

$$\therefore (4k)^6 = 4^6 \times k^6 = 4$$



8. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is _____.

Ans. (45)

Sol.

$$\begin{aligned} \therefore 7^n &= (10-3)^n = 10K + (-3)^n \\ \therefore 7^n + 3^n &= 10K + (-3)^n + 3^n \end{aligned} \quad \begin{cases} \rightarrow 10K \text{ if } n = \text{odd} \\ \rightarrow 10K + 2 \cdot 3^n \text{ if } n = \text{even} \end{cases}$$

Let $n = 2t$; $t \in \mathbb{N}$

$$3^n = 3^{2t} = (10-1)^t$$

$$= 10p + (-1)^t$$

$$= 10p \pm 1$$

\therefore if $n = \text{even}$ then $7^n + 3^n$ will not be multiple of 10

So if n is odd then only $7^n + 3^n$ will be multiple of 10

$$\therefore n = 11, 13, 15, \dots, 99$$

$$\therefore 45$$

9. Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors \vec{a} and \vec{b} is $8\sqrt{3}$ square units, then $\vec{a} \cdot \vec{b}$ is equal to _____.

Ans. (2)

Sol. $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$

$$\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$$

$$\text{Area of parallelogram} = |\vec{a} \times \vec{b}|$$

$$= |(\hat{i} + \alpha\hat{j} + 3\hat{k}) \times (3\hat{i} - \alpha\hat{j} + \hat{k})|$$

$$8\sqrt{3} = |(4\alpha)\hat{i} + 8\hat{j} - (4\alpha)\hat{k}|$$

$$(64)(3) = 16\alpha^2 + 64 + 16\alpha^2$$

$$\Rightarrow \alpha^2 = 4$$

$$\text{Now, } \vec{a} \cdot \vec{b} = 3 - \alpha^2 + 3$$

$$= 6 - \alpha^2$$

$$= 6 - 4$$

$$= 2$$



10. If the curve $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines, $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to _____.

Ans. 1

Sol. Given,

$$(2xy^2 - y)dx + xdy = 0$$

$$\Rightarrow \frac{dy}{dx} + 2y^2 - \frac{y}{x} = 0$$

$$\Rightarrow \frac{1}{y^2} \frac{dy}{dx} + \frac{1}{y} \left(\frac{1}{x}\right) = 2$$

$$\text{Let, } \frac{1}{y} = z$$

$$\frac{1}{y^2} \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow \frac{dz}{dx} + z \left(\frac{1}{x}\right) = 2$$

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = x$$

$$\therefore z(x) = \int 2(x) dx = x^2 + c$$

$$\Rightarrow \frac{x}{y} = x^2 + c$$

As it passes through P(2, 1)

[Point of intersection of $2x - 3y = 1$ and $3x + 2y = 8$]

$$\therefore \frac{2}{1} = 4 + c$$

$$\Rightarrow c = -2$$

$$\Rightarrow \frac{x}{y} = x^2 - 2$$

Put $x = 1$

$$\frac{1}{y} = 1 - 2 = -1$$

$$\Rightarrow y(1) = -1$$

$$\Rightarrow |y(1)| = 1$$