1. Which of the following examples represents periodic motion?

(a) A swimmer completing one (return) trip from one bank of a river to the other and back.

(b) A freely suspended bar magnet displaced from its N-S direction and released.

(c) A hydrogen molecule rotating about its centre of mass.

(d) An arrow released from a bow.

Solution:

(a) The swimmers motion is not periodic. The motion of the swimmer between the banks of a river is to and fro. However, it does not have a definite period. This is because the time taken by the swimmer during his back and forth journey may not be the same.

(b) The motion of a freely-suspended magnet, if displaced from its N-S direction and released, is periodic. Because the magnet oscillates about its position with a definite period of time.

(c) A hydrogen molecule rotating about its centre of mass is periodic. This is because when a hydrogen molecule rotates about its centre of mass, it comes to the same position again and again after an equal interval of time.

(d) An arrow released from a bow moves only in the forward direction. It does not come backward. Therefore, this motion is not periodic.

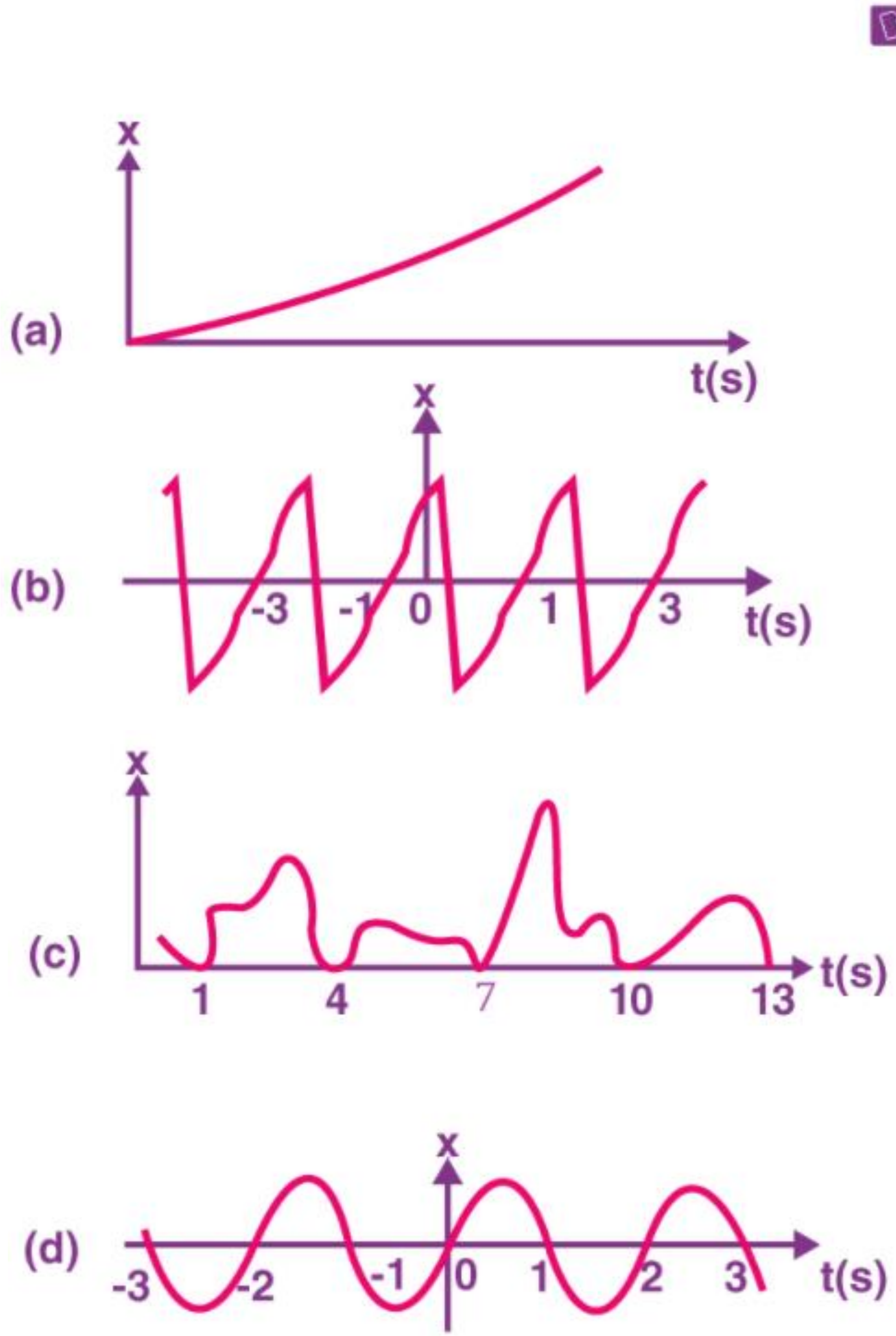
2. Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?
(a) the rotation of earth about its axis.
(b) motion of an oscillating mercury column in a U-tube.
(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.

(d) general vibrations of a polyatomic molecule about its equilibrium position.

Solution:

(a) Rotation of the earth is not to and fro motion about a fixed point. Therefore, it is periodic but not S.H.M.
(b) Simple harmonic motion
(c) Simple harmonic motion
(d) General vibrations of a polyatomic molecule about its equilibrium position is periodic but not SHM. A polyatomic molecule has a number of natural frequencies. Therefore its vibration is a superposition of simple harmonic motions of a number of different frequencies.

3. Fig. 14.23 depicts four x-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?



Solution:

(a) It is not periodic motion because motion does not repeat itself after a regular interval of time

(b) The given graph illustrates a periodic motion, which is repeating itself after every 2 seconds

(c) The given graph does not exhibit a periodic motion because the motion is repeated in one position only. For a periodic motion, the entire motion during one period must be repeated successively

(d) The given graph illustrates a periodic motion, which is repeating itself in every 2 seconds

4. Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):
(a) sin ωt – cos ωt
(b) $\sin^3 \omega t$
(c) 3 cos (π/4 – 2ωt)
(d) cos ωt + cos 3ωt + cos 5ωt
(e) $\exp(-\omega^2 t^2)$
(f) $1 + \omega t + \omega^2 t^2$

Solution:

(a) sinωt−cosωt
=√2 [(1/√2) sinωt− (1/√2) cosωt]
= √2 [sinωt×cos (π/4) −cosωt×sin (π/4)]

= √2 [sinωt – (π/4)]
It is a simple harmonic motion and its period is 2π/4

(b) $\sin^3\omega t=1/4[3\sin \omega t-\sin 3\omega t]$

Here sin ωt and sin 3ωt individually represent SHM. The superposition of two SHM is periodic but not simple harmonic.
Its time period is 2π/ω

(c) 3cos[4π −2ωt] = 3cos[2ωt−π/4]

The equation can be written in the form cos(ωt+ϕ). It is S.H.M with the period 2π/2ω=π/ω

(d) cosωt+cos3ωt+cos5ωt.

Each of the cosine function represents a simple harmonic motion. However, the superposition of three simple harmonic motions is periodic, but not simple harmonic. Its time period is 2π/ω

(e) $\exp(-\omega^2 t^2)$

It is an exponential function that does not repeat itself. Therefore, it is a non-periodic motion.

(f) The given function $1+\omega t+\omega^2 t^2$

It is non-periodic.

5. A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(a) at the end A,

(b) at the end B,

(c) at the mid-point of AB going towards A,

(d) at 2 cm away from B going towards A,

(e) at 3 cm away from A going towards B, and

(f) at 4 cm away from B going towards A.

Solution:

(a) Zero, Positive, Positive

(b) Zero, Negative, Negative

(c) Negative, Zero, Zero

(d) Negative, Negative, Negative

(e) Positive, Positive, Positive

(f) Negative, Negative, Negative

Explanation:

(a), (b) The given situation is shown in the following figure. Points A and B are the two end points, with AB = 10 cm, where 'O' is the midpoint of the path.

A particle is in linear simple harmonic motion between the end points. At the extreme point A, the particle is at rest momentarily. Therefore, its velocity is zero at this point. Its acceleration is positive as it is directed along AO. Force is also positive in this case as the particle is directed rightward.

At the extreme point B, the particle is at rest momentarily. Therefore, its velocity is zero at this point

(c) The particle is executing a simple harmonic motion. 'O' is the mean position of the particle. Its velocity at the mean position O is the maximum. The value for velocity is negative since the particle is directed leftward. The acceleration and force of a particle executing SHM is zero at the mean position.

(d) The particle is moving toward point O from the end B. This direction of motion is opposite to the conventional positive direction, which is from A and B. Therefore, the particle's velocity and acceleration, and the force on it are all negative.

(e) The particle is moving toward point O from the end A. This direction of motion is from A to B, which is the conventional positive direction. Therefore, the values for velocity, acceleration and force are all positive.

(f) This case is similar to the one mentioned in (d)

6. Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?

(a) a = 0.7x

(b) a = –200x^2

(c) a = –10x

(d) a = 100x^3

Solution:

Condition of SHM

Acceleration is directly proportional to negative of displacement of particle

If ‘a’ is acceleration

x is displacement

Then, for Simple Harmonic Motion,

a = – kx where k is constant

(a) a = 0.7x

This is not in the form of a = -kx

Hence, this is not SHM

(b) $a = -200x^2$

Clearly, it is not SHM

(c) a = -10x

This is in the form of a = -kx

Hence, this is SHM

(d) $a = 100x^3$

It’s clear it is not SHM

7. The motion of a particle executing simple harmonic motion is described by the displacement function,
x(t) = A cos (ωt + φ).
If the initial (t = 0) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is π s^{-1}. If instead of the cosine function, we choose the sine function to describe the SHM: x = B sin (ωt + α), what are the amplitude and initial phase of the particle with the above initial conditions.

Solution:

Displacement function x(t) = A cos (ωt + φ)

At position, t = 0;
Displacement, x(0)=1 cm
Initial velocity, v=ω cm/s
Angular frequency, $\omega=\pi\ s^{-1}$
The given function is x(t)=Acos(ωt+ϕ) ———(1)

1=Acos(ω×0+ϕ)=Acosϕ
Acosϕ=1 —————(2)

Differentiating equation (1) w.r.t “t”

Velocity, v=dx/dt
v=−Aωsin(ωt+ϕ) ——(3)

t = 0 and v = ω
1=−Asin(ω×0+ϕ)=−Asinϕ
Asinϕ=−1 ———-(4)

Squaring and adding equations (2) and (4), we get:
$A^2(\sin^2\phi+\cos^2\phi)=1+1$

A = √2 cm
Dividing equation (4) by equation (2),

(A sin ϕ/A cos ϕ) = -1/1

tanϕ=−1

⇒ ϕ=3π/4, 7π/4

If sine function is used

x=Bsin(ωt+α)

At t = 0, x = 1 and v = ω we get

1=Bsin(ω×0+α]=1+1
Bsinα=1 —- (5)
Velocity, v= dx/dt = Bωcos(ωt+α)

taking v = ω

1 = Bcos(ω(0)+α) = Bcos α——(6)

Squaring and adding equations(5) and (6), we get:
$B^2[\sin^2\alpha+\cos^2\alpha]=1+1$
$B^2=2$

B= √2 cm
Dividing equation (5) by equation (6), we get:
Bsinα/Bcosα=1/1
tanα=1

Therefore, α=π/4,5π/4,……

8. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

Solution:

Given

Maximum mass that the scale can read, M = 50 kg

Maximum displacement of the spring = Length of the scale, l = 20 cm

= 0.2 m

Time period, T = 0.6 s

Maximum force exerted on the spring, F = mg

Where,

g = acceleration due to gravity = 9.8 m/s^2

F = 50 x 9.8 = 490

Hence,

Spring constant, k = F / l

= 490 / 0.2

We get,

= 2450 N m^{-1}

Mass m is suspended from the balance.

Time period, t = 2π√m / k

Therefore,

m = $(T / 2\pi)^2$ x k

= $\{0.6 / (2 \times 3.14)\}^2$ x 2450

We get,

= 22.36 kg

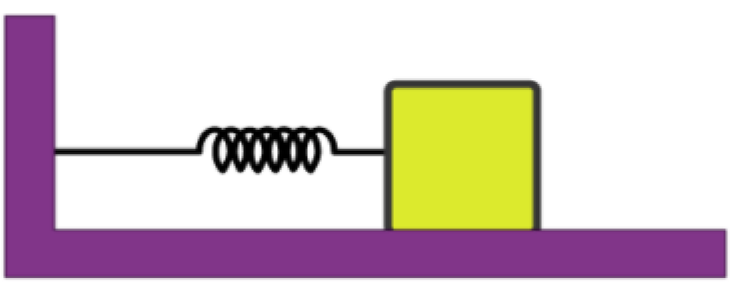
Hence, weight of the body = mg = 22.36 x 9.8

On calculation, we get,

= 219.13 N

Therefore, the weight of the body is about 219 N

9. A spring having with a spring constant 1200 N m^{-1} is mounted on a horizontal table as shown in Fig. 14.24. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.



Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.

Solution:

Given,

Spring constant, $k = 1200\ N\ m^{-1}$

Mass, m = 3 kg

Displacement, A = 2.0 cm

= 0.02 m

(i) Frequency of oscillation 'v' is given by the relation:

v = (1 / T)

= (1 / 2 π) (√k / m)

Where,

T is the time period

So,

v = {1 / (2 x 3.14)}√1200 / 3

On calculating further, we get,

= 3.18 m / s

Therefore, the frequency of oscillations is 3.18 cycles per second.

(ii) Maximum acceleration (a) is given by the relation:

$a = \omega^2 A$

Where,

ω = Angular frequency = √k / m

A = Maximum displacement

Hence,

a = (k / m) A

a = (1200 x 0.02) / 3

We get,

$= 8\ m\ s^{-2}$

Therefore, the maximum acceleration of the mass is $8.0\ m / s^2$

(iii) Maximum velocity, $v_{max} = A\omega$

On substituting, we get,

= A √k / m

= 0.02 x ($\sqrt{1200 / 3}$)

= 0.4 m / s

Therefore, the maximum velocity of the mass is 0.4 m / s

10. In Exercise 14.9, let us take the position of mass when the spring is unstreched as x = 0, and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is

(a) at the mean position,

(b) at the maximum stretched position, and

(c) at the maximum compressed position. In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

Solution:

Distance travelled by the mass sideways, a = 2.0 cm

Angular frequency of oscillation:

$\omega = \sqrt{k / m}$

$= \sqrt{1200 / 3}$

$= \sqrt{400}$

We get,

= 20 rad s^{-1}

(a) As time is noted from the mean position,

Hence, using

$x = a \sin \omega t$

We have,

$x = 2 \sin 20 t$

(b) At maximum stretched position, the body is at the extreme right position, with an initial phase of π / 2 rad. Then,

$x = a \sin (\omega t + \pi / 2)$

$= a \cos \omega t$

$= 2 \cos 20 t$

(c) At maximum compressed position, the body is at left position, with an initial phase of 3π / 2 rad.

Then,

$x = a \sin (\omega t + 3\pi / 2)$

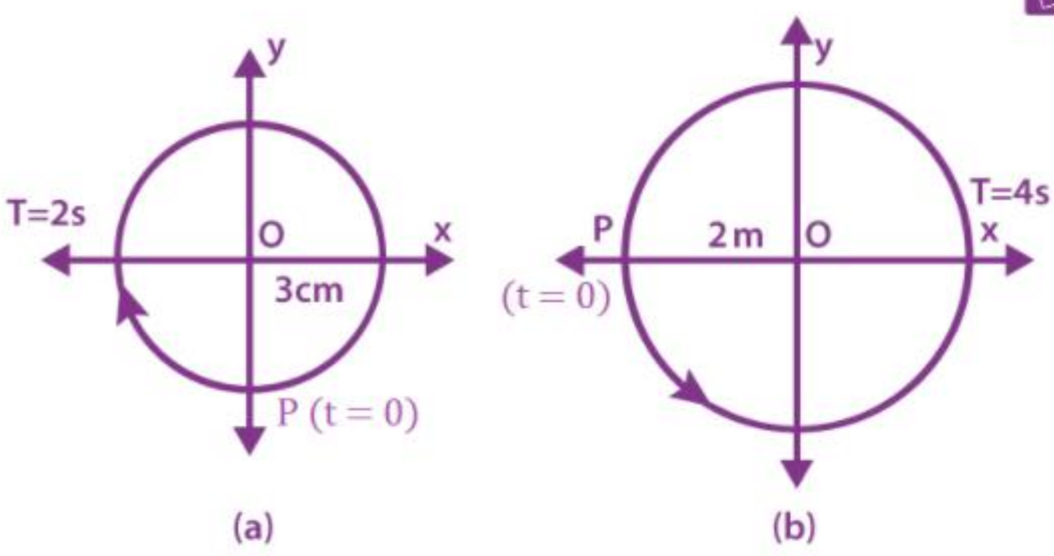
$= - a \cos \omega t$

= – 2 cos 20 t

Therefore,

The functions neither differ in amplitude nor in frequency. They differ in initial phase.

11. Figures 14.25 correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure.



Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

Solution:

(a) Time period, t = 2 s

Amplitude, A = 3 cm

At time, t = 0, the radius vector OP makes an angle π / 2 with the positive x -axis,

i.e,

Phase angle **ϕ = +** π / 2

Hence, the equation of simple harmonic motion for the x – projection of OP, at the time t, is given by the displacement equation:

x = A cos [(2πt / T) + ϕ]

= 3 cos [(2πt / 2) + π / 2]

= – 3 sin (2πt / 2)

Therefore,

$x = -3 \sin(\pi t)$ cm

(b) Time period, t = 4 s

Amplitude, a = 2 m

At time t = 0, OP makes an angle π with the x-axis, in the anticlockwise direction

Thus,

Phase angle **$\phi = +\pi$**

Hence, the equation of simple harmonic motion for the x-projection of OP, at the time t, is given as:

$x = a \cos[(2\pi t / T) + \phi]$

$= 2 \cos[(2\pi t / 4) + \pi]$

Hence,

$x = -2 \cos\{(\pi / 2) t\}$m

12. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t =0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).
(a) $x = -2 \sin(3t + \pi/3)$
(b) $x = \cos(\pi/6 - t)$
(c) $x = 3 \sin(2\pi t + \pi/4)$
(d) $x = 2 \cos \pi t$

Solution:

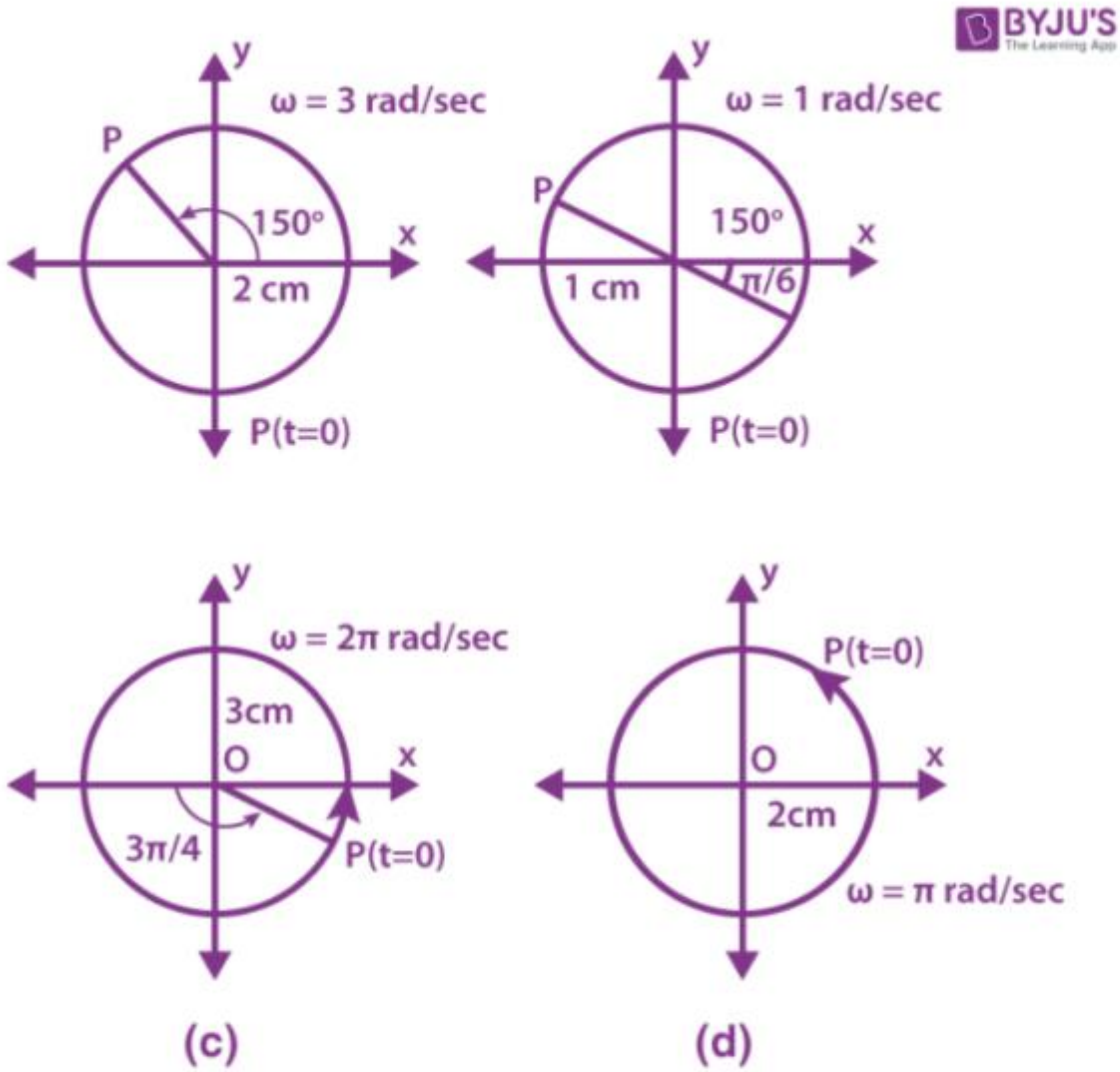
(a) $x = -2\sin(3t + \pi/3)$
$= 2\cos(3t + \pi/3 + \pi/2)$
$= 2\cos(3t + 5\pi/6)$
The above equation can be compared with the standard equation, $x = A \cos(\omega t + \Phi)$
Amplitude, A = 2 cm (radius of the circle)
Angular velocity, ω = 3 rad/s
Phase angle, $\Phi = 5\pi/6 = 150^0$

(b) $x = \cos(\pi/6 - t)$
$= \cos(t - \pi/6)$
Comparing this equation with $A\cos(\omega t + \Phi)$
Phase angle, $\Phi = -\pi/6 = -30^0$
Amplitude, A = 1 cm (radius of the circle)
Angular velocity, ω = 1 rad/s

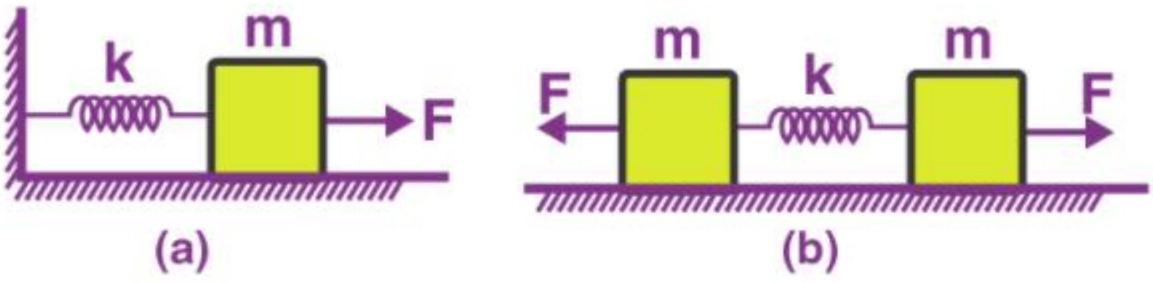
(c) $x = 3 \sin(2\pi t + \pi/4)$
$= -3 \cos(2\pi t + \pi/4 + \pi/2)$
$= -3 \cos(2\pi t + 3\pi/4)$
$=-3 \cos(2\pi t + 3\pi/4)$
Comparing with the standard equation $A\cos(\omega t + \Phi)$
Amplitude, A =- 3cm

Angular velocity, ω = 2π rad/s
Phase angle, Φ = 3π/4 rad

(d) x = 2 cosπt
Amplitude, A = 2
Angular velocity, ω = π rad/sec = 180^0
Phase angle, Φ = 0



13. Figure (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. (b) is stretched by the same force F.



(a) What is the maximum extension of the spring in the two cases?
(b) If the mass in Fig. (a) and the two masses in Fig. (b) is released, what is the period of oscillation in each case?

Solution:

(a) The maximum extension of the spring in fig (a) is x. The F = kx Therefore, x = F/k

The force on each mass acts as the reaction force acting on the other mass. The two mass behaves as if it is fixed with respect to the other.

Therefore, x = F/k

here k is the spring constant

(b) In figure (a) the restoring force on the mass is F = -kx, here x is the extension of the spring.

For the mass (m) of the block, force is written as

$F = ma = m (d^2x/dt^2)$

Therefore, $m (d^2x/dt^2) = -kx$

$(d^2x/dt^2) = (-k/m)x = -\omega^2x$

Here, angular frequency of oscillation, $\omega = \sqrt{k}/\sqrt{m}$

Time period of oscillation, $T = 2\pi/\omega$

$= 2\pi (\sqrt{m}/\sqrt{k})$

In Figure (b), the centre of the system is O and there are two springs. Each spring is of length l/2 and it is attached to two masses

Therefore $F = -2kx$

here x is the extension of the spring.

$F = ma = m (d^2x/dt^2)$

$m (d^2x/dt^2) = -2kx$

$(d^2x/dt^2) = (-2k/m)x = -\omega^2 x$
$\omega = \sqrt{(2k/m)}$

$T = 2\pi (\sqrt{m}/\sqrt{2k})$

14. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad/min, what is its maximum speed?

Given

Angular frequency of the piston, ω = 200 rad/ min

Stroke = 1.0 m

Amplitude, A = 1.0 / 2

= 0.5 m

The maximum speed (v_{max}) of the piston is given by the relation:

$v_{max} = A\omega$

= 200 x 0.5

We get,

= 100 m/ min

Therefore, its maximum speed is 100 m / min

15. The acceleration due to gravity on the surface of moon is 1.7 m s^{-2}. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (g on the surface of earth is 9.8 m s^{-2})

Solution:

Given

Acceleration due to gravity on the surface of moon, $g' = 1.7$ m s^{-2}

Acceleration due to gravity on the surface of earth, g = 9.8 m s^{-2}

Time period of a simple pendulum on earth, T = 3.5 s

$T = 2\pi\sqrt{l / g}$

Where,

l is the length of the pendulum

Therefore,

$l = \{T^2 / (2\pi)^2\} \times 2$

On substituting, we get,

$l = (3.5)^2 / \{4 \times (3.14)^2\} \times 9.8$ m

The length of the pendulum remains constant

On moon's surface, time period, $T' = 2\pi\sqrt{l / g'}$

$= 2\pi [\sqrt{(3.5)^2 / 4 \times (3.14)^2 \times 9.8 / 1.7}]$

We get,

= 8.4 s

Therefore, the time period of the simple pendulum on the surface of the moon is 8.4 s

16. Answer the following questions:
(a) Time period of a particle in SHM depends on the force constant k and mass m of the particle: $T = = 2\pi(\sqrt{m}/\sqrt{k})$
A simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?

(b) The motion of a simple pendulum is approximately simple harmonic for small-angle oscillations. For larger angles of oscillation, a more involved analysis shows that T is greater than $2\pi(\sqrt{l}/\sqrt{g})$. Think of a qualitative argument to appreciate this result.

(c) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give the correct time during the free fall?

(d) What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?

Solution:

(a) In the case of a simple pendulum, the spring constant k is proportional to the mass. The m is the numerator and the denominator will cancel each other. Therefore, the time period of the simple pendulum is independent of the mass of the bob.

(b) The restoring force acting on the bob of a simple pendulum is given by the expression

$F=-mg\sin\theta$

F is the restoring force

m is the mass of the bob

g is the acceleration due to gravity

θ is the angle of displacement

When θ is small, $\sin\theta \approx \theta$.

Then the expression for the time period of a simple pendulum is given by $T=2\pi(\sqrt{l}/\sqrt{g})$

When θ is large, $\sin\theta < \theta$. Therefore, the above equation will not be valid. There will be an increase in the time period T.

(c) Wristwatch works on spring action and does not depend on the acceleration due to gravity. Therefore, the watch will show the correct time.

(d) During the free fall of the cabin, the acceleration due to gravity will be zero. Therefore the frequency of oscillation of the simple pendulum will also be zero.

17. A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

Solution:

The bob of the simple pendulum will experience the centripetal acceleration provided by the circular motion of the car and the acceleration due to gravity.

Acceleration due to gravity = g

Centripetal acceleration = v^2 / R

Where,

v is the uniform speed of the car

R is the radius of the track

Effective acceleration (g') is given as

$g' = \sqrt{g^2 + (v^4 / R^2)}$

Hence,

Time period, $t = 2\pi\sqrt{l / g'}$

$= 2\pi \sqrt{(l / \sqrt{g^2 + v^4 / R^2})}$

Therefore, its time period will be $2\pi \sqrt{(l / \sqrt{g^2 + v^4 / R^2})}$

18. A cylindrical piece of cork of density of base area A and height h floats in a liquid of density ρ_1. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi \sqrt{h\rho / \rho_1 g}$ where ρ is the density of cork. (Ignore damping due to viscosity of the liquid)

Solution:

Given

Base area of the cork = A

Height of the cork = h

Density of the liquid = ρ_1

Density of the cork = ρ

In equilibrium:

Weight of the cork = Weight of the liquid displaced by the floating cork

Let the cork be depressed slightly by x. As a result, some excess water of a certain volume is displaced.

Thus, an extra up-thrust acts upward and provides the restoring force to the cork

Up-thrust = Restoring force, F = Weight of the extra water displaced

F = – (Volume x Density x g)

Volume = Area x Distance through which the cork is depressed

Volume = Ax

Therefore,

F = – Ax x ρ_1g …. (1)

According to the force law:

F = kx

k = F / x

where,

k is constant

k = F / x = – Aρ_1g ……. (2)

The time period of the oscillations of the cork:

T = 2π (√m / k) ……. (3)

Where,

m = Mass of the cork

= Volume of the cork x Density

= Base area of the cork x Height of the cork x Density of the cork

= Ahρ

Therefore, the expression for the time period becomes:

T = 2π √(Ahρ / Aρ_1g)

T = 2π √(hρ / ρ_1g)

19. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.

Solution:

Area of cross-section of the U-tube = A

Density of the mercury column **=** ρ

Acceleration due to gravity = g

Restoring force, F = Weight of the mercury column of a certain height

F = – (Volume x Density x g)

F = – (A x 2h x ρ x g)

= – 2Aρgh

= – k x Displacement in one of the arms (h)

Where,

2h is the height of the mercury column in the two arms

k is a constant, given by

k = – F / h

We get,

k = 2Aρg

Time period, T = 2π √(m / k)

On substituting k value, we get,

Time period, T = 2π √(m / 2Aρg)

Where,

m is the mass of the mercury column

Let l be the length of the total mercury in the U-tube

Mass of mercury, m = volume of mercury x Density of mercury

= Alρ

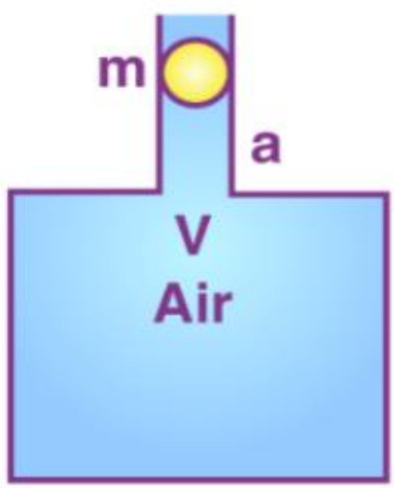
Hence,

T = 2π √(Alρ / 2Aρg)

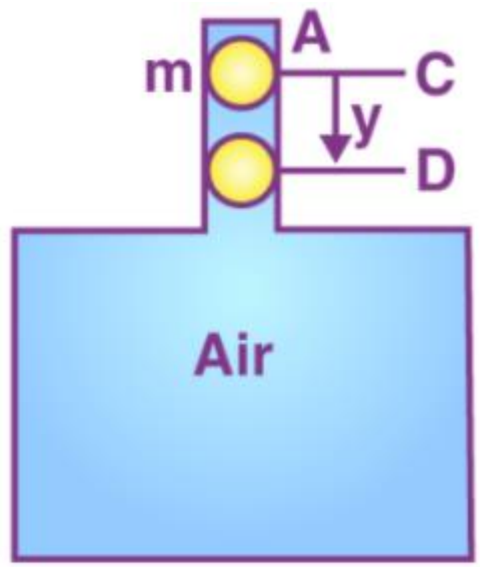
T = 2π √(l / 2g)

Therefore, the mercury column executes simple harmonic motion with time period 2π √(l / 2g)

20. An air chamber of volume V has a neck area of cross-section into which a ball of mass m just fits and can move up and down without any friction. Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal [see Figure]



Solution:



Volume of the air chamber = V

Cross-sectional area of the neck = A

Mass of the ball = m

The ball is fitted in the neck at position C

The pressure of the air below the ball in the chamber is equal to the atmospheric pressure.

The ball is pressed down a little by increasing the pressure by a small amount p, so the ball moves down to position D.

Increase the pressure on the ball by a little amount p, so that the ball is depressed to position D

The distance CD =y
The volume of the air chamber decreases and the pressure increases.

There will be a decrease in volume and hence increase in pressure of air inside the chamber. The decrease in volume of the air inside the chamber, ΔV=Ay
Volumetric strain = change in volume/ original volume
= ΔV/V = Ay/V
Bulk Modulus of elasticity, B = Stress/ volumetric strain

= -p/(Ay/V)

= -pV/Ay

p = – BAy/V

The restoring force on the ball due to the excess pressure

F = p x A = (- BAy/V) x A = – (BA^2/V).y ——(1)

F ∝ y and the negative sign indicates that the force is directed towards the equilibrium position.

If the increased pressure is removed the ball will execute simple harmonic motion in the neck of the chamber with C as the mean position.
In S.H.M., the restoring force, F=−ky ———(2)
Comparing (1) and (2),
-ky = – (BA^2/V).y

k = (BA^2/V)

Inertia factor = mass of ball =m
Time period, T = 2π√inertia factor/√spring factor

T = 2π √m/√k

$$T = 2\pi\sqrt{\frac{m}{\frac{EA^2}{V}}} = \frac{2\pi}{A}\sqrt{\frac{mV}{E}}$$

Frequency, $\nu = \frac{1}{T} = \frac{A}{2\pi}\sqrt{\frac{E}{mV}}$

21. You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant k and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

Solution:

(a) Mass of the automobile = 3000 kg

The suspension sags by 15 cm

Decrease in amplitude =50% during one complete oscillation

Let k be the spring constant of each spring, then the spring constant of the four springs in parallel is

K= 4k

Since F = 4kx

Mg = 4kx

⇒ k = Mg/4x = (3000 x 10)/(4 x 0.15) = 5 x 10^4 N

(b) Each wheel supports 750 kg weight

t = 2π√m/√k = 2 x 3.14 x (√750/√5 x 10^4) = 0.77 sec

Using, $x = x_0 e^{-\frac{bt}{2m}}$,

we get

$$\frac{50}{100}x_0 = x_0 e^{-\frac{b\times 0.77}{2\times 750}}$$

$$log_e 2 = (b \times 0.77)/(1500) log_e e$$

$$b = \frac{(1500) log_e 2}{0.77}$$

b = (1500 x 0.6931)/0.77 = 1350.2 kg/s

22. Show that for a particle in linear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

Solution:

Let m be the mass of the particle executing simple harmonic motion. The displacement of the particle at an instant t is given by

x = A sin ωt

Velocity of the particle, v= dx/dt = Aωcos ωt

Instantaneous Kinetic Energy, K = (1/2) mv^2

= (1/2) m $(A\omega \cos \omega t)^2$

= (1/2) m $(A^2\omega^2\cos^2\omega t)$

Average value of kinetic energy over one complete cycle

$$K_{av} = \frac{1}{T}\int_0^T \frac{1}{2} mA^2\omega^2 cos^2\omega t dt = \frac{mA^2\omega^2}{2T}\int_0^T cos^2 \omega t dt = \frac{mA^2\omega^2}{2T}\int_0^T \frac{(1+cos2\omega t)}{2} dt$$

$$= \frac{mA^2\omega^2}{4T}\left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{mA^2\omega^2}{4T}\left[(T-0) + \left(\frac{\sin 2\omega t - \sin 0}{2\omega}\right)\right]$$

$$= \frac{1}{4} m A^2 w^2$$

Average instantaneous potential energy, U = (1/2)kx^2 = (1/2) $m\omega^2 x^2$= (1/2)$m\omega^2 A^2 \sin^2 \omega t$

Average value of potential energy over one complete cycle

$$U_{av} = \frac{1}{T}\int_0^T \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t = \frac{m\omega^2 A^2}{2T}\int_0^T \sin^2 \omega t dt$$

$$= \frac{m\omega^2 A^2}{2T}\int_0^T \frac{(1-\cos 2\omega t)}{2} dt$$

$$= \frac{m\omega^2 A^2}{4T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{m\omega^2 A^2}{4T}\left[(T-0) - \frac{(\sin 2\omega t - \sin 0)}{2\omega}\right]$$
$$= \frac{1}{4} m\omega^2 A^2$$

Kinetic energy = Potential energy

23. A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constant α is defined by the relation J = –α θ, where J is the restoring couple and θ the angle of twist).

Solution:

Mass of the circular disc = 10 kg

Period of torsional oscillation = 1.5 s

Radius of the disc = 15 cm = 0.15 m

Restoring couple, J = –α θ

Moment of inertia, I = (1/2) mR^2

= (1/2) x 10 x $(0.15)^2$

= 0.1125 kgm^2

Time period is given by the relation

$$T = 2\pi\sqrt{\frac{I}{\alpha}}$$

So, $\alpha = \frac{4\pi^2 I}{T^2}$ $\alpha = \frac{4\times(3.14)^2\times 0.1125}{(1.5)^2}$

= 4.44/2.25

= 1.97 Nm/rad

24. A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is (a) 5 cm (b) 3 cm (c) 0 cm.

Solution:

Amplitude = 5 cm = 0.05 m

Time period = 0.2 s

When the displacement is y, then

acceleration, $A = -\omega^2 y$

Velocity, $v = \omega\sqrt{r^2 - y^2}$

$\omega = 2\pi/T$

$= 2\pi/0.2 = 10\pi$ rad/s

(a) When the displacement y = 5 cm = 0.05 m

Acceleration, $A = -(10\pi)^2(0.05) = 5\pi^2$ m/s^2

Velocity, $V = 10\pi\sqrt{(0.05)^2 - (0.05)^2} = 0$

(b) When the displacement y = 3 cm = 0.03 m

Acceleration, $A = -(10\pi)^2(0.03) = 3\pi^2$ m/s^2

Velocity, $V = 10\pi\sqrt{(0.05)^2 - (0.03)^2} = 10\pi \times 0.04 = 0.4\pi m/s$

(c) When the displacement y = 0

Acceleration, $A = -(10\pi)^2(0) = 0$

Velocity, $V = 10\pi\sqrt{(0.05)^2 - (0)^2} = 10\pi \times 0.05 = 0.5\pi m/s$

25. A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time t = 0. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0. [Hint: Start with the equation x = a cos (ωt+θ) and note that the initial velocity is negative.]

Solution:

The angular velocity of the spring = ω

$x = a\cos(\omega t+\theta)$

At t = 0, $x = x_0$

Substituting these values in the above equation we get

$x_0 = A\cos\theta$ ——(1)

Velocity, $v = dx/dt = -A\omega\sin(\omega t+\theta)$

At t = 0, $v = -v_0$

Substituting these values in the above equation we get

$-v_0 = -A\omega\sin\theta$

Asin θ = $v_{0/\omega}$———-(2)

Squaring and adding (1) and (2) we get

$$A^2(cos^2\theta + \ sin^2\theta) = \ x_0^2 + \ \frac{v_0^2}{\omega^2}$$

$$A = \ \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$$