Q-1: State whether the following physical quantities are scalar or vector.

(i) Mass

(ii) Volume

(iii) Speed

(iv) Acceleration

(v) Density

(vi) Number of moles

(vii) Velocity

(viii) Angular frequency

(ix) Displacement

(x) Angular velocity

Solution:

Scalar: Density, mass, speed, volume, angular frequency, number of moles.

Vector: Velocity, acceleration, angular velocity, displacement.

A scalar quantity depends only on the magnitude and it is independent of the direction. Density, mass, speed, volume, angular frequency and number of moles are scalar quantities.

A vector quantity depends on the magnitude as well as the direction. Velocity, acceleration, angular velocity, displacement comes under this.

Q-2: From the following pick any two scalar quantities:

Force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

Solution:

The dot product of force and displacement is the work done. Work is a scalar quantity since dot product of two quantities is always scalar.

Current is a scalar quantity as it is described only by its magnitude and it is independent of direction.

Q-3: From the following identify the vector quantities:

Pressure, temperature, energy, time, gravitational potential, power, total path length, charge, coefficient of friction, impulse.

Solution:

Impulse is the product of force and time. Since force is a vector quantity, its product with time which is a scalar quantity gives a vector quantity.

Q-4: State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful:

(a) Addition of any two scalars

(b) Adding a scalar to a vector which has the same dimensions

(c) Multiplying a vector by any scalar

(d) Multiplying any two scalars

(e) Adding any two vectors

(f) Addition of a vector component to the same vector.

Solution:

(a) Meaningful:

The addition of two scalar quantities is meaningful only if they both represent the same physical quantity.

(b) Not Meaningful:

The addition of a vector quantity with a scalar quantity is not meaningful

(c) Meaningful:

A scalar can be multiplied with a vector. For example, force is multiplied with time to give impulse.

(d) Meaningful:

A scalar, irrespective of the physical quantity it represents, can be multiplied by another scalar having the same or different dimensions.

(e) Meaningful:

The addition of two vector quantities is meaningful only if they both represent the same physical quantity.

(f) Meaningful:

A component of a vector can be added to the same vector as they both have the same dimensions.

Q-5: Read each statement below carefully and state with reasons, if it is true or false:

(a) The magnitude of a vector is always a scalar

(b) Each component of a vector is always a scalar

(c) The total path length is always equal to the magnitude of the displacement vector of a particle

(d) The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of the average velocity of the particle over the same interval of time

(e) Three vectors not lying in a plane can never add up to give a null vector.

Solution:

(a) True:

The magnitude of a vector is a number. So, it is a scalar.

(b) False:

Each component of a vector is also a vector.

(c) False:

The total path length is a scalar quantity, whereas displacement is a vector quantity. Hence, the total path length is always greater than the magnitude of displacement. It becomes equal to the magnitude of displacement only when a particle is moving in a straight line.

(d) True:

It is because of the fact that the total path length is always greater than or equal to the magnitude of displacement of a particle.

(e) True:

Three vectors, which do not lie in a plane, cannot be represented by the sides of a triangle taken in the same order.

Q-6: Establish the following vector inequalities geometrically or otherwise:

(a) $|a+b| \leq |a| + |b|$

(b) $|a- b| \geq ||a| - |b||$

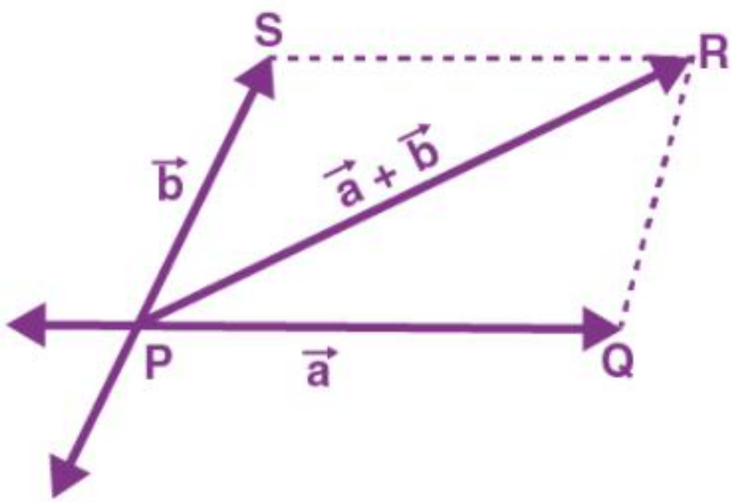
(c) $|a–b| \leq |a| + |b|$

(d) $|a–b| \geq ||a|–|b||$

When does the equality sign above apply?

Solution:

(a) Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram PQRS, as given in the figure.



Here,

$\left|\vec{QR}\right| = |\vec{a}|$ — (i)

$\left|\vec{RS}\right| = \left|\vec{QP}\right| = \left|\vec{b}\right|$ — (ii)

$\left|\vec{QS}\right| = \left|\vec{a} + \vec{b}\right|$ — (iii)

Each side in a triangle is smaller than the sum of the other two sides.

Therefore, in ΔQRS ,

QS < (QR + RS)

$\left|\vec{a} + \vec{b}\right| < |\vec{a}| + \left|\vec{b}\right|$ — (iv)

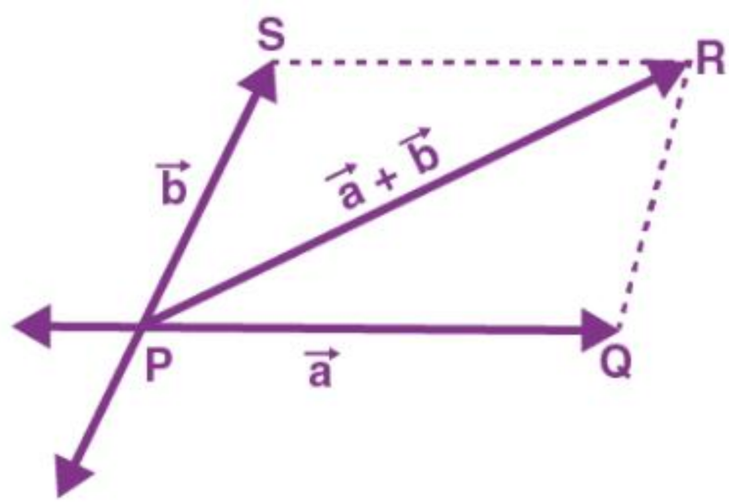
If the two vectors $\vec{a}$ and $\vec{b}$ act along a straight line in the same direction, then:

$\left|\vec{a} + \vec{b}\right| = |\vec{a}| + \left|\vec{b}\right|$ — (v)

Combine equation (iv) and (v),

$$\left|\vec{a}+\vec{b}\right| \leq |\vec{a}| + \left|\vec{b}\right|$$

(b) Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a **parallelogram PQRS,** as given in the figure.



Here,

$$\left|\vec{QR}\right| = |\vec{a}| \text{ — (i)}$$

$$\left|\vec{RS}\right| = \left|\vec{QP}\right| = \left|\vec{b}\right| \text{ — (ii)}$$

$$\left|\vec{QS}\right| = \left|\vec{a}+\vec{b}\right| \text{ — (iii)}$$

Each side in a triangle is smaller than the sum of the other two sides.

Therefore, in ΔQRS ,

QS + RS > QR

QS + QR > RS

$\left|\vec{QS}\right| > \left|\vec{QR} – \vec{QP}\right|$ (QP = RS)

$|\vec{a} + \vec{b}| > ||\vec{a}| – |\vec{b}||$——$(iv)$

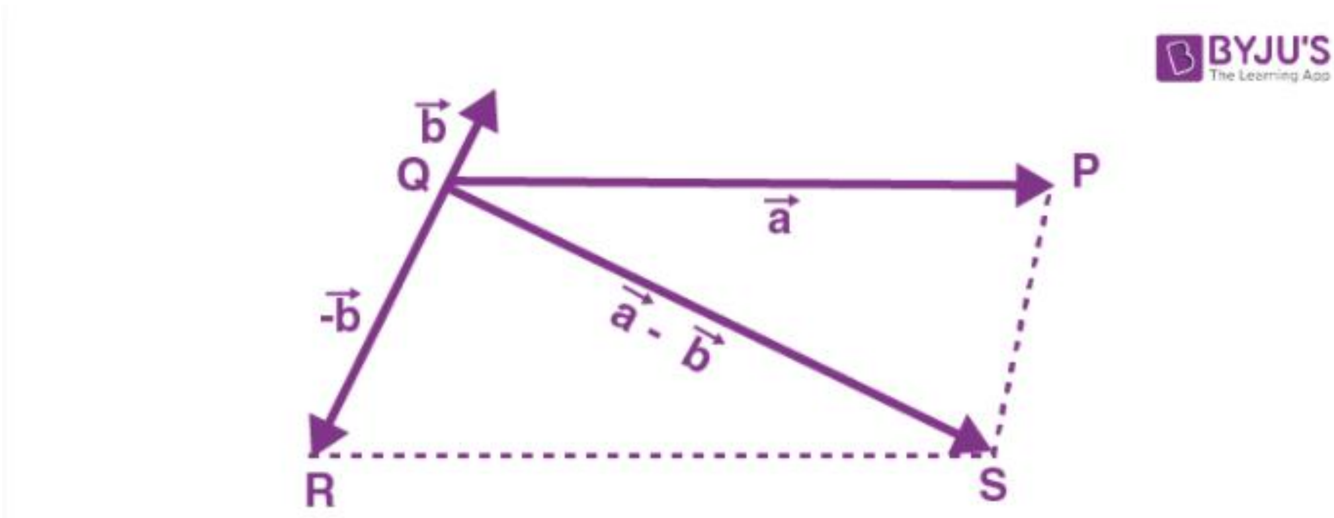
If the two vectors $\vec{a}$ and $\vec{b}$ act along a straight line in the same direction, then:

$|\vec{a} + \vec{b}| = ||\vec{a}| – |\vec{b}||$——$(v)$

Combine equation (iv) and (v):

$|\vec{a} + \vec{b}| \geq ||\vec{a}| – |\vec{b}||$

(c) Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a **parallelogram PQRS,** as given in the figure.



Here,

$\left|\vec{PQ}\right| = \left|\vec{SR}\right| = \left|\vec{b}\right|$ **— (i)**

$\left|\vec{PS}\right| = |\vec{a}|$ **— (ii)**

Each side in a triangle is smaller than the sum of the other two sides.

Therefore, in ΔPSR,

PR < PS + SR

$\left|\vec{a}-\vec{b}\right| < |\vec{a}| + \left|-\vec{b}\right|$ $\left|\vec{a}-\vec{b}\right| < |\vec{a}| + \left|\vec{b}\right|$ — **(iii)**

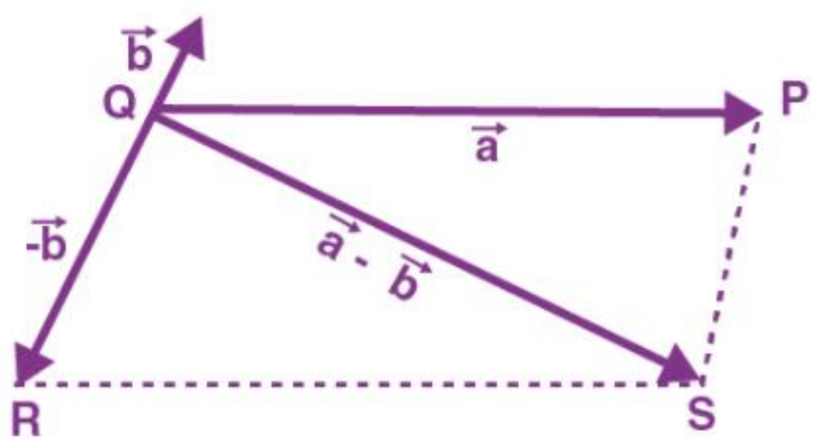
If the two vectors act along a straight line in the opposite direction, then:

$\left|\vec{a}-\vec{b}\right| = |\vec{a}| + \left|\vec{b}\right|$ — **(iv)**

Combine (iii) and (iv),

$\left|\vec{a}-\vec{b}\right| \leq |\vec{a}| + \left|\vec{b}\right|$

(d) Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallelogram PQRS, as given in the figure.



Here,

PR + SR > PS — **(i)**
PR > PS – SR — **(ii)**

$\left|\vec{a}-\vec{b}\right| > |\vec{a}| - \left|\vec{b}\right|$ — **(iii)**

The quantity on the left hand side is always positive and that on the right hand side can be positive or negative.

We take modulus on both the sides to make both quantities positive:

$$\left|\left|\vec{a}-\vec{b}\right|\right| > \left||\vec{a}|-\left|\vec{b}\right|\right| \quad \left|\vec{a}-\vec{b}\right| > \left||\vec{a}|-\left|\vec{b}\right|\right| \text{ — (iv)}$$

If the two vectors act along a straight line in the opposite direction, then:

$$\left|\vec{a}-\vec{b}\right| = \left||\vec{a}|-\left|\vec{b}\right|\right| \text{ — (v)}$$

Combine (iv) and (v):

$$\left|\vec{a}-\vec{b}\right| \geq \left||\vec{a}|-\left|\vec{b}\right|\right|$$

Q-7: Given that l + m + n + o = 0, which of the given statements are true:

(a) l, m, n and o each must be a null vector.

(b) The magnitude of (l + n) equals the magnitude of (m+ o).

(c) The magnitude of l can never be greater than the sum of the magnitudes of m, n and o.

(d) m + n must lie in the plane of l and o if l and o are not collinear, and in the line of l and o, if they are collinear?

Solution:

(a) False

In order to make l + m + n + o = 0, it is not necessary to have all the four given vectors to be null vectors. There are other combinations which can give the sum zero.

(b) True

l + m + n + o = 0

l + n = – (m + o)

Taking mode on both the sides,

$$|l+n| = |-(m+o)| = |m+o|$$

Therefore, the magnitude of (l + n) is the same as the magnitude of (m + o).

(c) True

l + m + n + o = 0

l = (m + n + o)

Taking mode on both the sides,

$|l| = \ |m+n+o| \ \ |l| \leq \ \ |l| + \ |m| + \ |n|$ **—- (i)**

Equation (i) shows the magnitude of l is equal to or less than the sum of the magnitudes of m, n and o.

(d) True

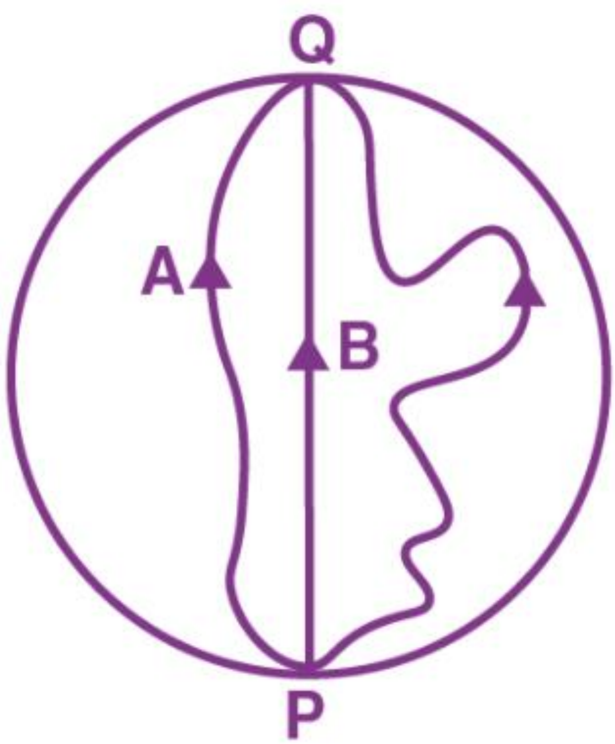
For,

l + m + n + o = 0

The resultant sum of the three vectors l, (m + n), and o can be zero only if (m + n) lie in a plane containing l and o, assuming that these three vectors are represented by the three sides of a triangle.

If l and o are collinear, then it implies that the vector (m + n) is in the line of l and o. This implication holds only then the vector sum of all the vectors will be zero.

Q-8: Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. 4.20. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skate?



Solution:

The distance between the initial and the final position of the particle is called the displacement. All the three girls reach from point P to Q. The diameter of the ground is the magnitude of displacement.

Radius = 200 m

Diameter = 200 x 2 = 400 m

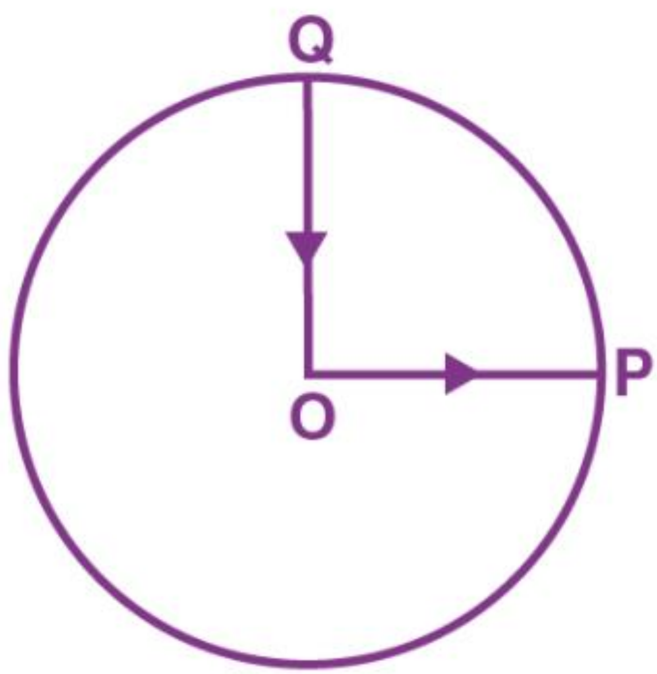
Hence, the magnitude of displacement is 400 m for each girl. This magnitude is equal to the path skated by girl B.

Q-9: A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. 4.21. If the round trip takes 10 min, what is the

(i) Net displacement

(ii) Average velocity and

(iii) The average speed of the cyclist.



Solution:

(i) The distance between the initial and final position of the body is called displacement. The cyclist comes back to the place where he had started in 20 minutes. So, the displacement is zero.

(ii) Average Velocity = $\frac{net\ displacement}{time\ taken}$

As the displacement is zero, the average velocity is zero.

(iii) Average speed = distance travelled/time taken

= OP + Distance PQ + QO/ 10 minutes

= {1 km + (1/4) x 2 x (22/7) x 1km + 1m}/ (10/60) h

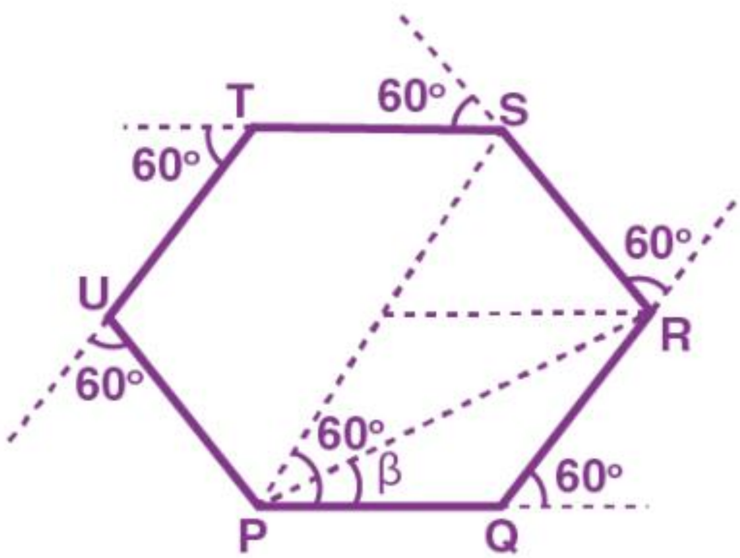
= 6 (2 + 22/14)

= 6 (50/14) = 21.43 km/h

Q-10: On an open ground, a motorist follows a track that turns to his left by an angle of 60^0 after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Solution:

The path followed by the motorist is a regular hexagon with side 500 m as given in the figure.



Let the motorist start from point P.

The motorist takes the third turn at S.

Therefore,

Magnitude of the displacement = PS = PV + VS

= 500 + 500 = 1000 m

Total path length = PQ + QR + RS

= 500 + 500 + 500 = 1500 m

The motorist takes the 6^{th} turn at point P, which is the starting point.

Therefore,

Magnitude of displacement = 0

Total path length = PQ + QR + RS + ST + TU + UP

= 500 + 500 + 500 + 500 + 500 + 500 = 3000 m

The motorist takes the eight turn at point R.

Magnitude of displacement = PR

$$= \sqrt{PQ^2 + QR^2 + 2\,(PQ) \times (QR)\cos 60^\circ}$$

$$= \sqrt{500^2 + 500^2 + (2\,(500) \times (500)\cos 60^\circ)}$$

$$= \sqrt{250000 + 250000 + \left(500000 \times \tfrac{1}{2}\right)}$$

= 866.03 m

$$\beta = \tan^{-1}\left(\frac{500\sin 60^\circ}{500 + 500\cos 60^\circ}\right)$$

= 30°

Therefore, the magnitude of displacement is 866.03 m at an angle of 30° with PR.

Total path length = Circumference of the hexagon + PQ + QR

= 6 × 500 + 500 + 500 = 4000 m

The magnitude of displacement and the total path length corresponding to the required turns is shown in the following table:

Turn	The magnitude of displacement (m)	Total path length (m)
3^{rd}	1000	1500
6^{th}	0	3000
8^{th}	866.03; 30°	4000

Q-11: A passenger arriving in a new town wants to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min.

(a) What is the average speed of the taxi?

(b) What is the magnitude of average velocity? Are the two equal?

Solution:

(a) Total distance travelled = 23 km

Total time taken = 28 min = $\frac{28}{60}$ h

Therefore,

Average speed = $\frac{Total\ distance\ travelled}{Total\ time\ taken}$

= $\frac{23}{\frac{28}{60}}$ = 49.29 km/h

(b) Distance between the hotel and the station = 10 km = Displacement of the car

Therefore,

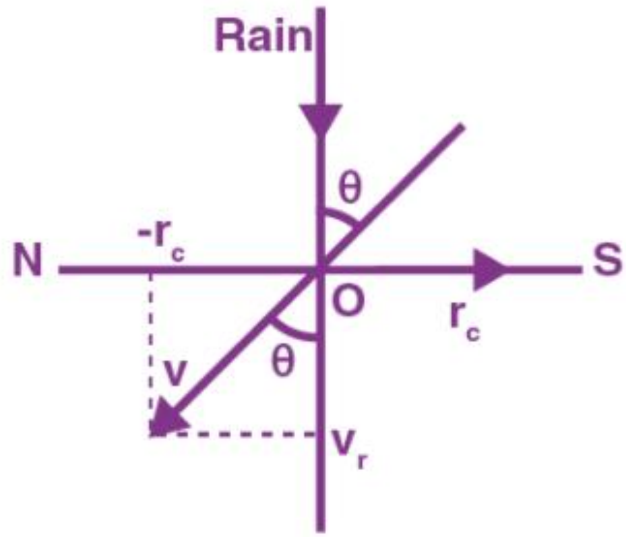
Average velocity = $\frac{10}{\frac{28}{60}}$ = 21.43 km/h

The two physical quantities are not equal.

Q-12: Rain is falling vertically with a speed of 30 ms^{-1}. A woman rides a bicycle with a speed of 10 ms^{-1} in the north to south direction. What is the direction in which she should hold her umbrella?

Solution:

The described situation is shown in the given figure



Here,

v_c = Velocity of the cyclist

v_r = Velocity of falling rain

In order to protect herself from the rain, the woman must hold her umbrella in the direction of the relative velocity (v) of the rain with respect to the woman.

$v = v_r + (-v_c)$

= 30 + (-10) = 20 m/s

$\tan\theta = \frac{v_c}{v_r}$

= $\frac{10}{30}$ $\theta = \tan^{-1}\left(\frac{1}{3}\right)$

= $\tan^{-1}(0.333)$ **= 18°C**

Hence, the woman must hold the umbrella toward the south, at an angle of nearly 18° with the vertical.

Q-13: A man can swim with a speed of 4 km/h in still water. How long does he take to cross a river 1 km wide if the river flows steadily at 3 km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

Solution:

Speed of the man v_m = 4 km/h

Width of the river = 1 km

Time taken to cross the river = $\frac{Width\ of\ the\ river}{Speed\ of\ the\ river}$

= $\frac{1}{4}$ h

= $\frac{1}{4} \times 60$ = 15 min

Speed of the river, v_r = 3 $\frac{km}{h}$

Distance covered with flow of the river = $v_r \times t$

= $3 \times \frac{1}{4}$

= $\frac{3}{4}$

= $\frac{3}{4} \times 1000$ **= 750 m**

Q-14: In a harbour, the wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat?

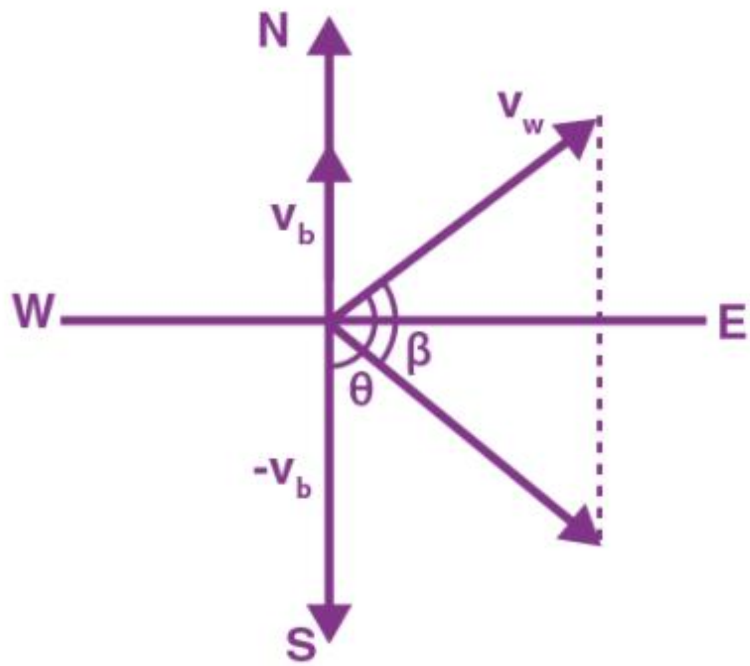
Solution:

Velocity of the boat = v_b = 51 km/h

Velocity of the wind = v_w = 72 km/h

The flag is fluttering in the northeast direction. It shows that the wind is blowing toward the north-east direction.

When the ship begins sailing toward the north, the flag will move along the direction of the relative velocity (v_{wb}) of the wind with respect to the boat.



The angle between v_w and $(-v_b)$ = $90^\circ + 45^\circ$ $\tan\beta = \frac{51\sin(90+45)}{72+51\cos(90+45)}$

= $\frac{51\sin 45}{72+51(-\cos 45)}$

= $\frac{51\times\frac{1}{\sqrt{2}}}{72-51\times\frac{1}{\sqrt{2}}}$

= $\frac{51}{72\sqrt{2}-51}$

= $\frac{51}{72\times 1.414-51}$

$= \frac{51}{50.800}$

$= \tan^{-1}(1.0038) = 45.11°$

Angle with respect to the east direction = 45.11° – 45° = 0.11°

Hence, the flag will flutter almost due east.

Q-15: The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball is thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the hall?

Solution:

Speed of the ball, u = 40 ms^{-1}

Maximum height, h = 25 m

In projectile motion, the maximum height achieved by a body projected at an angle θ, is given as:

$h = \frac{u^2 \sin^2\theta}{2g}$ $25 = \frac{(40)^2 \sin^2\theta}{2\times9.8}$ $\sin^2\theta = 0.30625$ $\sin\theta = 0.5534$ $\theta = \sin^{-1}(0.5534)$ = 33.60^0

Horizontal range, R:

$= \frac{u^2 \sin 2\theta}{g}$

$= \frac{(40)^2 \sin 2\times33.60}{9.8}$

$= \frac{1600\times\sin 67.2}{9.8}$

$= \frac{1600\times0.922}{9.8}$ **= 150.53 m**

Q-16: A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

Solution:

Maximum horizontal distance, **R = 100 m**

The cricketer will only be able to throw the ball to the maximum horizontal distance when the angle of projection is 45° i.e., θ **= 33.60°**

The horizontal range for a projection velocity v, is given as:

R = $\frac{u^2 \sin 2\theta}{g}$ $100 = \frac{u^2}{g} \sin 90^\circ$ $\frac{u^2}{g} = 100$ —— **(i)**

The ball will achieve the max height when it is thrown vertically upward. For such motion, the final velocity v is 0 at the max height H.

Acceleration, a = – g

Using the 3rd equation of motion:

$$v^2 - u^2 = -2gH \quad H = \frac{1}{2} \times \frac{u^2}{g}$$

= $H = \frac{1}{2} \times 100$ **= 50 m**

Q-17: A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the direction and magnitude of the acceleration of the stone?

Solution:

Length of the string, l = 80 cm = 0.8 m

No. of revolutions = 14

Time taken = 25 s

Frequency, v = $\frac{No.\ of\ revolution}{Time\ taken}$ = $\frac{14}{25}$ Hz

Angular frequency ω ,

= $2\pi v$

= $2 \times \frac{22}{7} \times \frac{14}{25}$

= $\frac{88}{25}\ rad\ s^{-1}$

Centripetal acceleration:

$a_c = \omega^2 r = \left(\frac{88}{25}\right)^2 \times 0.8 = 9.91\ ms^{-2}$

The direction of centripetal acceleration is always directed along the string, toward the centre, at all points.

Q-18: An aircraft executes a horizontal loop of radius 1 km with a steady speed of 900 km h^{-1}. Compare its centripetal acceleration with the acceleration due to gravity.

Solution:

Radius of the loop, r = 1 km **= 1000 m**

Speed, v = 900 km h^{-1} = $900 \times \frac{5}{18}$ **= 250 ms^{-1}**

Centripetal acceleration: $a_c = \frac{v^2}{r}$

= $\frac{(250)^2}{1000}$ = 62.5 ms^{-2}

Acceleration due to gravity, g = 9.8 ms^{-2}

$\frac{a_c}{g} = \frac{62.5}{9.8}$ = 6.38

$a_c = 6.38\ g$

Q-19: Read each statement below carefully and state, with reasons, if it is true or false:

(a) The net acceleration of a particle in a circular motion is always along the radius of the circle towards the centre.

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.

(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.

Solution:

(a) False

The net acceleration of a particle in a circular motion is always directed along the radius of the circle toward the centre only in the case of uniform circular motion.

(b) True

At a point on a circular path, a particle appears to move tangentially to the circular path.

(c) True

In uniform circular motion (UCM), the direction of the acceleration vector points toward the centre of the circle. However, it constantly changes with time. The average of these vectors over one cycle is a null vector.

Q-20: The position of a particle is given by

$$r = 3.0t\ \hat{i} – 2.0t^2\ \hat{j} + 4.0\hat{k}\ \ m$$

Where t is in seconds and the coefficients have the proper units for r to be in meters.

(a) Find the 'v' and 'a' of the particle?

(b) What is the magnitude and direction of the velocity of the particle at t = 2.0 s?

Solution:

(a) The position of the particle is given by:

$$\vec{r} = 3.0t\ \hat{i} – 2.0t^2\ \hat{j} + 4.0\hat{k}$$

Velocity $\vec{v}$, of the particle is given as:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(3.0t\ \hat{i} – 2.0t^2\ \hat{j} + 4.0\hat{k}\right)$$

$$\vec{v} = 3.0\ \hat{i} – 4.0t\ \hat{j}$$

Acceleration $\vec{a}$, of the particle is given as:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(3.0\,\hat{i} – 4.0t\,\hat{j}\right)$$

$$\vec{a} = -4.0\hat{j}$$

8.54 m/s, 69.45° below the x – axis

(b) we have velocity vector, $\vec{v} = 3.0\,\hat{i} – 4.0t\,\hat{j}$

At t = 2.0 s:

$$\vec{v} = 3.0\,\hat{i} – 8.0\,\hat{j}$$

The magnitude of velocity is given by:

$$|\vec{v}| = \sqrt{3.0^2 + -8.0^2} = \sqrt{73} = 8.54\,m/s$$

Direction, $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

$$= \tan^{-1}\left(\frac{-8}{3}\right) = -\tan^{-1}(2.667)$$

$$= 69.45^{\circ}$$

The negative sign indicates that the direction of velocity is below the x – axis.

Q-21: A particle starts from the origin at t = 0 s with the velocity of $10\,\hat{j}\,m\,s^{-1}$ and moves in the x –y plane with a constant acceleration of $\left(8.0\,\hat{i} + 2.0\,\hat{j}\right)\,m\,s^{-2}$.

(a) At what time is the x-coordinate of the particle 16 m? What is the y-coordinate of the particle at that time?

(b) What is the speed of the particle at the time?

Solution:

(a) Velocity of the particle = $10\,\hat{j}\,m\,s^{-1}$

Acceleration of the particle = $\left(8.0\ \hat{i} + 2.0\ \hat{j}\right)\ m\ s^{-2}$

But, $\vec{a} = \frac{d\vec{v}}{dt} = 8.0\ \hat{i} + 2.0\ \hat{j}\ \ d\vec{v} = \left(8.0\ \hat{i} + 2.0\ \hat{j}\right)\ ; dt$

Integrating both the sides:

$\vec{v}\,(t) = 8.0t\ \hat{i} + 2.0t\ \hat{j} + \vec{u}$

Where, $\vec{u}$ = velocity vector of the particle at t = 0

$\vec{v}$ = velocity vector of the particle at time t

But, $\vec{v} = \frac{d\vec{r}}{dt}\ \ d\vec{r} = \vec{v}\ dt$

= $\left(8.0t\ \hat{i} + 2.0t\ \hat{j} + \vec{u}\right)\ dt$

Integrating the equations with the conditions:

At t = 0; r = 0 and at t = t; r = r.

$\vec{r} = \vec{u}t + \frac{1}{2}8.0t^2\ \hat{i} + \frac{1}{2} \times 2.0t^2\ \hat{j}\ \ \vec{r} = \vec{u}t + 4.0t^2\ \hat{i} + t^2\ \hat{j}\ \ \vec{r} = \left(10.0\ \hat{j}\right)t +$

$4.0t^2\ \hat{i} + t^2\ \hat{j}\ \ x\ \hat{i} + y\ \hat{j} = 4.0t^2\ \hat{i} + \left(10.0\ t + t^2\right)\ \hat{j}$

Since the motion of the particle is confined to the x-y plane, on equating the coefficients of $\hat{i}\ and\ \hat{j}$, we get:

$x = 4t^2$

t = $\left(\frac{x}{4}\right)^{\frac{1}{2}}$

y = $10t + t^2$

When x = 16 m:

$t = \left(\frac{16}{4}\right)^{\frac{1}{2}}$ = 2 s

Therefore, y = 10 × 2 + $(2)^2$ **= 24 m**

(b) Velocity of the particle:

$$\vec{v}(t) = 8.0t\ \hat{i} + 2.0t\ \hat{j} + \vec{u}$$

At t = 2 s:

$$\vec{v}(t) = 8.0 \times 2\ \hat{i} + 2.0 \times 2\ \hat{j} + 20\ \hat{j}\ \ \vec{v}(t) = 16\ \hat{i} + 14\ \hat{j}$$

Therefore, Speed of the particle:

$$|\vec{v}| = \sqrt{(16)^2 + (14)^2}\ \ |\vec{v}| = \sqrt{256 + 196}\ \ |\vec{v}| = \sqrt{452}\ \ |\vec{v}| = 21.26\ m\ s^{-1}$$

Q-22: $\vec{i}$ and $\vec{j}$ are unit vectors along x- and y-axis respectively. What is the magnitude and direction of the vectors $\vec{i}$ + $\vec{j}$ and $\vec{i}$ − $\vec{j}$? What are the components of a vector $\vec{A}$ = 2 $\vec{i}$ and 3 $\vec{j}$ along the directions of $\vec{i}$ + $\vec{j}$ and $\vec{i}$ − $\vec{j}$? [You may use graphical method]

Solution:

Consider a vector $\vec{P}$, given as below,

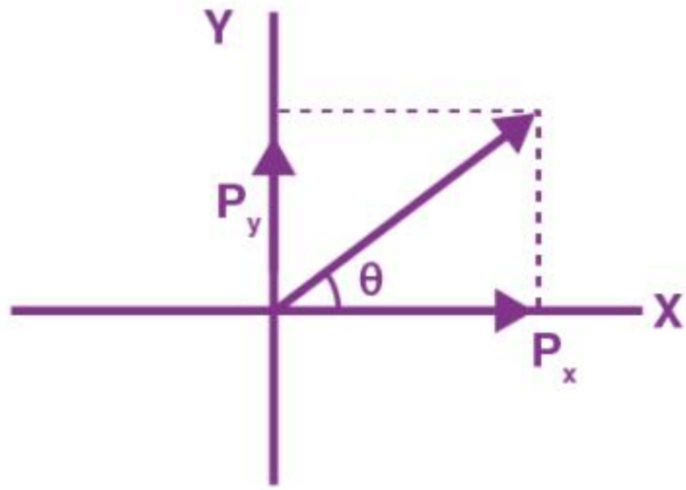
$$\vec{P} = \hat{i} + \hat{j}\ \ P_x\ \hat{i} + P_y\ \hat{j} = \hat{i} + \hat{j}$$

Comparing the components on both the sides, we get:

$P_x = P_y = 1 \; \left|\vec{P}\right| = \sqrt{P_x^2 + P_y^2} \; \left|\vec{P}\right| = \sqrt{1^2 + 1^2} \; \left|\vec{P}\right| = \sqrt{2}$ —— (i)

Therefore, the magnitude of the vector $\vec{i} + \vec{j}$ is $\sqrt{2}$.

Let θ be the angle made by the vector $\vec{P}$, with the x – axis as given in the figure below.



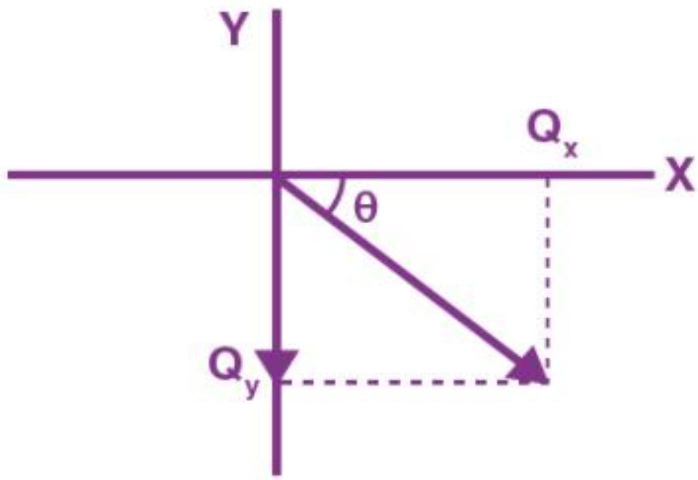
$\tan\theta = \left(\frac{P_x}{P_y}\right) \; \theta = \tan^{-1}\left(\frac{1}{1}\right) \; \theta = 45^\circ$ — (ii)

Therefore, the vector $\vec{i} - \vec{j}$ makes an angle of 45° with the x axis.

Let $\vec{Q} = \hat{i} - \hat{j} \; Q_x \hat{i} - Q_y \hat{j} = \hat{i} - \hat{j} \; Q_x = Q_y = 1 \; \left|\vec{Q}\right| = \sqrt{Q_x^2 + Q_y^2} \; \left|\vec{Q}\right| = \sqrt{2}$ —— (iii)

Therefore, the magnitude of the vector $\hat{1} - \hat{j}$ is $\sqrt{2}$.

Let θ be the angle by the vector $\vec{Q}$, with the x – axis as given in the figure below,



$\tan\theta = \left(\frac{Q_x}{Q_y}\right)$ $\theta = -\tan^{-1}\left(-\frac{1}{1}\right)$ $\theta = -45^\circ$ —— (iv)

Therefore, the vector $\vec{i}-\vec{j}$ makes an angle of -45° with the x axis.

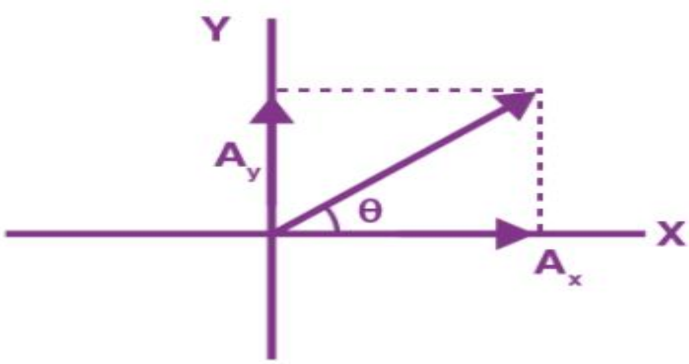
It is given that,

$\vec{A} = 2\hat{i} + 3\hat{j}$ $A_x\,\hat{i} + A_y\,\hat{j} = 2\hat{i} + 3\hat{j}$

Comparing the components on both the sides, we get:

$A_x = 2\ and\ A_y = 3$ $\left|\vec{A}\right| = \sqrt{2^2+3^2}$ $\left|\vec{A}\right| = \sqrt{13}$

Let $\vec{A_x}$ make an angle θ with the x- axis, as it is shown in the figure,



$\tan\theta = \left(\frac{A_x}{A_y}\right)$ $\theta = \tan^{-1}\left(\frac{3}{2}\right)$ $\theta = \tan^{-1}(1.5)$ $\theta = 56.31^{\circ}$

Angle between the vectors $\left(2\hat{i} + 3\hat{j}\right)$ and $\left(\hat{i} + \hat{j}\right)$,

$\theta' = 56.31 – 45 = 11.31^{\circ}$

Components of vector $\vec{A}$, along the direction of $\vec{P}$, making an angle θ.

= $(A\cos\theta')\,\hat{P}$

= $(A\cos 11.31)\,\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

= $\sqrt{13} \times \frac{0.9806}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$

= $2.5\left(\hat{i} + \hat{j}\right)$

= $\frac{25}{10} \times \sqrt{2}$

= $\frac{5}{\sqrt{2}}$ — (v)

Let θ' be the angle between the vectors $\left(2\hat{i} + 3\hat{j}\right)$ and $\left(\hat{i} – \hat{j}\right)$ $\theta'' = 45 + 56.31 = 101.31^{\circ}$

Component of vector $\vec{A}$, along the direction of $\vec{Q}$, making an angle θ.

= $(A\cos\theta'')\,\vec{Q}$

= $(A\cos\theta'')\,\frac{\hat{i}–\hat{j}}{\sqrt{2}}$

= $\sqrt{13}\cos\left(901.31^{\circ}\right)\frac{\left(\hat{i}-\hat{j}\right)}{\sqrt{2}}$

= $-\sqrt{\frac{13}{2}}\sin 11.30^{\circ}\left(\hat{i}-\hat{j}\right)$

= $-2.550\times\ 0.1961\left(\hat{i}-\hat{j}\right)$

= $-0.5\left(\hat{i}-\hat{j}\right)$

= $-\frac{5}{10}\times\ \sqrt{2}$

= $-\frac{1}{\sqrt{2}}$ — (iv)

Q-23: Which of the given relations are true for any arbitrary motion in space?

(a) $v_{average}=\ \left(\frac{1}{2}\right)\left(v\left(t_1\right)+v\left(t_2\right)\right)$

(b) $v_{average}=\ \frac{\left[r\left(t_2\right)-r\left(t_1\right)\right]}{\left(t_2-t_1\right)}$

(c) $v(t)=\ v\left(0\right)+\ at$

(d) $r(t)=\ r(0)+\ v(0)t+\ \left(\frac{1}{2}\right)at^2$

(e) $a_{average}=\ \frac{\left[v(t_2)-v(t_1)\right]}{\left(t_2-t_1\right)}$

Solution:

(a) False

It is given that the motion of the particle is arbitrary. Therefore, the average velocity of the particle cannot be given by this equation.

(b) True

The arbitrary motion of the particle can be represented by this equation.

(c) False

The motion of the particle is arbitrary. The acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of the particle in space.

(d) False

The motion of the particle is arbitrary; acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of a particle in space.

(e) True

The arbitrary motion of the particle can be represented by this equation.

Q-24: Read each statement below carefully and state, with reasons and examples, if it is true or false:

A scalar quantity is one that

(a) is conserved in a process

(b) can never take negative values

(c) must be dimensionless

(d) does not vary from one point to another in space

(e) has the same value for observers with different orientations of axes

Solution:

(a) False

Despite being a scalar quantity, energy is not conserved in inelastic collisions.

b) False

Despite being a scalar quantity, the temperature can take negative values.

c) False

The total path length is a scalar quantity. Yet it has the dimension of length.

d) False

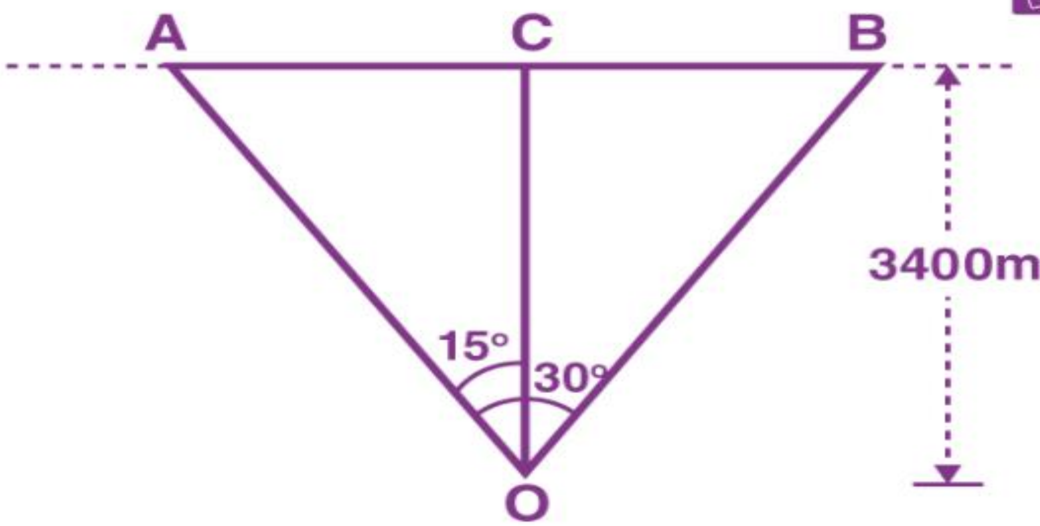
A scalar quantity such as gravitational potential can vary from one point to another in space.

e) True

The value of a scalar does not vary for observers with different orientations of axes.

Q.25. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30°, what is the speed of the aircraft?

Solution:



Height at which the aircraft is flying = 3400 m

Let A and B be the positions of the aircraft making an angle ∠AOB = 30^0. The perpendicular OC is drawn on AB. Here OC is the height of the aircraft which is equal to 3400 m and ∠AOC = ∠COB = 15^0.

In the ΔAOC, AC = OC tan 15^0 = 3400 x 0.267 = 910.86 m

AB = AC + CB = AC + AC = 2 AC = 2 x 910.86 m

Speed of the aircraft = distance AB/time = (2 x 910.86)/10= 182.17 m/s =182.2 m/s

Q-26: Does a vector have a location in space? Will it fluctuate with time? Can two equivalent vectors x and y at various locations in space fundamentally have indistinguishable physical effects? Give cases in support of your answer.

Solution:

No, Yes and No.

A vector in space has no distinct location. The reason behind this is that a vector stays invariant when it displaces in a way that its direction and magnitude does not change. Although, a position vector has a distinct location in space.

A vector change with time. For instance, the velocity vector of a ball moving with a specific speed fluctuates with time.

Two equivalent vectors situated at different locations in space do not generate the same physical effect. For instance, two equivalent forces acting at different points on a body tend to rotate the body, but the combination will not generate the equivalent turning effect.

Q-27: As a vector is having both direction and magnitude, then is it necessary that if anything is having direction and magnitude it is termed as a vector? The rotation of an object is defined by the angle of rotation about the axis and the direction of rotation of the axis. Will it be a rotation of a vector?

Solution:

No and no

A physical quantity which is having both direction and magnitude is not necessarily a vector. For instance, in spite of having direction and magnitude, the current is a scalar quantity. The basic necessity for a physical quantity to fall in a vector category is that it ought to follow the "law of vector addition."

As the rotation of a body about an axis does not follow the basic necessity to be a vector i.e, it does not follow the "law of vector addition", so it is not a vector quantity. Although in some cases rotation of a body about an axis by a small angle follows the law of vector addition so it is termed as a vector.

Q-28: Can we associate a vector with

(i) a sphere

(ii) the length of a wire bent into a loop

(iii) a plane area

Clarify for the same.

Solution:

No, No, Yes

(i) We can't associate the volume of a sphere with a vector, but the area of a sphere can be associated with an area vector.

(ii) We can't associate the length of a wire bent into a loop with a vector.

(iii) We can associate a plane area with a vector.

Q. 29. A bullet fired at an angle of 30° with the horizontal hits the ground 3.0 km away. By adjusting its angle of projection, can one hope to hit a target 5.0 km away? Assume the muzzle speed to be fixed, and neglect air resistance.

Solution:

Bullet is fired at an angle = 30°

The bullet hits the ground at a distance of 3km = 3000 m

Horizontal range, $R = u^2 \sin 2\theta / g$

$3000 = u^2 \sin 60^0/9.8$

$u^2 = (3000 \times 9.8)/(\sqrt{3}/2)$

$= 2\,(3000 \times 9.8)/\sqrt{3} = 58800/1.732 = 33949$

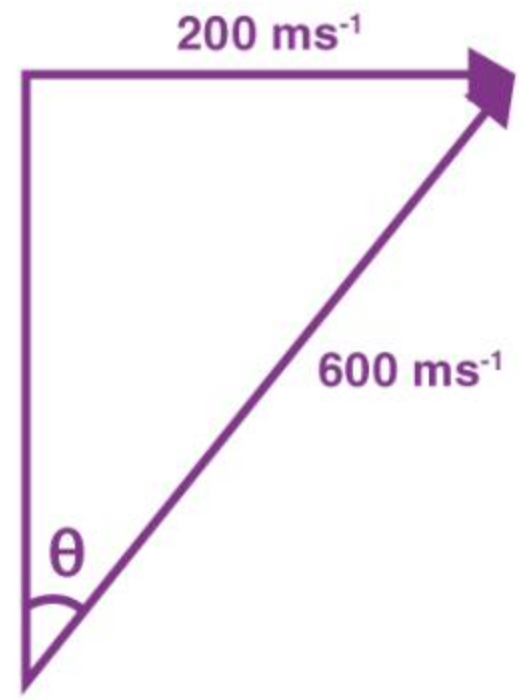
Also, R’ = $u^2 \sin 2\theta/g \Rightarrow 5000 = (33949 \times \sin 2\theta)/9.8$

Sin 2θ = (5000 x 9.8)/33949 = 49000/33949 = 1.44

Since sine of an angle cannot be more than 1. Therefore, a target 5 km away cannot be hit

Q. 30. A fighter plane flying horizontally at an altitude of 1.5 km with a speed of 720 km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 m s^{-1} to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take g = 10 m s^{-2}).

Solution:



Speed of the fighter plane = 720 km/h = 720 x (5/18) = 200 m/s

The altitude of the plane = 1.5 km

Velocity of the shell = 600 m/s

Sin θ = 200/600 = 1/3

$\theta = \sin^{-1}(1/3) = 19.47^0$

Let H be the minimum altitude

Using equation,

$H = u^2 \sin^2 (90 - \theta)/2g$

$= (600^2 \cos^2\theta)/2g$

$= 600^2 \cos^2\theta/(2 \times 9.8) = \{360000[(1+\cos 2\theta)/2]\}/2g$

= 360000[1+cos2 (19.470)/2]/2g

= 360000[(1 + cos 38.94)/2]/(2 x 9.8)

= 360000 [(1 + 0.778)/2]/19.6

= 360000 [(1.778/2)]/19.6

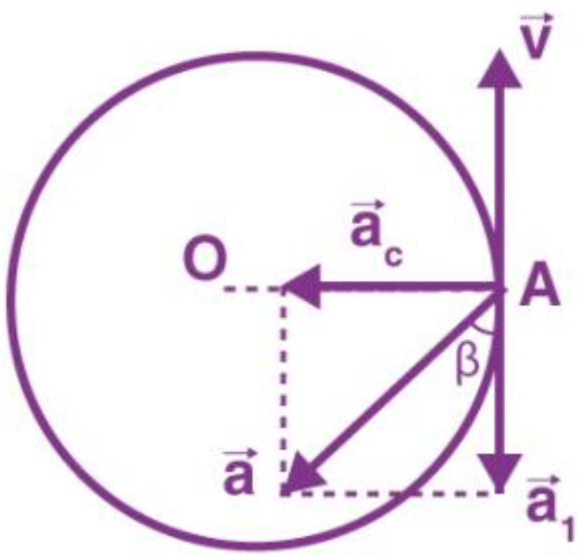
= (360000 x 0.889) /19.6

= 320040/19.6

= 16328 m = 16.328 km

Q. 31. A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of a radius of 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

Solution:



Speed of the cyclist = 27 km/h = 27 x (5/18) = 7.5 m/s

Radius of the road = 80 m

The net acceleration is due to the braking and the centripetal acceleration

Due to braking, a = 0.50 m/s^2

Centripetal acceleration, a = v^2/2 = $(7.5)^2$/80 = 0.70 m/s^2

since the angle between a_c and a_T is 90^o, the resultant acceleration is given by

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(0.5)^2 + (0.7)^2}$$

$= 0.86\ m/s^2$

and $\tan \beta = a_c/a_t = 0.7/0.5 = 1.4$

$\beta = \tan^{-1} (1.4) = 54.5^0$ from the direction of velocity

Q.32. (a) Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

$$\theta(t) = \ tan^{-1}(\frac{v_{0y-gt}}{v_{ox}})$$

(b) Shows that the projection angle θ_0 for a projectile launched from the origin is given by

$$\theta_0 = \ tan^{-1}(\frac{4h_m}{R})$$

where the symbols have their usual meaning

Solution:

(a) Let θ be the angle at which the projectile is fired w.r.t the x-axis

θ depends on t

Therefore, $\tan \theta(t) = v_x/v_y = (v_{oy} - gt)/v_{0x}$ (since $v_y = v_{0y}$ -gt and $v_x = v_{ox}$)

$\theta(t) = \tan^{-1} ((v_{oy} - gt)/v_{0x})$

(b) Since, $h_{max} = u^2 \sin^2\theta/2g$ ——(1)

$R = u^2 \sin 2\theta/g$———(2)

Dividing (1) by (2)

$(h_{max}/R) = [u^2 \sin^2\theta/2g]/[u^2 \sin 2\theta/g] = \tan \theta/4$

$\theta = \tan^{-1} (4 h_{max}/R)$