Q.1: Explain why
(a) The blood pressure in humans is greater at the feet than at the brain
(b) Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100 km
(c) Hydrostatic pressure is a scalar quantity even though the pressure is force divided by area

Solution:

(i). The blood column to the feet is at a greater height than the head, thus the blood pressure in the feet is greater than that in the brain.

(ii). The density of the atmosphere does not decrease linearly with the increase in altitude, in fact, most of the air molecules are close to the surface. Thus, there is this nonlinear variation of atmospheric pressure.

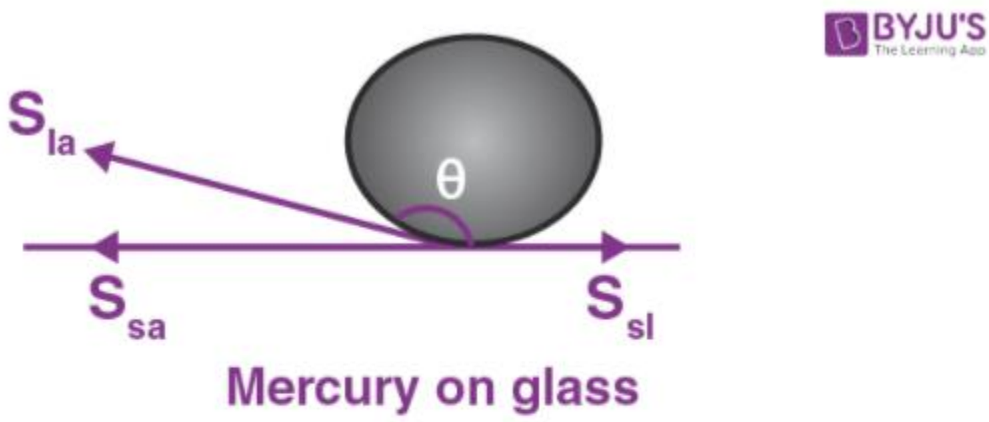
(iii). In hydrostatic pressure the force is transmitted equally in all direction in the liquid, thus there is no fixed direction of pressure making it a scalar quantity.

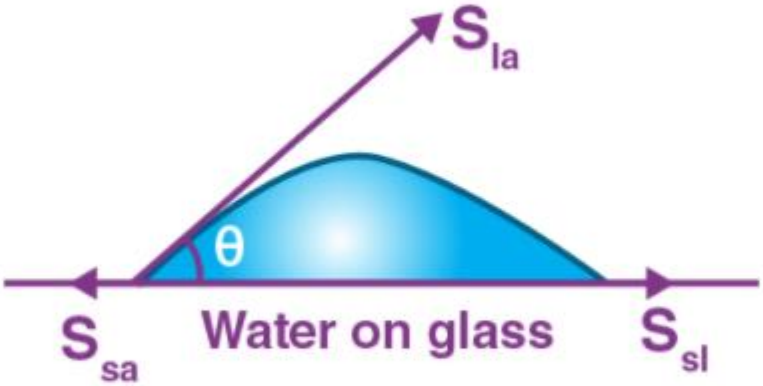
Q.2: Explain why
(a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.
(b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not.)
(c) Surface tension of a liquid is independent of the area of the surface
(d) Water with detergent disolved in it should have small angles of contact.
(e) A drop of liquid under no external forces is always spherical in shape

Solution:

(a) Water molecules have weak intermolecular forces and a strong force of attraction towards solids. Thus, they spill out. Whereas mercury molecules have a stronger intermolecular force of attraction and a weak attraction force towards solids, thus they form droplets.

(b) The angle of contact is the angle between the line tangent to the liquid surface at the point of contact and the surface of the liquid. It is donated by θ in the following diagram:





In the diagram $\mathbf{S_{sl}}$**,** $\mathbf{S_{la}}$ **and** $\mathbf{S_{sa}}$, are the respective interfacial tensions between the liquid-solid, liquid-air, and solid-air interfaces. At the line of contact, the surface forces between the three media are in equilibrium, i.e.,

$$cos\Theta = \left(\frac{S_{sa} - S_{la}}{S_{la}}\right)$$

Thus, for mercury, the angle of contact θ, is obtuse because $\mathbf{S_{sa} < S_{la}}$. And for water, the angle is acute because $\mathbf{S_{sl} < S_{la}}$

(c) A liquid always tends to acquire minimum surface area because of the presence of surface tension. And as a sphere always has the smallest surface area for a given volume, a liquid drop will always take the shape of a sphere under zero external forces.

(d) Surface tension is independent of the area of the liquid surface because it is a force depending upon the unit length of the interface between the liquid and the other surface, not the area of the liquid.

(e) Clothes have narrow pores that behave like capillaries, now we know that the rise of liquid in a capillary tube is directly proportional to **cos θ**. So a soap decreases the value of **θ** in order to increase the value of **cos θ**, allowing the faster rise of water through the pores of the clothes.

Q.3: Fill in the blanks using the word(s) from the list appended with each statement:
(a) The surface tension of liquids generally … with temperatures (increases/decreases)
(b) The viscosity of gases … with temperature, whereas the viscosity of liquids … with temperature (increases/decreases)
(c) For solids with an elastic modulus of rigidity, the shearing force is proportional to … , while for fluids, it is proportional to … (shear strain/rate of shear strain)
(d) For a fluid in a steady flow, the increase in flow speed at a constriction follows (conservation of mass / Bernoulli's principle)
(e) For the model of a plane in a wind tunnel, turbulence occurs at a … speed for turbulence for an actual plane (greater / smaller)

Solution:

(a) Decreases

(b) increases; decreases

(c) shear strain; rate of shear strain

(d) conservation of mass; Bernoulli's principle

(e) greater.

Q.4: Explain why
(a) To keep a piece of paper horizontal, you should blow over, not under, it
(b) When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers
(c) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection
(d) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel
(e) A spinning cricket ball in air does not follow a parabolic trajectory

Solution:

(a) If we blow over a piece of paper, velocity of air above the paper becomes more than that below it. As K.E. of air above the paper increases, so in accordance with Bernoulli's theorem its pressure energy and hence its pressure decreases.

Due to greater value of pressure below the piece of paper = atmospheric pressure, it remains horizontal and does not fall.

(b) As per the equation of continuity area × velocity = constant. When we try to close a water tap with our fingers, the area of cross-section of the outlet of water jet is reduced considerably as the openings between our fingers provide constriction (regions of smaller area)
Thus, velocity of water increases greatly and fast jets of water come through the openings between our fingers.

(c) The size of the needle controls the velocity of flow and the thumb pressure controls pressure. According to the Bernoulli's theorem $P + \frac{1}{2}pv^2$ = Constant

In this equation, the pressure P occurs with a single power whereas the velocity occurs with a square power. Therefore, the velocity has more effect compared to the pressure. It is for this reason that needle of the syringe controls flow rate better than the thumb pressure exerted by the doctor.

(d) This is because of principle of conservation of momentum. While the flowing fluid carries forward momentum, the vessel gets a backward momentum.
(e) A spinning cricket ball would have followed a parabolic trajectory has there been no air. But because of air the Magnus effect takes place. Due to the Magnus effect the spinning cricket ball deviates from its parabolic trajectory.

Q.5: A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter of 1.0 cm. What is the pressure exerted by the heel on the horizontal floor?

Solution:

Given:

Radius of the heel, r = $\frac{d}{2}$ = **0.005 m**

Mass of the lady, **m = 50 kg**

Area of the heel, **A** = $\pi r^2 = \pi (0.005)^2$= **7.85×10^{-5} m^2**

Force on the floor due to the heel: F = mg = 50 × 9.8 = **490 N**

Pressure exerted by the heel on the floor:

$$P = \frac{F}{A} = \frac{490}{7.85 \times 10^{-5}}$$

P = 6.24×10^6 **Nm^{-2}**

Q-6: Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m^{-3}. Determine the height of the wine column for normal atmospheric pressure

Solution:

We know:
Density of mercury, **$\rho_1 = 13.6 \times 10^3$ kg/m^3**
Height of the mercury column, h_1 = 0.76 m
Density of French wine, **ρ_2 = 984 kg/m^3**

Let the height of the French wine column **= h_2**
Acceleration due to gravity, **g = 9.8 m/s^2.**

We know that:
Pressure in the mercury column = Pressure in the wine column
$\rho_1 h_1 g = \rho_2 h_2 g$

$$\Rightarrow h_2 = \frac{\rho_1 h_1}{\rho_2} \Rightarrow h_2 = \frac{13.6 \times 10^3 \times 0.76}{984} = 10.5 \text{ m}$$

Q-7: A vertical off-shore structure is built to withstand a maximum stress of 10^9 Pa. Is the structure suitable for putting up on top of an oil well in the ocean ? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents

Solution:

Given:

The maximum stress the structure can handle, **P = 10^9 Pa**
Depth of the sea, **d =** 3 km **= 3×10^3 m**
Density of water, **$\rho = 10^3$ kg/m^3**
Acceleration due to gravity, **g = 9.8 m/s^2**

We know:
The pressure exerted by the seawater at depth, **d = ρdg** = 10^3 x 3 × 10^3 × 9.8 **= 2.94 × 10^7 Pa**

As the sea exerts a pressure lesser than the maximum stress the structure can handle, the structure can survive on the oil well in the sea.

Q-8: A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear?

Solution:

Given:
Maximum mass that can be lifted, **m = 3000 kg**
Area of cross-section of the load-carrying piston, **A =** 425 cm^2**= 425 × 10^{-4} m^2**

The maximum force exerted by the load,
F = **mg =** 3000 x 9.8 **= 29400 N**
The maximum pressure on the load carrying piston, **P = F / A**

$$P = \frac{29400}{425 \times 10^{-4}} = 6.917 \times 10^5 \text{ Pa}$$

In a liquid, the pressure is transmitted equally in all directions. Therefore, the maximum pressure on the smaller is 6.917 × 10^5 Pa

Q-9: A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit?

Solution:

Given:

Height of the spirit column, **h_1 =** 12.5 cm **= 0.125 m**

Height of the water column, **h_2 =** 10 cm **= 0.1 m**

Let, A and B be the points of contact between spirit and mercury and water and mercury, respectively.
P_0 = Atmospheric pressure
ρ_1 = Density of spirit
ρ_2 = Density of water

Pressure a **point A** = $P_0 + \rho_1 h_1 g$

Pressure at **point B** = $P_0 + \rho_2 h_2 g$

We know pressure at B and D is the same so;

$P_0 + \rho_1 h_1 g = P_0 + \rho_2 h_2 g$

$$\frac{\rho_1}{\rho_2} = \frac{h_2}{h_1} = \frac{10}{12.5} = 0.8$$

Therefore the specific gravity of water is 0.8.

Q-10: In the previous problem, if 15.0 cm of water and spirit each is further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms? (Specific gravity of mercury = 13.6)

Solution:

Given:

h_1 = 10.0 cm
ρ_1 = 1 g cm^{-3}
For spirit column in other arm of U-tube,
h_2 = 12.5 cm
p_2 = ?

Let h be the difference between the levels of mercury in the two arms.
Pressure exerted by height h, of the mercury column:
= hρg
= h × 13.6g … (i)
Difference between the pressures exerted by water and spirit:
= $\rho_1 h_1 g - \rho_2 h_2 g$
= g(25 × 1 – 27.5 × 0.8)
= 3g … (ii)
Equating equations (i) and (ii), we get:
13.6 hg = 3g
h = 0.220588 ≈ 0.221 cm

Q-11: Can Bernoulli's equation be used to describe the flow of water through a rapid in a river? Explain.

Solution:

Bernoulli's equation cannot be applied to the water flowing in a river because it is applicable only for ideal liquids in a streamlined flow and the water in a stream is turbulent.

Q-12: Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation? Explain

Solution:

No, it does not matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation, provided the atmospheric pressure at the two points where Bernoulli's equation applied to the system are significantly different.

Q-13: Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is 4.0 × 10^{-3} kg s^{-1}, what is the pressure difference between the two ends of the tube? (Density of glycerine = 1.3 × 10^3 kg m^{-3} and

viscosity of glycerine = 0.83 Pa s). [You may also like to check if the assumption of laminar flow in the tube is correct].

Solution:

Given:
Length of the horizontal tube, **l = 1.5 m**

Radius of the tube, r = 1 cm **= 0.01 m**

Diameter of the tube, d = 2r **= 0.02 m**

Glycerine is flowing at the rate of 4**.0 × 10^{-3} kg/s**

M = 4.0 × 10^{-3} kg/s

Density of glycerine, ρ = 1.3 × 10^3 kg m^{-3}

Viscosity of glycerine, η = 0.83 Pa s

We know, volume of glycerine flowing per sec:

$$V = \frac{M}{density} = \frac{4 \times 10^{-3}}{1.3 \times 10^{3}} = 3.08 \times 10^{-6}\ m^3/s$$

Using Poiseville's formula, we get:

$$V = \frac{\pi p' r^4}{8\,\eta\, l} \quad p' = \frac{V\, 8\,\eta\, l}{\pi r^4}$$

Where **p'** is the pressure difference between the two ends of the pipe.

$$p' = \frac{3.08 \times 10^{-6} \times 8 \times 0.83 \times 2}{\pi \times (0.01)^4} = 9.8 \times 10^2\ Pa$$

And, we know:

$R = \frac{4\,\rho\, V}{\pi\, d\, \eta}$, [Where R = Reynolds's number]

$$R = \frac{4 \times 1.3 \times 10^3 \times 3.08 \times 10^{-6}}{\pi \times 0.83 \times 0.02} = 0.3$$

Since the Reynolds's number is 0.3 which is way smaller than 2000, the flow of glycerine in the pipe is laminar.

Q-14: In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 m s^{-1} and 63 m s^{-1} respectively. What is the lift on the wing if its area is 2.5 m^2? Take the density of air to be 1.3 kg m^{-3}.

Solution:

Given:
Speed of wind on the upper side of the wing, **V_1 = 70 m/s**
Speed of wind on the lower side of the wing, **V_2 = 63 m/s**
Area of the wing, **A = 2.5 m^2**
Density of air, **ρ = 1.3 kg m^{-3}**

Using Bernoulli's theorem, we get:

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_2 - P_1 = \frac{1}{2}\left(\rho V_2^2 - \rho V_1^2\right)$$

Where, P_1 = Pressure on the upper side of the wing
P_2 = Pressure on the lower side of the wing

Now the lift on the wing = ($P_2 - P_1$) x Area

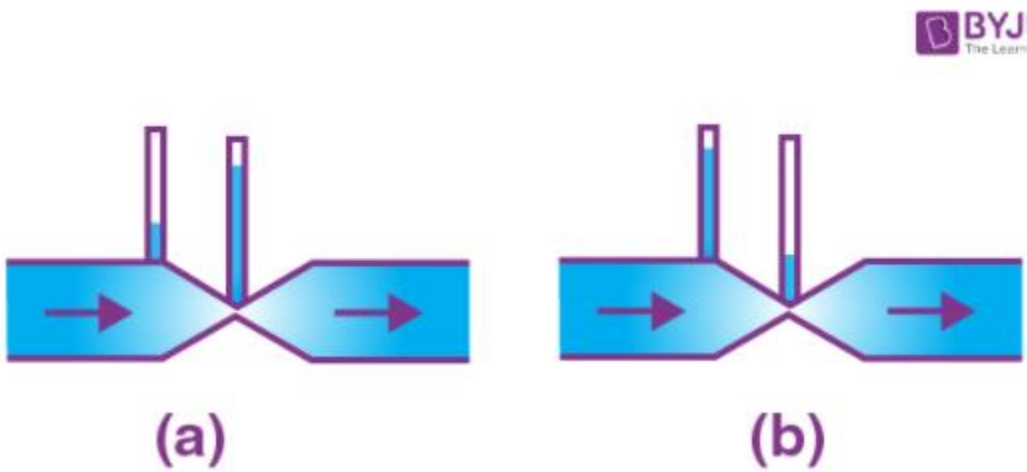
$$= \frac{1}{2}\rho\left(V_1^2 - V_2^2\right) \times A$$

$$= \frac{1}{2} \times 1.3\left((70)^2 - (63)^2\right) \times 2.5$$

= 1.51×10^3 N

Therefore the lift experienced by the wings of the air craft is 1.51×10^3 N.

Q-15: Figures (a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect ? Why ?



Solution:

Figure (a) is incorrect. The reason is that at the kink, the velocity of flow of liquid is large and hence using the Bernoulli's theorem the pressure is less. As a result, the water should not rise higher in the tube where there is a kink (i.e., where the area of cross-section is small).

Q-16: The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2 one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1}, what is the speed of ejection of the liquid through the holes?

Solution:

Given:
Number of holes, n = 40
Cross-sectional area of the spray pump, A_1 = 8 cm^{-2}**= 8 × 10^{-4} m^{-2}**
Radius of each hole, **r = 0.5 × 10^{-3} m**
Cross-sectional area of each hole, a = πr^2 = **π (0.5 × 10^{-3} $)^2$ m^2**

Total area of 40 holes, A_2= n × a = $40 \times \pi \left(0.5 \times 10^{-3}\right)^2$ m^2 = 3.14 x 10^{-5} m^2

Speed of flow of water inside the tube, V_1 = 1.5 m/min **= 0.025 m/s**
Let, the water ejected through the holes at a speed **= V_2**

Using the law of continuity:
$A_1V_1 = A_2V_2$

$$V_2 = \frac{A_1V_1}{A_2} = \frac{8\times10^{-4}\times0.025}{3.14\times10^{-5}}$$

Therefore, V_2 = 0.636 m/s

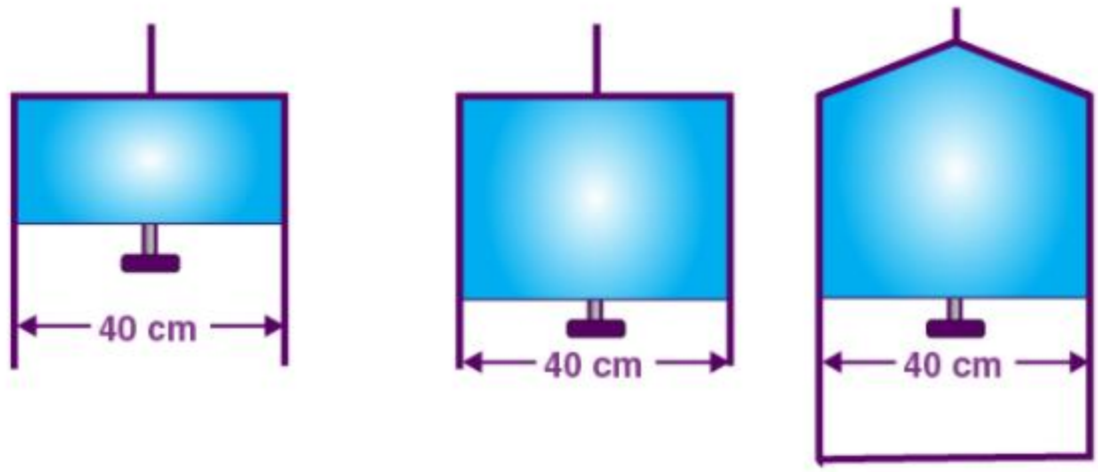
Q-17: A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and the light slider supports a weight of 1.5 × 10^{-2} N (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film?

Solution:

Given:
The maximum weight the film can support, **W = 1.5 × 10^{-2} N**
Length of the slider, l = 30 cm **= 0.3 m**

Total length of liquid film, l = 2 x 30 cm = 60 cm = 0.6 m because the liquid film has two surfaces.
Surface tension, T = F/l =1.5 x 10^{-2} N/0.6m =2.5 x 10^{-2} Nm^{-1}

Q-18: Figure (a) shows a thin liquid film supporting a small weight = 4.5 × 10^{-2} N. What is the weight supported by a film of the same liquid at the same temperature in Fig. (b) and (c)? Explain your answer physically.



Solution:

Case (a):

(a) Given, the length of the film supporting the weight = 40 cm = 0.4 m.

Total weight supported (or force) = 4.5×10^{-2} N.
Film has two free surfaces, Surface tension, S =$4.5 \times 10^{-2}/2 \times 0.4$ =5.625×10^{-2} Nm^{-1}
Since the liquid is same for all the given cases (a), (b) and (c), and temperature is also the same, therefore surface tension for cases (b) and (c) will also be the same = 5.625×10^{-2}.

In Fig. (b) and (c), the length of the film supporting the weight is also the same as that of (a), hence the total weight supported in each case is 4.5×10^{-2} N.

Q-19: What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature? Surface tension of mercury at that temperature (20 °C) is 4.65×10^{-1} N m^{-1}. The atmospheric pressure is 1.01×10^5 Pa. Also give the excess pressure inside the drop.

Solution:

Given:
Surface tension of mercury, S = 4.65×10^{-1} N m^{-1}
Radius of the mercury drop, r = 3.00 mm = 3×10^{-3} m
Atmospheric pressure, P_0 = 1.01×10^5 Pa

We know:

Total pressure inside the mercury drop = Excess pressure inside mercury + Atmospheric pressure

= 2S/r + P_0 = $\left(\frac{2\times4.65\times10^{-1}}{3\times10^{-3}}\right) + 1.01\times10^{5}$ = 1.0131 x 10^5 Pa

= 1.01 x 10^5 Pa

Excess pressure = $\frac{2S}{r}$

= $\left(\frac{2\times4.65\times10^{-1}}{3\times10^{-3}}\right)$ **= 310 Pa**

Q-20: What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature (20 °C) is 2.50 × 10^{-2} N m^{-1}? If an air bubble of the same dimension were formed at depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atmospheric pressure is 1.01 × 10^5 Pa)

Solution:

Given:
Surface tension of the soap solution, **S = 2.50 × 10^{-2} N/m**
r = 5.00 mm = 5 × 10^{-3} m
Density of the soap solution, **ρ = 1.2 × 10^3 kg/m^3**
Relative density of the soap solution **= 1.20**
Air bubble is at a depth, h = 30 cm **= 0.3 m**
Radius of the air bubble, r = 4 mm **= 5× 10^{-3} m**
Acceleration due to gravity, **g = 9.8 m/s^2**
1 atmospheric pressure = **1.01 × 10^5 Pa**
We know;

P = $\frac{4S}{r}$

= $\left(\frac{4\times2.5\times10^{-2}}{5\times10^{-3}}\right)$ = 20 Pa

Thus the excess pressure inside the soap bubble is 20 Pa.

Now for the excess pressure inside the air bubble, P' = $\frac{2S}{r}$

P' = $\left(\frac{2\times2.5\times10^{-2}}{5\times10^{-3}}\right)$ = 10 Pa

Thus, the excess pressure inside the air bubble is 10 Pa

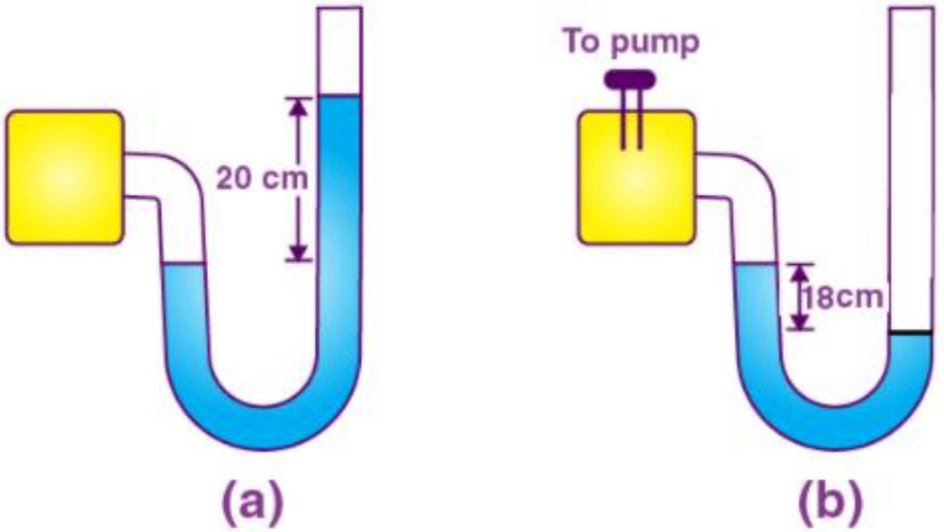
At the depth of 0.4 m, the total pressure inside the air bubble = Atmospheric pressure + hρg + P'
$= 1.01 \times 10^5 + 0.4 \times 1.2 \times 10^3 \times 9.8 + 10$
$= 1.06 \times 10^5$ Pa.

Q-21: A tank with a square base of area 1.0 m^2 is divided by a vertical partition in the middle. The bottom of the partition has a small-hinged door of area 20 cm^2. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. compute the force necessary to keep the door close.

Solution:

Given:
Area of the hinged door, a = 20 cm^2**$= 20 \times 10^{-4}$ m**
Base area of the given tank, **A = 2 m^2**
Density of water, **$\rho_1 = 10^3$ kg/m^3**
Density of acid, **$\rho_2 = 1.7 \times 10^3$ kg/m^3**
Height of the water column, **h_1 = 4 m**
Height of the acid column, **h_2 = 4 m**
Acceleration due to gravity, **g = 9.8 ms^{-2}**
Pressure exerted by water , **$P_1 = h_1\rho_1 g$** $= 4 \times 10^3 \times 9.8$ **$= 3.92 \times 10^4$ Pa**
the pressure exerted by acid , **$P_2 = h_2\rho_2 g$** $= 4 \times 1.7 \times 10^3 \times 9.8$ **$= 6.664 \times 10^4$ Pa**
Pressure difference between the above two :
$\Delta P = P_2 - P_1$
$= (6.664 - 3.92) \times 10^4$**$= 2.744 \times 10^4$Pa**
Thus, the force on the door = ΔP x a
$=2.744 \times 10^4 \times 20 \times 10^{-4}$**= 54.88 N**
Hence the force required to keep the door closed is 54.88N

Q-22: A manometer reads the pressure of a gas in an enclosure as shown in Fig. 10.25 (a) When a pump removes some of the gas, the manometer reads as in Fig. 10.25 (b) The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.
(a) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b), in units of cm of mercury.
(b) How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) is poured into the right limb of the manometer? (Ignore the small change in the volume of the gas).



Solution:

(a) For diagram (a):
Given, Atmospheric pressure, P_0 = 76 cm of Hg
The difference between the levels of mercury in the two arms is gauge pressure.
Thus, gauge pressure is 20 cm of Hg.
We know, Absolute pressure = Atmospheric pressure + Gauge pressure

= 76 + 20 = 96 cm of Hg

For diagram (b):
Difference between the levels of mercury in the two arms = –18 cm
Hence, gauge pressure is –18 cm of Hg.
And, Absolute pressure = Atmospheric pressure + Gauge pressure
= 76 cm – 18 cm = 58 cm

(b) It is given that 13.6 cm of water is poured into the right arm of figure (b).
We know that relative density of mercury = 13.6
=> A 13.6 cm column of water is equivalent to 1 cm of mercury.
Let, h be the difference in the mercury levels of the two arms.
Now, pressure in the right arm P_R = Atmospheric pressure + 1 cm of Hg

= 76 + 1 = 77 cm of Hg . . . . . . (a)

The mercury column rises in the left arm, thus the pressure in the left limb, **P_L = 58 + h . . . . . . (b)**
Equating equations (a) and (b) we get :

77 = 58 + h

Therefore the difference in the mercury levels of the two arms, h = 19 cm

Q-23: Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases ? If so, why do the vessels filled with water to that same height give different readings on a weighing scale ?

Solution:

As the base area is the same the pressure and thus the force acting on the two vessels will also be the same. However, force is also exerted on the walls of the vessel, which have a nonvertical component when the walls are not perpendicular to the base. The net non-vertical component on the sides of the vessel is lesser for the second vessel than the first. Therefore, **the vessels have different weights despite having the same force on the base.**

Q-24: During a blood transfusion, the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein? [Use the density of whole blood from Table 10.1]

Solution:

Given:
Density of whole blood, $\rho = 1.06 \times 10^{3}$ **kg** m^{-3}
Gauge pressure, **P = 2000 Pa**
Acceleration due to gravity, **g = 9.8** m/s^{2}
$=P/\rho g = 200/(1.06 \times 10^{3} \times 9.8) = 0.1925$ **m**
The blood may just enter the vein if the height at which the blood container be kept must be slightly greater than 0.1925 m i.e„ 0.2 m.

Q-25: In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy. (a) What is the largest average velocity of blood flow in an artery of diameter 2×10^{-3} m if the flow must remain laminar? (b) Do the dissipative forces become more important as the fluid velocity increases? Discuss qualitatively.

Solution:

(a) If dissipative forces are present, then some forces in liquid flow due to pressure difference is spent against dissipative forces, due to which the pressure drop becomes large.
(b) The dissipative forces become more important with increasing flow velocity, because of turbulence.

Q-26: (a) What is the largest average velocity of blood flow in an artery of radius 2×10^{-3}m if the flow must remain lanimar? (b) What is the corresponding flow rate ? (Take viscosity of blood to be 2.084×10^{-3} Pa s).

Solution:

Given:
Radius of the vein, $r = 2 \times 10^{-3}$ **m**
Diameter of the vein, $d = 2 \times 1 \times 10^{-3}$ m $= 2 \times 10^{-3}$ **m**
Viscosity of blood ,$\eta = 2.08 \times 10^{-3}$ **m**
Density of blood, $\rho = 1.06 \times 10^{3}$ kg/m^{3}

(a) We know, Reynolds' number for laminar flow, N_R = 2000
Therefore, greatest average velocity of blood is:

$V_{AVG} = \frac{N_R \eta}{\rho d}$

$= \frac{2000 \times 2.084 \times 10^{-3}}{1.06 \times 10^3 \times 4 \times 10^{-3}}$

= 0.983m/s

(b) And, flow rate **$R = V_{AVG} \pi r^2$** = 0.983 x 3.14 x (10^{-3})^2**= 1.235 x 10^{-6} m^3/s**

Q.27: A plane is in level flight at constant speed and each of its wings has an area of 25 m^2. If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. (Take air density to be 1 kg/m^3), g = 9.8 m/s^2

Solution:

Area of the wings of the plane, A=2×25=50 m^2
Speed of air over the lower wing, V_1
=180km/h= 180 x (5/18) = 50 m/s

Speed of air over the upper wing, V_2
=234km/h= 234 x (5/18) = 65 m/s

Density of air, =1kg/m^3
Pressure of air over the lower wing =P_1
Pressure of air over the upper wing =P_2
Pressure difference, $\Delta P = P_1 - P_2 = (1/2)\ \rho\ (V_2^2 - V_1^2)$

= (1/2) x 1 x ($65^2 - 50^2$) = 862.5 Pa

The net upward force F=ΔP x A
The upward forces balances the weight of the plane

mg = ΔP x A

m = (ΔP x A)/g

= (862.5 x 50)/9.8
=4400kg

The mass of the plane is 4400kg

Q-28: In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius 2.0 × 10^{-5} m and density 1.2 × 10^3 kg m^{-3}. Take the viscosity of air at the temperature of the experiment to be 1.8 × 10^{-5} Pa s. How much is the viscous force on the drop at that speed? Neglect buoyancy of the drop due to air.

Solution:

Given:
Acceleration due to gravity, **g = 9.8 m/s^2**
Radius of the uncharged drop, **r = 2.0 × 10^{-5} m**
Density of the uncharged drop, **ρ = 1.2 × 10^3 kg m^{-3}**
Viscosity of air, **η = 1.8 × 10^{-5} Pa s**
We consider the density of air to be zero in order to neglect the buoyancy of air.
Therefore terminal velocity (v) is:

$$v = \frac{2r^2 g\rho}{9\eta}$$

$$= \frac{2\left(2.0\times10^{-5}\right)^2\times9.8\times1.2\times10^3}{9\times1.8\times10^{-5}} = 5.8cm^{-1}$$

And the viscous force on the drop is:
F = 6πηrv
= 6 x 3.14 x 1.8 × 10^{-5}x2 x 10^{-5} x 5.8 10^{-2}
= 3.91 x 10^{-10} N

Q-29: Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside ? Surface tension of mercury at the temperature of the experiment is 0.465 N m^{-1}. Density of mercury = 13.6 × 10^3 kg m^{-3}

Solution:

Given:
Density of mercury, **ρ =13.6 × 10^3 kg/m^3**
Angle of contact between mercury and soda lime glass, **θ = 140°**
Surface tension of mercury at that temperature, **s = 0.465 N m^{-3}**
Radius of the narrow tube, r = 2/2 = 1 mm **= 1 × 10^{-3} m**
Let, the dip in the depth of mercury = h
Acceleration due to gravity, **g = 9.8 m/s^2**

We know, surface tension $S = \frac{hg\rho r}{2cos\Theta}$

$$\Rightarrow h = \frac{2Scos\Theta}{g\rho r} \Rightarrow h = \frac{2\times0.465\times cos140^{\circ}}{13.6\times9.8} = -5.34 \text{ mm}$$

The negative sign indicates the falling level of mercury. Thus, the mercury dips by 5.34 mm.

Q-30: Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3 × 10^{-2} N m^{-1}. Take the angle of contact to be zero and density of water to be 1.0 × 10^3 kg m^{-3} (g = 9.8 m s^{-2}).

Solution:

Given:
Diameter of the first bore, d_1 = 3.0 mm **= 3 × 10^{-3} m**
Radius of the first bore, r_1 = 3/2 **= 1.5 x 10^{-3} m.**
Diameter of the second bore, **d_2 =6mm**
Radius of the second bore, r_2 = 6/2 **= 3 x 10^{-3} mm**
Surface tension of water, **s = 7.3 × 10^{-2} N /m**
Angle of contact between the bore surface and water, **θ= 0**
Density of water, **ρ =1.0 × 10^3 kg/m^{-3}**
Acceleration due to gravity, **g = 9.8 m/s^2**
Let, h_1 and h_2 be the heights to which water rises in the first and second tubes respectively.
Thus, the difference in the height:

$$h_1 - h_2 = \frac{2sCos\Theta}{r_1\rho g} - \frac{2sCos\Theta}{r_2\rho g}$$

Since, $h = \frac{2sCos\Theta}{r\rho g}$

$$h_1 - h_2 = \frac{2sCos\Theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{2\times7.3\times10^{-2}\times1}{10^3\times9.8}\left[\frac{1}{1.5\times10^{-3}} - \frac{1}{3\times10^{-3}}\right] = 4.97 \text{ mm}$$

Therefore, the difference in the water levels of the two arms =2.482 mm.

Q-31: (a) According to the law of atmospheres density of air decreases with increase in height y as $\rho_o e^{\frac{-y}{y_0}}$. Where ρ_0 =1.25 kg m^{-3} is the density of air at sea level and y_0 is a constant. Derive this equation/law considering that the atmosphere and acceleration due to gravity remain constant.

(b) A zeppelin of volume 1500 m^3 is filled with helium and it is lifting a mass of 400 kg. Assuming that the radius of the zeppelin remains constant as it ascends. How high will it rise?[y_0 = 8000 m and ρH_e =0.18 kg m^{-3}].

Solution:

(a). We know that rate of decrease of density ρ of air is directly proportional to the height y.

i.e., $\frac{d\rho}{dy} = -\frac{\rho}{y_0}$ . . . . (1)

Where y is the constant of proportionality and the –ve sign indicates the decrease in density with increase in height.

Integrating equation (1), we get:

$$\int_{\rho_0}^{\rho} \frac{d}{\rho} = -\int_0^y \frac{1}{y_0} dy$$

$$[log\rho]_{\rho_0}^{\rho} = -[\frac{y}{y_0}]_0^y$$

Where ρ_0 = density of air at sea level ie y =0
Or, $\log_e(\rho_0 / \rho) = -y/y_0$

Therefore, $\rho = \rho_0 \, e^{\frac{y}{y_0}}$

(b). Given:
Volume of zeppelin = 1500 m^3
Mass of payload, m = 400 kg
y_0 = 8000 m
ρ_0 =1.25 kg m^{-3}
density of helium, $\rho\, H_e$ =0.18 kg m^{-3}

Density ρ = $\frac{Mass}{Volume}$ = $\frac{Mass\ of\ payload + Mass\ of\ helium}{Volume}$

= $\frac{400+1500\times 0.18}{1500}$ [Mass = volume × density]

= 0.45 kg m^{-3}
Using equation (1), we will get:

$$\rho = \rho_0 e^{\frac{y}{y_0}}$$

$$\Rightarrow log_e(\frac{\rho_0}{\rho}) = \frac{y_0}{y}$$

Or,

$$y = \frac{y_0}{log_e(\frac{\rho_0}{\rho})}$$

$$\Rightarrow y = \frac{8000}{log_e(\frac{1.25}{0.45})}$$

Therefore, y ≈ 8km