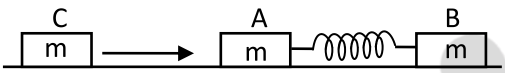
JEE Main 2021 Paper

Date: 17th March 2021 (Shift-2)

Subject: Physics

Section - A

1. Two identical blocks A and B each of mass m resting on the smooth horizontal floor are connected by a light spring of natural length L and spring constant K. A third block C of mass m moving with a speed v along the line joining A and B collides elastically with A. The maximum compression in the spring is



a. $\sqrt{\frac{mv}{2K}}$

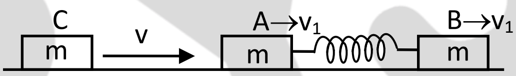
b. $\sqrt{\frac{m}{2K}}$

c. $\sqrt{\frac{mv}{K}}$

d. $v\sqrt{\frac{m}{2K}}$

Answer (d)

Sol.

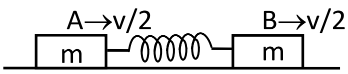


Let v_1 be the common velocity of blocks A and B at the maximum compression of the spring

From conservation of momentum

$mv = mv_1 + mv_1$

$v_1 = \frac{V}{2}$



& from energy conservation

$$\frac{1}{2}mv^2 = \left(\frac{1}{2}\times m\left(\frac{v}{2}\right)^2\right)\times 2 + \frac{1}{2}Kx^2$$

$$\frac{mv^2}{2} - \frac{mv^2}{4} = \frac{1}{2}kx^2$$

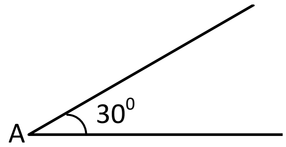
$$\frac{mv^2}{4} = \frac{1}{2}kx^2$$

Then the maximum compression in the spring is

$$x = \sqrt{\frac{mv^2}{2K}}$$

$$x = v\sqrt{\frac{m}{2K}}$$

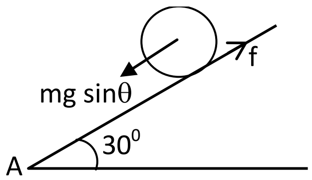
2. **A solid sphere of mass $2\ kg$ radius $0.5m$ is rolling with an initial speed of $1\ ms^{-1}$ goes up an inclined plane which makes an angle of 30^0 with the horizontal plane, without slipping. How long will the sphere take to return to the starting point A ?**



a. 0.80 s
b. 0.60 s
c. 0.52 s
d. 0.57 s

Answer (d)

Sol.



$a = \frac{g\sin\theta}{1+c}$

For a solid sphere c = $\frac{2}{5}$

$a = \frac{9.8 \sin 30^{\circ}}{1+\frac{2}{5}}$

a = 3.5 m/sec^2

Time of ascent is given by

v = u + at

0 = 1 –3.5 t

t = $\frac{1}{3.5}$ sec.

Time of decent

t = $\frac{1}{3.5}$ sec. [due to symmetry of motion]

Total time $T = \frac{2}{3.5}$ =0.57 sec.

3. **If one mole of a polyatomic gas has two vibrational modes and β is the ratio of molar specific heats for polyatomic gas $\left(\beta = \frac{C_p}{C_v}\right)$ then the value of β is :**

a. 1.35
b. 1.02
c. 1.25
d. 1.2

Answer (d)

Sol.

Degree of freedom of polyatomic gas

Here, no of translational dof = 3,

no of rotational dof = 3

and no of vibrational dof = 2×2 = 4

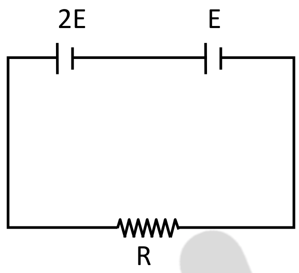
Therefore, total no of dof

f = 3 + 3 + 4 = 10

$\beta = \frac{C_P}{C_V} = 1 + \frac{2}{f} = 1 + \frac{2}{10}$

$\beta = \frac{12}{10} = 1.2$

4. **Two cells of emf $2E$ and E with internal resistance r_1 and r_2 respectively are connected in series to an external resistor R (see figure). The value of R, at which the potential difference across the terminals of the first cell becomes zero is**



a. $r_1 - r_2$

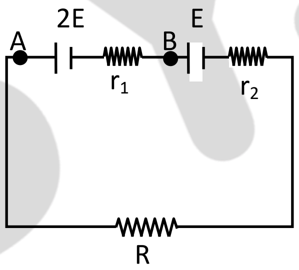
b. $r_1 + r_2$

c. $\frac{r_1}{2} + r_2$

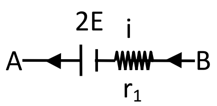
d. $\frac{r_1}{2} - r_2$

Answer (d)

Sol.



$i = \frac{3E}{R+r_1+r_2}$



If potential difference across terminals of first cell is zero

$V_A = V_B$

$2E = i r_1$

$2E = \frac{3E}{R+r_1+r_2} r_1$

$2R + 2r_1 + 2r_2 = 3r_1$

$R = \frac{r_1}{2} - r_2$

5. **A sound wave of frequency 245 Hz travels with the speed of 300 ms^{-1} along the positive x-axis. Each point of the medium moves to and fro through a total**

distance of 6 cm. What will be the mathematical expression of the travelling wave ?

a. $Y(x,t) = 0.03[sin5.1x - (0.2 \times 10^3)t]$

b. $Y(x,t) = 0.06[sin5.1x - (1.5 \times 10^3)t]$

c. $Y(x,t) = 0.06[sin0.8x - (0.5 \times 10^3)t]$

d. $Y(x,t) = 0.03[sin5.1x - (1.5 \times 10^3)t]$

Answer (d)

Sol.

General expression for wave travelling along positive x axis is of the form

Y = A sin (kx - ωt)

Here

A = $\frac{6}{2}$ = 3cm = 0.03 m

ω = 2π f = 2π ×245

ω = 1.54 ×10^3 $rad\ s^{-1}$

k = $\frac{\omega}{v} = \frac{1.54\times10^3}{300}$

k = 5.1 m^{-1}

y = 0.03 sin (5.1x – 1.5 × 10^3t)

6. A carrier signal $C(t) = 25\, sin\,(2.512 \times 10^{10}t)$ is amplitude modulated by a message signal $m(t) = 5\, sin\,(1.57 \times 10^8 t)$ and transmitted through an antenna. What will be bandwidth of the modulated signal ?

a. 1987.5 MHz

b. 2.01 GHz

c. 50 MHz

d. 8 GHz

Answer (c)

Sol.

Bandwidth of modulated signal is given by,

β = $2f_{m(t)}$

β = 2× $\frac{1.57\times10^8}{2\pi}$

β = 50 MHz

7. Two particles A and B of equal masses are suspended from two massless springs of spring constants K_1 and K_2 respectively. If the maximum velocities during oscillations are equal, the ratio of the amplitude of A and B is :

a. $\frac{K_1}{K_2}$

b. $\sqrt{\frac{K_1}{K_2}}$

c. $\sqrt{\frac{K_2}{K_1}}$

d. $\frac{K_2}{K_1}$

Answer (c)

Sol.

$\because V_{max} = A\omega$

Given $\omega_1 A_1 = \omega_2 A_2$

We know that $\omega = \sqrt{\frac{K}{m}}$

$\sqrt{\frac{k_1}{m}}A_1 = \sqrt{\frac{k_2}{m}}A_2$

$\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$

8. Match List I with List II

List I		List II	
(a)	**Phase difference between current and voltage in a purely resistive AC circuit**	**(i)**	$\frac{\pi}{2}$; **current leads voltage**
(b)	**Phase difference between current and voltage in a pure inductive AC circuit**	**(ii)**	**zero**
(c)	**Phase difference between current and voltage in a pure capacitive AC circuit**	**(iii)**	$\frac{\pi}{2}$; **current lags voltage**
(d)	**Phase difference between current and voltage in an LCR series circuit**	**(iv)**	$tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

Choose the most appropriate answer from the options given below :

a. (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

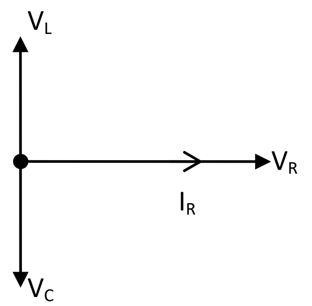
b. (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

c. (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

d. (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

Answer (d)

Sol.



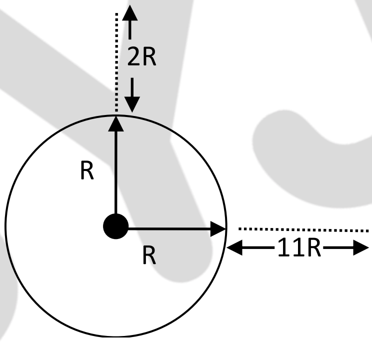
(a) phase difference b/w current & voltage in a purely resistive AC circuit is zero
(b) phase difference b/w current & voltage in a pure inductive AC circuit is $\frac{\pi}{2}$; current lags voltage.
(c) phase different b/w current & voltage in a pure capacitive AC circuit is $\frac{\pi}{2}$; current lead voltage.
(d) phase difference b/w current & voltage in an LCR series circuit is = $tan^{-1}\left|\left(\frac{X_C-X_L}{R}\right)\right|$

9. A geostationary satellite is orbiting around an arbitrary planet 'P' at a height of $11R$ above the surface of 'P', R being the radius of 'P'. The time period of another satellite in hours at a height of $2R$ from the surface of 'P' is ____. 'P' has the time period of rotation of 24 hours.

a. $\frac{6}{\sqrt{2}}$
b. 3
c. $6\sqrt{2}$
d. 5

Answer (b)

Sol.



From Kepler's law
$T^2 \propto R^3$
Since time period of a geostationary satellite is equal to the time period of rotation of planet i.e. 24 hours,
Using Kepler's law
$\left(\frac{24}{T}\right)^2 = \left(\frac{12R}{3R}\right)^3$
T = 3 sec

10. The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is $x = 0$ at $t = 0$; then its displacement after time ($t = 1$) is :

a. $v_0 + \frac{g}{2} + F$
b. $v_0+2g+3F$
c. v_0+g+F
d. $v_0 + \frac{g}{2} + \frac{F}{3}$

Answer (d)

Sol.

$v = v_0 + gt + Ft^2$

$\frac{dx}{dt} = v_0 + gt + Ft^2$

$\int_{x=0}^{x} dx = \int_{t=0}^{t=1} (v_0 + gt + Ft^2)\, dt$

$x = \left[v_0 t + \frac{gt^2}{2} + \frac{Ft^3}{3}\right]_{t=0}^{t=1}$

$x = v_0 + \frac{g}{2} + \frac{F}{3}$

11. A block of mass 1 kg attached to a spring is made to oscillate with an initial amplitude of 12 cm. After 2 minutes the amplitude decreases to 6 cm. Determine the value of the damping constant for this motion. (take $ln2 = 0.693$).

a. (1) 3.3×10^{2} kg s^{-1}
b. 5.7×10^{-3} kg s^{-1}
c. 1.16×10^{-2} kg s^{-1}
d. 0.69×10^{2} kg s^{-1}

Answer (c)

Sol.

Amplitude of a damped oscillation is given by

$A = A_o\, e^{\frac{-b}{2m}t}$

$6 = 12\, e^{\frac{-b}{2\times1}\times120}$

$6 = 12\, e^{-b\times60}$

$\frac{1}{2} = e^{-60b}$

In(2) = 60b

$b = \frac{ln(2)}{60} = 1.16\times10^{-2} Kg/s$

12. An object is located at 2 km beneath the surface of the water. If the fractional compression $\frac{\Delta V}{V}$ is 1.36%, the ratio of hydraulic stress to the corresponding hydraulic strain will be ……..

[Given : density of water is 1000 $kgm-3$ and $g = 9.8\ ms-2$]

a. 2.26×10^{9} Nm^{-2}
b. 1.96×10^{7} Nm^{-2}
c. 1.44×10^{7} Nm^{-2}
d. 1.44×10^{9} Nm^{-2}

Answer (d)

Sol.

Bulk modulus

$\beta = \frac{\Delta p}{\frac{\Delta V}{V}}$

$$\beta = \frac{\rho g h}{\frac{\Delta V}{V}} = \frac{1000 \times 9.8 \times 2 \times 10^3}{\frac{1.36}{100}}$$

$\beta = 1.44 \times 10^9\ N/m^2$

13. Two identical photocathodes receive the light of frequencies f_1 and f_2 respectively. If the velocities of the photo-electrons coming out are v_1 and v_2 respectively, then

a. $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$

b. $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{\frac{1}{2}}$

c. $v_1^2 + v_2^2 = \frac{2h}{m}[f_1 + f_2]$

d. $v_1^2 - v_2^2 = \frac{2h}{m}[f_1 - f_2]$

Answer (d)

Sol.

Using expression for K.E of photoelectrons

$\frac{1}{2}mv_1^2 = hf_1 - \phi$ ____(1)

$\frac{1}{2}mv_2^2 = hf_2 - \phi$ ____(2)

[ϕ is the same for same material of photocathodes]

Subtracting equation (1) by equation (2)

$\frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = hf_1 - hf_2$

$v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$ $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$

14. The atomic hydrogen emits a line spectrum consisting of various series. Which series of hydrogen atomic spectra lies in the visible region ?

a. Balmer series

b. Lyman series

c. Brackett series

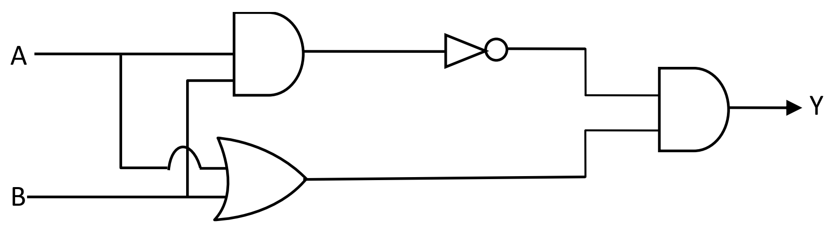
d. Paschen series

Answer (a)

Sol.

Balmer series of hydrogen atomic spectrum is lying in the visible region, when electron jumps from a higher energy level to n = 2 orbit.

15. Which one of the following will be the output of the given circuit ?



a. NAND Gate

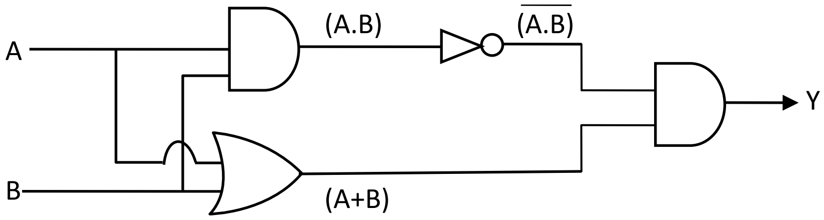
b. AND Gate

c. XOR Gate

d. NOR Gate

Answer (c)

Sol.



Output of the above circuit can be obtained as

y = $\left(\underline{A.B}\right).(A+B)$

=$(A^{-}+B^{-}).(A+B)$ [De Morgan's law]

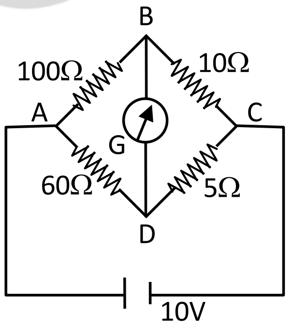
= $A^{-}\ A+A^{-}B+A\,B^{-}+B^{-}B$

= $0+A^{-}B+A\,B^{-}\ +0$

y = $A^{-}B+A\,B^{-}$

which is XOR gate

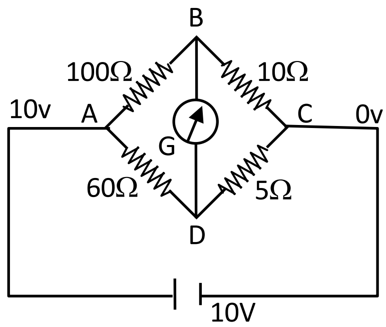
16. **The four arms of a wheatstone bridge have resistances as shown in the figure. A galvanometer of 15 Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC.**



a. 4.87 mA
b. 4.87 μA
c. 2.44 μA
d. 2.44 mA

Answer (a)

Sol.



Applying KCL for point B,

$$\frac{V_B-10}{100}+\frac{V_B-V_D}{15}+\frac{V_B-0}{10}=0$$

$$\frac{V_B-10}{20}+\frac{V_B-V_D}{3}+\frac{V_B}{2}=0$$

$$3V_B-30+20V_B-20V_D+30V_B=0$$

$53V_B - 20V_D = 30$ ____(1)

Similarly applying KCL for point D,

$$\frac{V_D-10}{60}+\frac{V_D-V_B}{15}+\frac{V_D-0}{5}=0$$

$V_D - 10 + 4V_D - 4V_B + 12V_D = 0$

$- 4V_B + 17V_D = 10$ _____(2)

after solving equation (1) & (2)

$V_D = 0.792$ volt

$V_B = 0.865$ volt

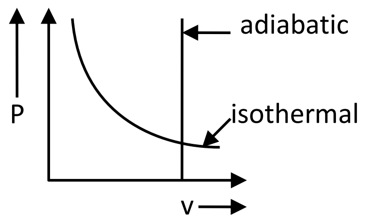
Then the current through the galvanometer

$= \frac{V_B-V_D}{R}$

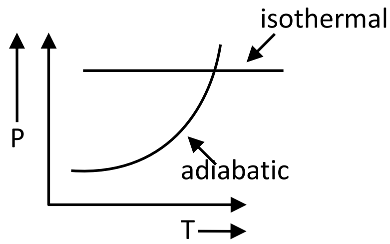
$= \frac{0.865-0.792}{15}$

= 4.67 mA

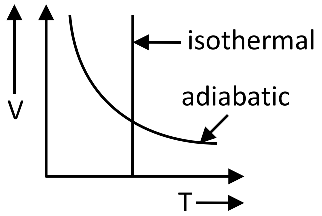
17. Which one is the correct option depicting the two different thermodynamic processes ?



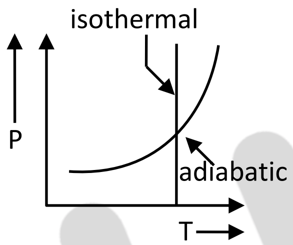
(a)



(b)



(c)



(d)

a. (c) and (d)
b. (b) and (c)
c. (a) only
d. (c) and (a)

Answer (a)

Sol.

Isothermal process means constant temperature which is only possible in graph (c) & (d)

for adiabatic process

pv^{γ} = constant ……(1)

$\because$ PV = nRT

$p\ \alpha \frac{T}{v}$

So $\frac{T}{v} v^{\gamma}$ = constant

Tv^{r-1} = constant ….(2)

Similarly,

$v \propto \frac{T}{p}$

$P\left(\frac{T}{P}\right)^{\gamma}$ =constant

$P^{1-r}\ T^{r}$ = constant ….(3)

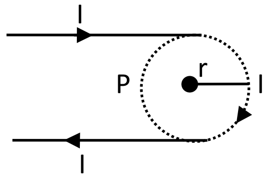
$\because$ differentiating equation (3) w.r.to temp.

$(P)^{1-\gamma}\gamma(T)^{\gamma-1}dT + (T)^{\gamma}(1-\gamma)(P)^{1-\gamma-1}dP = 0$

$\frac{dP}{dT} = -\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{P}{T}\right)$

$\frac{dP}{dT} = \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{P}{T}\right)$, It gives (+ve) slope.

18. A hairpin like shape as shown in figure is made by bending a long current carrying wire. What is the magnitude of a magnetic field at point P which lies on the centre of the semicircle ?



a. $\frac{\mu_0 I}{4\pi r}(2-\pi)$

b. $\frac{\mu_0 I}{4\pi r}(2+\pi)$

c. $\frac{\mu_0 I}{2\pi r}(2+\pi)$

d. $\frac{\mu_0 I}{2\pi r}(2-\pi)$

Answer (b)

Sol.

Magnetic field due to each of the straight wire = $\frac{\mu_0 I}{4\pi r}$

Magnetic field due to semicircular arc = $\frac{\mu_0 I}{4r}$

Thus, total magnetic field at point P

B = $\frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r}$

= $\frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{4r}$

B = $\frac{\mu_0 I}{4\pi r}(2+\pi)$

19. A rubber ball is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball. (Take $g = 10\ ms-2$)

a. 2.50 ms^{-1}

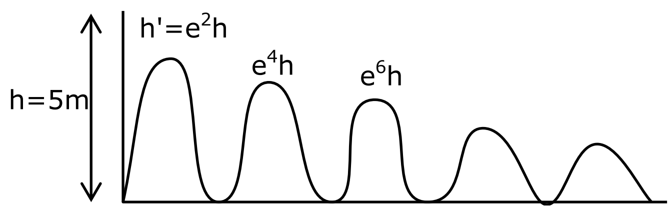
b. 3.50 ms^{-1}

c. 3.0 ms^{-1}

d. 2.0 ms^{-1}

Answer (a)

Sol.



Total distance d = h + $2e^2h + 2e^4h + 2e^6h + 2e^8h$ +....

d = h + $2e^2h\,(1+e^2+e^4+e^6+.....)$

d = h + $2e^2h\left(\frac{1}{1-e^2}\right)$

$d = \frac{(1-e^2)h+2e^2h}{1-e^2} = \frac{h(1+e^2)}{1-e^2}$

Total time = T + 2 eT + 2e^{2}T + 2e^3 T +…..

Total time = T + 2e T (1+e + e^2 + e^3+……)

= T + 2e.T $\left(\frac{1}{1-e}\right)$

Total time = $\frac{T(1+e)}{1-e}$

Average speed of the ball

$V_{avg} = \frac{h\frac{(1+e^2)}{(1-e^2)}}{T\left(\frac{1+e}{1-e}\right)}$

$= \frac{5}{1}\left(\frac{1+e^2}{(1+e)(1-e)}\frac{(1-e)}{(1+e)}\right)$

$V_{avg} = \frac{5(1+e^2)}{(1+e)^2}$

$\because$ h′ = e^{2}h

$\frac{81}{100} = e^2$

$e = \frac{9}{10} = 0.9$

$V_{avg} = \frac{5\left(1+\frac{81}{100}\right)}{(1+0.9)^2}$

= 2.50 m/sec.

20. What happens to the inductive reactance and the current in a purely inductive circuit if the frequency is halved ?

a. Both , including reactance and current will be doubled

b. Both, inductive reactance and current will be halved

c. Inductive reactance will be halved and current will be doubled

d. Inductive reactance will be doubled and current will be halved.

Answer (c)

Sol.

$X_L = \omega L$

If frequency is halved, $X_L' = \left(\frac{X_L}{2}\right)$

[inductive reactance is halved]

$\because\ I = \frac{V}{X_L}$

& I′ = $\frac{2V}{X_L} = 2I$ [current will be doubled]

SECTION –B

1. **The electric field intensity produced by the radiation coming from a 100 W bulb at a distance of 3 m is E. The electric filed intensity produced by the radiation coming from 60 W at the same distance is $\sqrt{\frac{x}{5}}E$. Where the value of x = _______**

Answer (3)

Sol.

Intensity of electromagnetic radiation

I = $\frac{1}{2} C \in_0 E^2$

where E is electric field intensity at a point

$E^2 \, \alpha \, I$

I = $\frac{Power}{Area}$

$E^2 \propto \frac{P}{A}$

$E \propto \sqrt{P}$

[at the same distance, A will be the same]

$\frac{E'}{E} = \sqrt{\frac{60}{100}}$

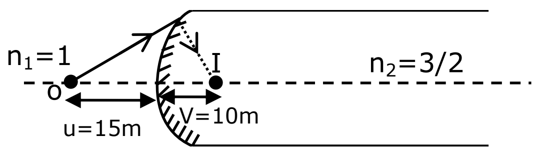
$E' = \sqrt{\frac{3}{5}}\, E$

So the value of x = 3

2. **The image of an object placed in air formed by convex refracting surface is at a distance of 10 m behind the surface. The image is real and is at $\frac{2^{nd}}{3}$ of the distance of the object from the surface. The wavelength of light inside the surface is $\frac{2}{3}$ times the wavelength in air, The radius of the curved surface is $\frac{x}{13}$ m . The value of 'x' is ____**

Answer: (30)

Sol.



$\frac{n_2}{v} - \frac{n_1}{u} = \left(\frac{n_2 - n_1}{R}\right)$

Since wavelength of light inside the surface is $\frac{3}{2}$ times the wavelength in air, $n_2 = \frac{3}{2} \times n_1 = \frac{3}{2}$

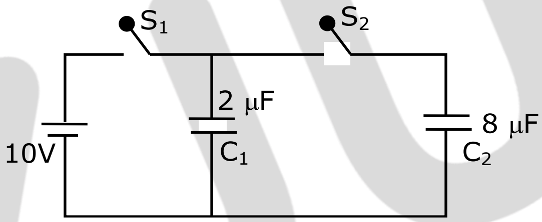
$\frac{\frac{3}{2}}{10} - \frac{1}{(-15)} = \frac{\left(\frac{3}{2}-1\right)}{R}$

$\frac{3}{20} + \frac{1}{15} = \frac{1}{2R}$

R = $\frac{150}{65}$

R = $\frac{30}{13}m$, Thus, the value of x = 30

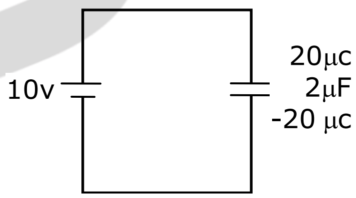
3. **A $2\mu F$ capacitor C_1 is first charged to a potential difference of $10V$ using a battery. Then the battery is removed and the capacitor is connected to an uncharged capacitor C_2 of 8 μF. The charge in C_2 on equilibrium condition is ________ μC. (Round off to the Nearest Integer)**

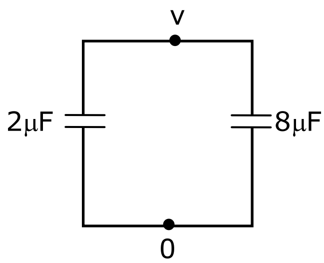


Answer (16)

Sol.

After capacitor C_1 is fully charged,





When battery is removed & the capacitor is connected

At equilibrium condition, let voltage across each capacitor be V.

Then, using conservation of charge

2V + 8V = 20

10V = 20

V = 2 volt

Q Q = CV

Q = 8×2 = 16μc

4. A particle of mass m moves in a circular orbit in a central potential field $U(r) = U_{0r^4}$. If Bohr's quantization conditions are applied, radii of possible orbital r_n vary with $n^{\frac{1}{\alpha}}$, where α is ____

Answer (3)

Sol.

$F^{\rightarrow} = -\frac{du^{\rightarrow}}{dr}$

$= — \frac{d}{dr}(U_0 r^4)$

$F^{\rightarrow} = -4U_0 r^3$

$\because \frac{mv^2}{r} = 4U_0 r^3$

$mv^2 = 4U_0 r^4$

Then $v \propto r^2$

$\because mvr = \frac{nh}{2\pi}$

Thus, $r^3 \alpha\ n$

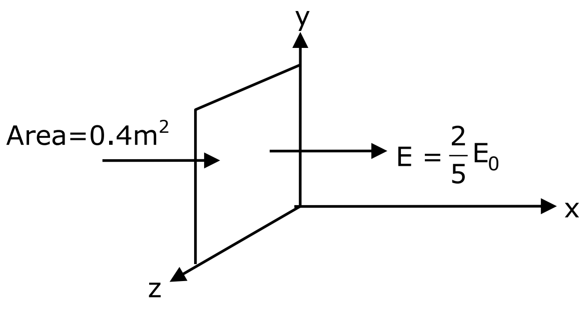
$r\ \alpha\ (n)^{\frac{1}{3}}$

So the value of $\alpha = 3$

5. The electric field in a region is given by $E^{\rightarrow} = \frac{2}{5}E_0 i^{\wedge} + \frac{3}{5}E_0 j^{\wedge}$ with $E_0 = 4.0 \times 10^3 \frac{N}{C}$. The flux of this field through a rectangular surface are $0.4\ m2$ parallel to $Y-Z$ plane is _______Nm^2C^{-1}.

Answer (640)

Sol.



From Gauss' law

$\phi = \oint E^{\rightarrow} . dA^{\rightarrow}$

$= \frac{2}{5}E_0 \times (0.4)$

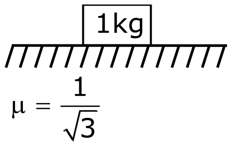
$= \frac{2}{5} \times 4 \times 10^3 \times 0.4$

ϕ = 640 Nm^2 c^{-1}

6. **A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force F N. The value of F will be _________. (Round off to the Nearest Integer)**
[Take $g = 10\ ms^{-2}$]

Answer (5)

Sol.



Minimum possible force ⇒

$F = \frac{\mu mg}{\sqrt{1+\mu^2}}$

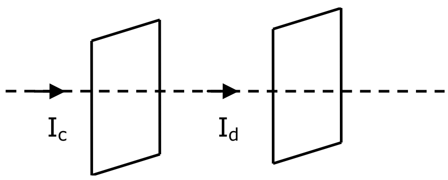
$F_{min} = \frac{\frac{1}{\sqrt{3}} \times 1 \times 10}{\sqrt{1+\frac{1}{3}}}$

F_{min} = 5 N

7. **Seawater at a frequency $f = 9 \times 10^2\ Hz$, has permittivity $\varepsilon = 80\epsilon_0$ and resistivity $\rho = 0.25\ \Omega m$. Imagine a parallel plate capacitor is immersed in seawater and is driven by an alternating voltage source $V(t) = V_0 \sin(2\pi f t)$. Then the conduction current density becomes 10x times the displacement current density after time $t = \frac{1}{800}$. The value of x is _________.**
(Given : $\frac{1}{4\pi\varepsilon_0}$ = $9 \times 10^9 Nm^2C^{-2}$)

Answer (6)

Sol.



Given:-
f = 9 × 10^2 Hz
$\in = \in_0 \in_r$
$\in = 80 \in_0$
So, $\in_r = 80$

ρ = 0.25 Ωm

V(t) = V_0 sin (2πft)

Displacement current, $I_d = \frac{dq}{dt} = \frac{cdv}{dt}$

$I_d = \frac{\epsilon_0 \epsilon_r A}{d} \frac{d}{dt}$ ($v_0$sin(2πft))

$I_d = \frac{\epsilon_0 \epsilon_r A}{d} V_0(2\pi f)$ cos(2πft) ……..(1)

Where di si the distance between plates & conduction current $I_c = \frac{V}{R}$

$I_c = \frac{V_0 sin(2\pi f t)}{\rho \frac{d}{A}} = \frac{A v_0 sin(2\pi f t)}{\rho d}$ ....(2)

Divide equation (1) and (2)

$\frac{I_d}{I_c} = \epsilon_0 \epsilon_r$ 2πf(ρ) cot (2πft)

$\frac{I_d}{I_c} = \frac{1}{4\pi \times 9 \times 10^9} \times 80 \times 2\pi \times 9 \times 10^2 \times (0.25) \times cot\left(2\pi \times 9 \times 10^2 \times \frac{1}{800}\right)$

$= \frac{10^3}{10^9}\left(cot\left(\frac{9\pi}{4}\right)\right)$

$= \frac{10^3}{10^9}$

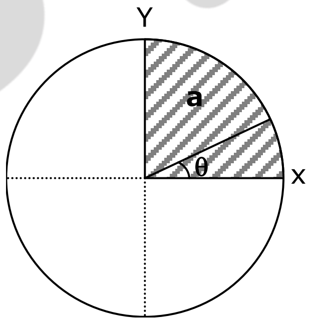
$\frac{I_d}{I_c} = \frac{1}{10^6}$

$I_c = 10^6 I_d$

So x = 6

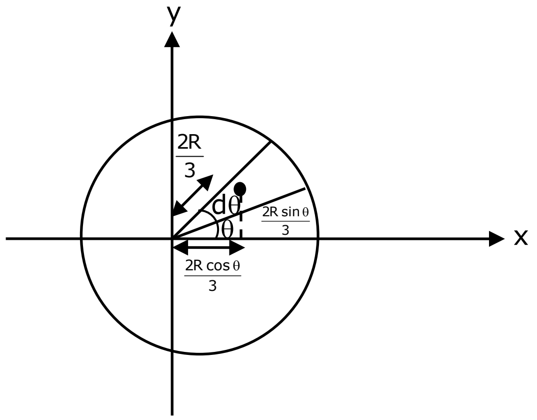
8. **The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{R}{\pi}, \frac{x}{3}\frac{R}{\pi}$. x is ____(Round off to the Nearest Integer)**

[a is an area as shown in the figure]



Answer (4)

Sol.



Each segment of disc subtending angle dθ can be considered as a triangle of height R and base Rdθ

Mass of a segment of disc subtending angle dθ

$dm = \sigma \frac{1}{2}\ R \times R d\theta$

$dm = \frac{\sigma R^2 d\theta}{2}$

Therefore, x coordinate of COM of quarter disc

$x_{cm} = \frac{\int x\, dm}{\int dm} = \frac{\int_0^{\pi/2} \frac{\sigma R^2}{2} d\theta \left(\frac{2R}{3} cos\theta\right)}{\int_0^{\pi/2} \frac{\sigma R^2}{2} d\theta}$

Here, x is the x-coordinate of COM of dm mass. And for a triangular section it's COM is at cross section of median which will b at a distance 2R/3 from vertex. That's why they have taken x-coordinate as $\frac{2R}{3} cos\theta$

$= \frac{2R}{3} \frac{\int_0^{\pi/2} cos\theta d\theta}{\int_0^{\pi/2} d\theta}$

$= \frac{2R}{3}\left(\frac{2}{\pi}\right)$

$= \frac{4R}{3\pi}$

So the value of x = 4

9. Suppose you have taken a dilute solution of oleic acid in such a way that its concentration becomes $0.01\ cm^3$ of oleic acid per cm^3 of the solution. Then you make a thin film of this solution (monomolecular thickness) of area $4\ cm^2$ by considering 100 spherical drops of radius $\left(\frac{3}{40\pi}\right)^{\frac{1}{3}} \times 10^{-3} cm$. Then the thickness of oleic acid layer will be $x \times 10^{-14}\ m$ where x is ________.

Answer (25)

Sol.

Volume of film = Area of film × thickness

Also, we know : Volume of film = Volume of 100 spherical drops

$4t_T = 100 \times \frac{4}{3}\pi r^3$

$= 100 \times \frac{4\pi}{3} \times \frac{3}{40\pi} \times 10^{-9}$

$=10^{-8}$ cm^3

Hence, thickness of film

$t_T = 25 \times 10^{-10} cm$

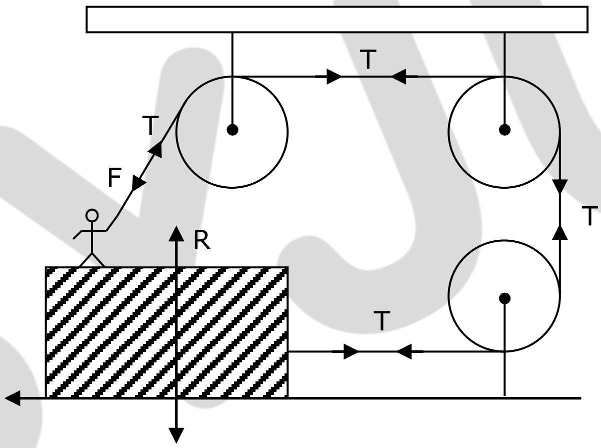
$=25\times10^{-12}$ m

& Thickness of oleic acid layer

$t_0 = 0.01\ t_T = 25 \times 10^{-14} m$

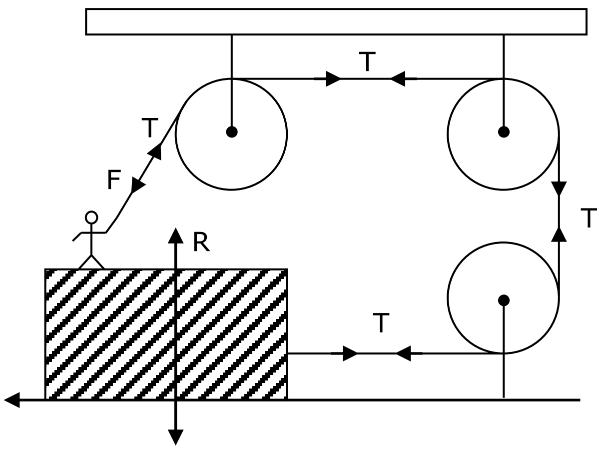
So, x = 25

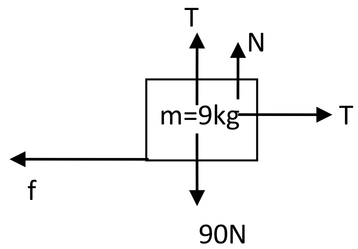
10. **A boy of mass 4 kg is standing on a piece of wood having mass 5 kg. If the coefficient of friction between the wood and the floor is 0.5, the maximum force that the boy can exert on the rope so that the piece of wood does not move from its place is _______N. (Round off to the Nearest Integer)**
[Take $g = 10\ ms^{-2}$]



Answer (30)

Sol.





[R is the reaction force between man and block while N from ground to block]

From FBD of [man + wooden block] system

$\because f = T$ [horizontal direction]

$\mu N = T$

$\mu(90-T) = T$

[Because T + N = 90 (in vertical direction)]

$0.5\,(90-T) = T$

$90-T = 2T$

$3T = 90 \Rightarrow T = 30$ N