

Q.1: The triple points of neon and carbon dioxide are 24.57 K and 216.55 K, respectively. Express these temperatures on the Celsius and Fahrenheit scales.

Solution:

Given: Kelvin and Celsius scales are related as:

$$T_C = T_K - 273.15 \dots\dots\dots (1)$$

We know:

$$T_F = \left(\frac{9}{5}\right) T_C + 32 \dots\dots\dots (2)$$

For neon:

$$T_K = 24.57 \text{ K}$$

$$\Rightarrow T_C = 24.57 - 273.15 = -248.58^\circ \text{ C}$$

$$T_F = \left(\frac{9}{5}\right) T_C + 32$$

$$= \left(\frac{9}{5}\right) \times (-248.58) + 32 = -415.44^\circ \text{ F}$$

For carbon dioxide:

$$T_K = 216.55 \text{ K}$$

$$\Rightarrow T_C = 216.55 - 273.15 = -56.60^\circ \text{ C}$$

$$T_F = \left(\frac{9}{5}\right) T_C + 32$$

$$= \left(\frac{9}{5}\right) \times (-56.60) + 32 = -69.88^\circ \text{ F}$$

Q.2: Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is the relation between T_A and T_B ?

Solution:

Given:

Triple point of water on **absolute scale B**, $T_2 = 400 \text{ B}$

Triple point of water on **absolute scale A**, $T_1 = 200 \text{ A}$

Triple point of water on **Kelvin scale**, $T_K = 273.15 \text{ K}$

273.15 K on the Kelvin scale is equivalent to 200 A on absolute scale A.

$$\Rightarrow T_1 = T_K$$

$$200 A = 273.15 K$$

$$\text{Thus, } A = \frac{273.15}{200}$$

273.15 K on the Kelvin scale is equivalent to 350 B on absolute scale B.

$$\Rightarrow T_2 = T_K$$

$$350 B = 273.15 K$$

$$\text{Thus, } B = \frac{273.15}{350}$$

Let, T_A and T_B be the triple point of water on scale A and B respectively.

Thus, we have:

$$273.15 \times \frac{T_A}{200} = 273.15 \times \frac{T_B}{350}$$

Therefore, $T_A : T_B = 4 : 7$

Q.3: The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law:

$$R = R_0[1 + \alpha (T - T_0)]$$

The resistance is 101.6Ω at the triple-point of water $273.16 K$, and 165.5Ω at the normal melting point of lead ($600.5 K$). What is the temperature when the resistance is 123.4Ω ?

Solution:

Triple point temperature, $T_0 = 273.16 K$

Resistance at the triple point, $R_0 = 101.6 \Omega$

Normal melting point of lead, $T_1 = 600.5 K$

Resistance at normal melting point, $R_1 = 165.5 \Omega$

According to approximate law

$$R_1 = R_0[1 + \alpha(T_1 - T_0)]$$

$$165.5 = 101.6[1 + \alpha(600.5 - 273.16)]$$

$$165.5 = 101.6[1 + \alpha(327.34)]$$

$$165.5 = 101.6 + \alpha(101.6)(327.34)$$

$$165.5 = 101.6 + \alpha(101.6 \times 327.34)$$

$$\alpha = (165.5 - 101.6) / (101.6 \times 327.34)$$

$$\alpha = 63.9 / (101.6 \times 327.34)$$

Now when resistance is 123.4Ω then temperature T_2 is:

$$R_2 = R_0[1 + \alpha(T_2 - T_0)]$$

$$123.4 = 101.6[1 + \alpha(T_2 - 273.16)]$$

$$123.4 = 101.6[1 + (63.9 / (101.6 \times 327.34))(T_2 - 273.16)]$$

$$123.4 = 101.6 + (63.9 / 327.34)(T_2 - 273.16)$$

$$123.4 = 101.6 + (0.195)(T_2) - (0.195)(273.16)$$

$$123.4 = 101.6 + (0.195)(T_2) - 53.32$$

$$T_2 = (123.4 - 101.6 + 53.32) / 0.195 = 75.12 / 0.195 = 385.23$$

Q.4: Answer the following:

(a) The triple-point of water is a standard fixed point in modern thermometer. Why? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale)?

(b) There were two fixed points in the original Celsius scale as mentioned above which were assigned the number 0 °C and 100 °C, respectively. On the absolute scale, one of the fixed points is the triple-point of water, which on the Kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (Kelvin) scale?

(c) The absolute temperature (Kelvin scale) T is related to the temperature t_c on the Celsius scale by $t_c = T - 273.15$. Why do we have 273.15 in this relation, and not 273.16?

(d) What is the temperature of the triple-point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale?

Solution:

(i) Melting and boiling points of water aren't considered as the standard fixed points because they vary with change in pressure, the temperature of the triple point of water is unique and it does not vary with pressure.

(ii) On the Kelvin's scale, there is only a lower fixed point which is 273.16 K, the upper fixed point is not there.

(iii) The relation is such because 273.15 K on the Kelvin's scale corresponds to the melting point of ice while 273.16 K is the triple point of water.

(iv) We know,

Relation between the Fahrenheit scale and Absolute scale :

i.e.

$$\frac{T_F - 32}{180} = \frac{T_K - 273}{100} \dots\dots\dots (1)$$

For another set of T'_F and T'_K

$$\frac{T'_F - 32}{180} = \frac{T'_K - 273}{100} \dots\dots\dots (2)$$

Subtracting Equation (1) and (2):

$$\frac{T'_F - T_F}{180} = \frac{T'_K - T_K}{100}$$

Therefore, $T'_F - T_F = 1.8(T'_K - T_K)$

For, $T'_K - T_K = 1\text{K}$

$T'_F - T_F = 1.8$

\Rightarrow For the triple point temperature = 273.16 K, the temperature on the new scale = $1.8 \times 273.16 = 491.688$

Units

Q.5. Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made:

Temperature	Pressure thermometer A	Pressure thermometer B
Triple-point of water	$1.250 \times 10^5 \text{Pa}$	$0.200 \times 10^5 \text{Pa}$
Normal melting point of sulphur	$1.797 \times 10^5 \text{Pa}$	$0.287 \times 10^5 \text{Pa}$

a) What is the absolute temperature of the normal melting point of sulphur as read by thermometers A and B?

(b) What do you think is the reason behind the slight difference in answers of thermometers A and B? (The thermometers are not faulty). What further procedure is needed in the experiment to reduce the discrepancy between the two readings?

Solution:

(a)

Triple point of water, $T = 273.16\text{K}$.

Pressure in thermometer A at the triple point, $P_A = 1.25 \times 10^5 \text{ Pa}$

Normal melting point of sulphur = T_1

Pressure in thermometer A at this temperature, $P_1 = 1.797 \times 10^5 \text{ Pa}$

According to Charles law, we have the relation:

$$P_A/T = P_1/T_1$$

$$T_1 = P_1 T/P_A$$

$$= (1.797 \times 10^5 \times 273.16) / 1.25 \times 10^5$$

$$= 392.69 \text{ K}$$

Therefore, the absolute temperature of the normal melting point of sulphur as read by thermometer A is 392.69 K.

(b) The pressure in thermometer B at the triple point of water, $P_B = 0.2 \times 10^5 \text{ Pa}$

The temperature in thermometer B at the normal melting point (T_1) of sulphur is, $P_1 = 0.287 \times 10^5 \text{ Pa}$

According to Charles law, we can write the relation:

$$P_B/T = P_1/T_1$$

$$T_1 = P_1 T/P_B$$

$$T_1 = (0.287 \times 10^5 \times 273.16)/0.2 \times 10^5 \\ = 391.98 \text{ K}$$

Therefore, the absolute temperature of the normal melting point of sulphur as read by thermometer B is 391.98 K.

(b) The reason for the discrepancy is that the gases do not behave like ideal gas in practice. The discrepancy can be reduced by taking the reading at very low pressure so that the gases show perfect behaviour.

Q.6. A steel tape 1m long is correctly calibrated for a temperature of 27.0 °C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0 °C. What is the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is 27.0 °C? Coefficient of linear expansion of steel = $1.20 \times 10^{-5} \text{ K}^{-1}$

Solution:

$$T = 27 \text{ }^\circ\text{C}$$

At the temperature of 27 °C the length of the tape is 1m = 100 cm

$$T_1 = 45 \text{ }^\circ\text{C}$$

At the temperature of 45 °C the length of the tape is 63 cm

$$\text{Coefficient of linear expansion of steel} = 1.20 \times 10^{-5} \text{ K}^{-1}$$

Let L be the actual length of the steel rod and L' is the length at 45 °C

$$L' = L [1 + \alpha(T_1 - T)]$$

$$= 100 [1 + 1.20 \times 10^{-5} (45 - 27)]$$

$$= 100 [1 + 21.6 \times 10^{-5}]$$

$$= 100 + 2160 \times 10^{-5}$$

$$= 100 + 0.02160 = 100.02160$$

The actual length of the rod at 45° C is

$$L_2 = (100.02160 / 100) \times 63 = 63.013608 \text{ cm}$$

The length of the rod at 27° C is 63.0 cm

Q.7. A large steel wheel is to be fitted on to a shaft of the same material. At 27 °C, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using 'dry ice'. At what temperature of the shaft does the wheel slip on the shaft? Assume the coefficient of linear expansion of the steel to be constant over the required temperature range: $\alpha_{\text{steel}} = 1.20 \times 10^{-5} \text{ K}^{-1}$.

Solution:

Temperature, $T = 27\text{ }^\circ\text{C}$

The outer diameter of the shaft at $27\text{ }^\circ\text{C}$ is $d_1 = 8.70\text{ cm}$

Diameter of the central hole in the wheel at $27\text{ }^\circ\text{C}$ is $d_2 = 8.69\text{ cm}$

Coefficient of linear expansion of steel, $\alpha_{\text{steel}} = 1.2 \times 10^{-5}\text{ K}^{-1}$

Temperature at which the wheel will slip on the shaft = T_1

Change due to cooling

$$d_2 = d_1(1 + \alpha\Delta T)$$

$$d_2 = d_1[1 + \alpha(T_1 - T)]$$

$$d_2 - d_1 = d_1 \alpha(T_1 - T)$$

$$8.69 - 8.70 = 8.70 \times 1.2 \times 10^{-5} \times (T_1 - 27)$$

$$-0.01 = 10.44 \times 10^{-5} \times (T_1 - 27)$$

$$-0.01 / (10.44 \times 10^{-5}) = T_1 - 27$$

$$T_1 = 27 - [0.01 / (10.44 \times 10^{-5})]$$

$$T_1 = 27 - 95.7 = -68.7$$

Therefore, the wheel will slip on the shaft when the temperature of the shaft is $-68.7\text{ }^\circ\text{C}$.

Q.8. A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at $27.0\text{ }^\circ\text{C}$. What is the change in the diameter of the hole when the sheet is heated to $227\text{ }^\circ\text{C}$? Coefficient of linear expansion of copper = $1.70 \times 10^{-5}\text{ K}^{-1}$.

Solution:

Diameter of hole (D_1) = 4.24 cm

Initial Temperature, $T_1 = 27.0\text{ }^\circ\text{C} = 27 + 273 = 300\text{ K}$

Final temperature, $T_2 = 227\text{ }^\circ\text{C} = 227 + 273 = 500\text{ K}$

Let the diameter of the hole at the final temperature be D_2

Coefficient of linear expansion of copper, $\alpha = 1.70 \times 10^{-5}\text{ K}^{-1}$

$$\text{Initial area of hole } (A_0) = \pi r^2 = \pi (4.24/2)^2$$

$$\text{coefficient of superficial expansion } (\beta) = 2 \times \text{coefficient of linear expansion of copper } (\alpha) = 2 \times 1.7 \times 10^{-5} = 3.4 \times 10^{-5}$$

Using the formula,

$$A = A_0(1 + \beta\Delta T)$$

$$A = \pi (4.24/2)^2 [1 + 3.4 \times 10^{-5} \times (500-300)]$$

$$= \pi (4.24/2)^2 [1 + 3.4 \times 10^{-5} \times 200]$$

$$= \pi (4.24/2)^2 [1 + 6.8 \times 10^{-3}]$$

$$= \pi (4.24/2)^2 [1 + 0.0068]$$

$$= \pi (4.24/2)^2 (1.0068)$$

$$\pi D_2^2/4 = \pi (4.24/2)^2 (1.0068)$$

$$D_2^2 = 17.97 (1.0068)$$

$$D_2^2 = 18.092$$

$$= 4.253$$

$$D_2 = 4.253 \text{ cm}$$

$$\text{Change in diameter } (\Delta D) = D_2 - D_1$$

$$= 4.253 - 4.24$$

$$= 0.013 \text{ cm}$$

Q.9. A brass wire 1.8 m long at 27 °C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of -39 °C, what is the tension developed in the wire, if its diameter is 2.0 mm? Co-efficient of linear expansion of brass = $2.0 \times 10^{-5} \text{ K}^{-1}$; Young's modulus of brass = $0.91 \times 10^{11} \text{ Pa}$.

Solution:

Initial temperature, $T_1 = 27^\circ\text{C}$

Length of the brass wire at 27°C , $l = 1.8\text{m}$

Final temperature, $T_2 = -39^\circ\text{C}$

Diameter of the wire, $d = 2.0\text{mm} = 2 \times 10^{-3}\text{m}$

Coefficient of linear expansion of brass, $\alpha = 2.0 \times 10^{-5} \text{K}^{-1}$

Young's modulus of brass, $Y = 0.91 \times 10^{11} \text{ Pa}$

Young's modulus is given by the relation:

$$Y = \text{Stress} / \text{Strain}$$

$$Y = (F/A) / (\Delta L/L)$$

$$Y = (F \times L) / (A \times \Delta L)$$

$$\Delta L = F \times L / (A \times Y) \dots\dots(1)$$

Where,

F = Tension developed in the wire

Cross-sectional area of the wire, $A = \pi d^2/4 = \pi (2 \times 10^{-3})^2/4$

The change in the length (ΔL) is given by the relation:

$$\Delta L = \alpha L(T_2 - T_1) \dots (2)$$

Equating equations (1) and (2), we get:

$$\alpha L(T_2 - T_1) = (F \times L) / (A \times Y)$$

$$F = [\alpha L(T_2 - T_1) \times A \times Y] / L$$

$$F = \alpha(T_2 - T_1) Y \pi (d/2)^2$$

$$= 2 \times 10^{-5} \times (-39 - 27) \times 0.91 \times 10^{11} \times 3.14 \times (2 \times 10^{-3} / 2)^2$$

$$= 2 \times 10^{-5} \times (-66) \times 0.91 \times 10^{11} \times 3.14 \times 1 \times 10^{-6}$$

$$= -377.18$$

$$= -3.77 \times 10^2 \text{ N}$$

The negative sign indicates that the tension is directed inward

Hence, the tension developed in the wire is $3.77 \times 10^2 \text{ N}$

Q. 10. A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250 °C, if the original lengths are at 40.0 °C? Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand (Co-efficient of linear expansion of brass = $2.0 \times 10^{-5} \text{ K}^{-1}$, steel = $1.2 \times 10^{-5} \text{ K}^{-1}$).

Solution:

Length of the brass rod = length of the steel rod = $L_0 = 50 \text{ cm}$

Diameter of the brass rod = diameter of the steel rod = 3 mm

Initial temperature (T_1) = 40°C

Final temperature (T_2) = 250°C

Therefore, the increase in temperature (ΔT) = 250 – 40 = 210°C

Coefficient of linear expansion of brass, $\alpha = 2 \times 10^{-5} \text{ K}^{-1}$

Coefficient of linear expansion of steel, $\beta = 1.2 \times 10^{-5} \text{ K}^{-1}$

Final length of brass, L_1

$$\begin{aligned} L_1 &= L_0(1 + \alpha \Delta T) \\ &= 50(1 + (2 \times 10^{-5} \times 210)) \\ &= 50(1 + 420 \times 10^{-5}) \\ &= 50 \times 1.00420 \\ &= 50.21 \text{ cm} \end{aligned}$$

Increase in length of brass (ΔL) = $L_2 - L_1$

$$\begin{aligned} &= 50.21 - 50 \\ &= 0.21 \text{ cm} \end{aligned}$$

Final length of steel, $L_2 = L_0(1 + \beta \Delta T)$

$$= 50(1 + (1.2 \times 10^{-5} \times 210))$$

$$= 50 \times 1.00252$$

$$= 50.126 \text{ cm}$$

Increase in length ($\Delta L'$) = 50.126 cm – 50 cm = 0.126 cm
 Total increase in the length = $\Delta L + \Delta L'$
 = 0.21 + 0.126
 = 0.336 cm

Q-11: The coefficient of volume expansion of glycerine is $49 \times 10^{-5} \text{ K}^{-1}$. What is the fractional change in its density for a 30°C rise in temperature?

Solution:

Given:

Coefficient of volume expansion of glycerine, $\alpha_v = 49 \times 10^{-5} \text{ K}^{-1}$
 Rise in temperature, $\Delta T = 30^\circ \text{C}$

$$\text{Fractional change in volume} = \frac{\Delta V}{V}$$

We know:

$$\alpha_v \Delta T = \frac{\Delta V}{V}$$

$$\text{Or, } V_{T_2} - V_{T_1} = V_{T_1} \alpha_v \Delta T$$

Or,

$$\frac{m}{\rho_{T_2}} - \frac{m}{\rho_{T_1}} = \frac{m}{\rho_{T_1}} \alpha_v \Delta T$$

Where, m = mass of glycerine

$$\rho_{T_2} = \text{Final density at } T_2$$

$$\rho_{T_1} = \text{Initial density at } T_1$$

$$\Rightarrow \frac{\rho_{T_1} - \rho_{T_2}}{\rho_{T_2}} = \text{Fractional change in density}$$

$$\text{Therefore, Fractional change in density} = 1.47 \times 10^{-2}$$

Q.12: A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings. Specific heat of aluminium = $0.91 \text{ J g}^{-1} \text{ K}^{-1}$.

Solution:

Power = 10 kW

Mass of the small aluminium block, $m = 8 \text{ kg} = 8 \times 10^3 \text{ g}$

Time = $2.5 \times 60 = 150 \text{ s}$

Specific heat of aluminium, $c = 0.91 \text{ J g}^{-1} \text{ K}^{-1}$.

Total energy = $P \times t = 10^4 \times 150 = 15 \times 10^5 \text{ J}$

As 50% of the energy is used in the heating or lost to the surrounding,

Therefore, thermal energy available, $\Delta Q = (1/2) \times 15 \times 10^5$
 $= 7.5 \times 10^5$

As $\Delta Q = mc\Delta T$

$\Delta T = \Delta Q/mc$

$= 7.5 \times 10^5 / (8 \times 10^3 \times 0.91)$

$= 7.5 \times 10^5 / 7.28 \times 10^3$

$= 1.03 \times 10^2$

Rise in the temperature of the block, $\Delta T = 103^\circ \text{C}$

Q.13. A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat of copper = $0.39 \text{ J g}^{-1} \text{ K}^{-1}$; the heat of fusion of water = 335 J g^{-1})

Solution:

Mass of the copper block, $m = 2.5 \text{ kg}$

Temperature of the block, $\Delta T = 500^\circ \text{C}$

Specific heat of copper, $c = 0.39 \text{ J g}^{-1} \text{ K}^{-1}$

Latent Heat of fusion of water, $L = 335 \text{ J g}^{-1}$

Let m' be the mass of the ice melted

Therefore, heat gained by ice = heat lost by copper

$m'L = mc\Delta T$

$m' = mc \Delta T/L$

$= (2500 \times 0.39 \times 500)/335 = 1500 \text{ g} = 1.5 \text{ kg}$

Q.14. In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm^3 of water at 27°C . The final temperature is 40°C . Compute the specific heat of the metal. If heat losses to the surroundings are not negligible, is your answer greater or smaller than the actual value for the specific heat of the metal?

Solution:

Mass of the metal block, $m = 0.20 \text{ kg} = 200 \text{ g}$

Initial temperature of the metal block, $T_1 = 150^\circ\text{C}$

Final temperature of the metal block, $T_2 = 40^\circ\text{C}$

Copper calorimeter has water equivalent of mass, $m_1 = 0.025 \text{ kg} = 25 \text{ g}$

Volume of water, $V = 150 \text{ cm}^3$

Mass (M) of water at temperature $T = 27^\circ\text{C}$ is $150 \times 1 = 150 \text{ g}$

Specific heat of water, $C_w = 4.186 \text{ J/g/K}$

Specific heat of the metal = c

Decrease in the temperature of the metal block

$$\Delta T_1 = T_1 - T_2 = 150 - 40 = 110^\circ\text{C}$$

Increase in the temperature of the water and calorimeter system, $\Delta T_2 = 40 - 27 = 13^\circ\text{C}$

Heat lost by the metal = Heat gained by the water + heat gained by the calorimeter

$$mC \Delta T_1 = (M+m_1)C_w \Delta T_2$$

$$C = [(M+m_1)C_w \Delta T_2] / m \Delta T_1$$

$$= [(150 + 25) \times 4.186 \times 13] / (200 \times 100)$$

$$= (175 \times 4.186 \times 13) / (200 \times 100)$$

$$= 9523.15 / 22000$$

$$= 0.43 \text{ Jg}^{-1}\text{k}^{-1}$$

If some heat is lost to the surroundings, specific heat of metal will be lesser than the actual value.

Q.15: Given below are observations on molar specific heats at room temperature of some common gases.

Gas	Molar Specific Heat C_v (cal mol ⁻¹ K ⁻¹)
Chlorine	6.17
Oxygen	5.02
Carbon monoxide	5.01
Nitric oxide	4.99
Nitrogen	4.97
Hydrogen	4.87

Generally, the specific heat of a monoatomic gas is $2.92 \text{ cal (mol K)}^{-1}$, which is significantly lower from the specific heat of the above gases. Explain.

It can be observed that chlorine has little larger value of specific heat, what could be the reason?

Solution:

The gases in the above list are all diatomic and a diatomic molecule has translational, vibrational and rotational motion. Whereas, a **monoatomic gas** only has translational motion. So, to increase the temperature of one mole of a diatomic gas by 1°C , heat needs to be supplied to increase translational, rotational and vibrational energy. Thus, the above gases have significantly higher specific heats than monoatomic gases.

Chlorine has little larger **specific heat** as compared to the others in the list because it possesses **vibrational motion** as well while the rest only have **rotational and translational motions**.

Q.16. A child running a temperature of 101°F is given an antipyryn (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98°F in 20 minutes, what is the average rate of extra evaporation caused, by the drug. Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water, and the latent heat of evaporation of water at that temperature is about 580 cal g^{-1} .

Solution:

Initial temperature of the child, $T_i = 101^\circ\text{F}$

Final temperature of the child, $T_f = 98^\circ\text{F}$

Decrease in the temperature, $\Delta T = (101 - 98) = 3^\circ\text{F} = 3 \times (5/9) = 1.67^\circ\text{C}$

Mass of the child, $m = 30 \text{ kg} = 30 \times 10^3 \text{ g}$

Time taken to reduce the temperature, $t = 20 \text{ min}$

Specific heat of the human body = Specific heat of water = $c = 1000 \text{ cal kg}^{-1}^\circ\text{C}^{-1}$

Latent heat of evaporation of water, $L = 580 \text{ cal g}^{-1}$

The heat lost by the child is given as:

$$\begin{aligned}\Delta\theta &= mc\Delta T \\ &= 30 \times 1000 \times 1.67 \\ &= 50100 \text{ cal}\end{aligned}$$

Let m' be the amount of water evaporated from the child's body in 20 min.

$$\begin{aligned}m' &= \Delta\theta/L \\ &= (50100/580) = 86.37 \text{ g}\end{aligned}$$

Therefore, the average rate of evaporation = $86.2/20$
= 4.3 g/min.

Q. 17: A 'thermacole' icebox is a cheap and an efficient method for storing small quantities of cooked food in summer in particular. A cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice is put in the box, estimate the amount of ice remaining after 6 h. The outside temperature is 45 °C, and the co-efficient of thermal conductivity of thermacole is $0.01 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$. [Heat of fusion of water = $335 \times 10^3 \text{ J kg}^{-1}$]

Solution:

Side of the cubical icebox, $s = 30 \text{ cm} = 3 \times 10^{-2} \text{ m}$

Thickness of the icebox, $L = 5.0 \text{ cm} = 0.05 \text{ m}$

Mass of ice kept in the icebox, $m = 4 \text{ kg}$

Time, $t = 6 \text{ h} = 6 \times 60 \times 60 = 21600$

Outside temperature, $T_1 = 45^\circ \text{ C}$

Temperature of the icebox = 0° C

Temperature difference = $T_1 - T_2 = 45^\circ \text{ C} - 0^\circ \text{ C}$

Surface area of the icebox = $6 \times (0.30)^2 = 0.54$

Coefficient of thermal conductivity of thermacole, $K = 0.01 \text{ Js}^{-1}\text{m}^{-1}\text{k}^{-1}$

Heat of fusion of water, $L = 335 \times 10^3 \text{ Jkg}^{-1}$

Total heat entering the icebox in 6 hours is

$$\begin{aligned}Q &= KA(T_1 - T_2)t/L \\ &= (0.01 \text{ Js}^{-1}\text{m}^{-1}\text{C}^{-1} \times 0.54 \text{ m}^2 \times 450 \text{ C} \times 21600 \text{ s})/0.05 \text{ m} \\ &= 1.05 \times 10^5 \text{ J}\end{aligned}$$

Let m be the total amount of ice that melts in 6 h.

But $Q = mL$

Therefore, $m = Q/L$

$$= 1.05 \times 10^5 / (335 \times 10^3) = 0.313 \text{ kg}$$

Amount of ice remaining after 6 h = $4 - 0.313 = 3.687 \text{ kg}$

Q.18. A brass boiler has a base area of 0.15 m^2 and thickness 1.0 cm. It boils water at the rate of 6.0 kg/min when placed on a gas stove. Estimate the temperature of the part of the flame in

contact with the boiler. Thermal conductivity of brass = $109 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$; Heat of vaporisation of water = $2256 \times 10^3 \text{ J kg}^{-1}$.

Solution:

Base area of the brass boiler, $A = 0.15 \text{ m}^2$

Thickness of the boiler, $d = 1.0 \text{ cm} = 0.01 \text{ m}$

Brass boiler boils water at the rate, $R = 6.0 \text{ kg/min}$

Time, $t = 60 \text{ sec}$

Mass = 6 kg

Thermal conductivity of brass, $K = 109 \text{ Js}^{-1} \text{ m}^{-1} \text{ k}^{-1}$

Heat of vaporisation of water, $L = 2256 \times 10^3 \text{ J kg}^{-1}$

Let T_1 Temperature of the flame in contact with the boiler

Let T_2 be the boiling point of water = $100 \text{ }^\circ\text{C}$

The Q be the amount of heat flowing into water through the base of the boiler

$$Q = KA(T_1 - T_2)t/d$$

$$Q = [109 \times 0.15 \times (T_1 - 100) \times 60] / 0.01 \text{ ---(1)}$$

Heat received by water:

$$Q = mL$$

$$Q = 6 \times 2256 \times 10^3 \text{ ---(2)}$$

Equating equations (1) and (2), we get:

$$mL = KA(T_1 - T_2)t/d$$

$$6 \times 2256 \times 10^3 = [109 \times 0.15 \times (T_1 - 100) \times 60] / 0.01$$

$$13536 \times 10^3 \times 0.01 = 981 (T_1 - 100)$$

$$T_1 - 100 = 13536 \times 10^3 \times 0.01 / 981$$

$$T_1 = 137.9 + 100$$

$$= 237.9 \text{ }^\circ\text{C}$$

Therefore, the temperature of the part of the flame in contact with the boiler is $237.98 \text{ }^\circ\text{C}$.

Q.19. Explain why:

(a) a body with large reflectivity is a poor emitter

(b) a brass tumbler feels much colder than a wooden tray on a chilly day

(c) an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace

(d) the earth without its atmosphere would be inhospitably cold

(e) heating systems based on the circulation of steam are more efficient in warming a building than those based on the circulation of hot water

Solution:

(a) A body with a large reflectivity is a poor absorber of heat radiation. A poor absorber will be a poor emitter of radiations. Therefore, a body with a large reflectivity is a poor emitter.

(b) Brass is a good conductor of heat and wood is a poor conductor of heat. When we touch a brass tumbler, heat is conducted from our hand to the tumbler and there is a drop in the body temperature. Therefore, we feel cold. When a wooden tray is touched on a chilly day, very less heat is conducted from the hand to the wooden tray and body temperature is not decreased much. So, we do not feel cold.

(c) The radiation energy from a red hot iron piece placed in a furnace is given by the relation

$$E = \sigma T^4$$

When the iron piece is placed in the open, the radiation energy is given by the relation

$$E = \sigma(T^4 - T_0^4)$$

Here, E is the energy radiation

T is the temperature of optical pyrometer

σ is a constant

T_0 is the temperature of the open space.

The increase in the temperature of the open space reduces the radiation energy.

(d) If there is no atmosphere, extra heat will not be trapped. All the heat from the sun will be radiated back from the surface of the earth. So without the atmosphere, the earth would be inhospitably cold.

(e) Steam at 100 °C is much hotter than water at 100 °C. This is because steam will have a lot of heat in the form of latent heat (540 cal/g). Therefore, heating systems based on the circulation of steam are more efficient in warming a building than those based on the circulation of hot water.

Q.20. A body cools from 80 °C to 50 °C in 5 minutes. Calculate the time it takes to cool from 60 °C to 30 °C. The temperature of the surroundings is 20 °C.

Solution:

Here,

Initial temperature of the body, $T_1 = 80$ °C

Final temperature of the body, $T_2 = 50$ °C

Average temperature, $(T_1 + T_2) / 2$

$$= (80 + 50) / 2 = 65$$
 °C

Temperature of the surrounding, $T_0 = 20$ °C

Temperature difference, $\Delta T = 65$ °C – 20 °C = 45 °C

t = 5 min

According to Newton's law of cooling, the rate of cooling is proportional to the difference in temperature

Change in temperature/ time = K ΔT

$$(T_2 - T_1)/t = K\Delta T$$
$$(80 - 50)/5 = K(45)$$

$$6 = K(45)$$

$$K = 6/45$$

$$K = 2/15$$

In second condition,

Initial temperature of the body, $T_1 = 60^\circ\text{C}$

Final temperature of the body, $T_2 = 30^\circ\text{C}$

Let t be the time taken for cooling

$$\text{Average temperature, } (T_1 + T_2)/2 = (60 + 30)/2 = 45^\circ\text{C}$$

$$\text{Temperature difference, } \Delta T = 45^\circ\text{C} - 20^\circ\text{C} = 25^\circ\text{C}$$

According to Newton's law of cooling, the rate of cooling is proportional to the difference in temperature

$$\text{Change in temperature/ time} = K \Delta T$$

$$(T_2 - T_1)/t = K\Delta T$$

$$(60 - 30)/t = (2/15) (25)$$

$$30/t = 3.33$$

$$t = 30/3.33 = 9 \text{ min}$$

Q.21. Answer the following questions based on the P-T phase diagram of carbon dioxide:

(a) At what temperature and pressure can the solid, liquid and vapour phases of CO_2 co-exist in equilibrium?

(b) What is the effect of the decrease of pressure on the fusion and boiling point of CO_2 ?

(c) What are the critical temperature and pressure for CO_2 ? What is its significance?

(d) Is CO_2 solid, liquid or gas at (a) -70°C under 1 atm, (b) -60°C under 10 atm, (c) 15°C under 56 atm?

Solution:

(a) Liquid and vapour phases of CO_2 co-exist at the triple point temperature = -56.6°C and pressure = 5.11 atm.

(b) Both the boiling point and freezing point of CO_2 decrease if pressure decreases.

(c) The critical temperature and pressure of CO_2 are 31.1°C and 73.0 atm, respectively.

Above this temperature, CO_2 will not liquefy even if compressed to high pressures.

(d) (a) vapour (b) solid (c) liquid

Q.22: A hot ball cools from 90°C to 10°C in 5 minutes. If the surrounding temperature is 20°C , what is the time taken to cool from 60°C to 30°C ?

Solution:

Using Newton's law of cooling, the cooling rate is directly proportional to the difference in temperature.

Here, average of 90°C and 40 °C = 50 °C

Surrounding temperature = 20 °C

Difference = 50 – 20= 30° C

Under the given conditions, the ball cools 80° C in 5 minutes

$$\text{Therefore, } \frac{\text{Change in temperature}}{\text{Time}} = k\Delta t \Rightarrow \frac{30}{5} = K \times 30 \dots\dots\dots (1)$$

Where the value of K is a constant.

The average of 60 °C and 30 °C = 45 °C

\Rightarrow 45 °C – 20 °C = 25 °C above the room temperature and the body cools by 30 °C [60 °C – 30 °C] within

time t (Assume)

$$\text{Therefore, } \frac{30}{t} = K \times 25 \dots\dots\dots (2)$$

Dividing equation (1) by (2), we have:

$$\frac{\frac{30}{5}}{\frac{30}{t}} = \frac{K \times 30}{K \times 25} \Rightarrow \frac{t}{5} = 1.2$$

Therefore, t = 6 mins.

