Q.1.The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:

(a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.
(b) work done by the gravitational force in the above case,
(c) work done by friction on a body sliding down an inclined plane,
(d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,
(e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest

Solution:

(a) It is clear that the direction of both the force and the displacement are the same and thus the work done on it is positive.

(b) It can be noted that the displacement of the object is in an upward direction whereas, the force due to gravity is in a downward direction. Hence, the work done is negative.

(c) It can be observed that the direction of motion of the object is opposite to the direction of the frictional force. So, the work done is negative.

(d) The object which is moving in a rough horizontal plane faces the frictional force which is opposite to the direction of the motion. To maintain a uniform velocity, a uniform force is applied on the object. So, the motion of the object and the applied force are in the same direction. Thus, the work done is positive.

(e) It is noted that the direction of the bob and the resistive force of air which is acting on it are in opposite directions. Thus, the work done is negative.

Q.2. A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with the coefficient of kinetic friction = 0.1. Compute the
(a) work done by the applied force in 10 s,
(b) work done by friction in 10 s,
(c) work done by the net force on the body in 10 s,
(d) change in kinetic energy of the body in 10 s,

Solution:

The mass of the body = 2 kg

Horizontal force applied = 7 N

Coefficient of kinetic friction = 0.1

Acceleration produced by the applied force, a_1= F/m = 7/2 = 3.5 m/s^2

Force of friction, f = μR = μmg = 0.1 x 2 x 9.8

Retardation produced by friction, a_2 = -f/m = -196/2 = -0.98

Net acceleration with which the body moves

a = a_1 + a_2 = 3.5 – 9.8 = 2.5

Distance moved by the body in 10 seconds,

s = ut + (1/2)at^2 = 0 + (1/2) x 2.52 x $(10)^2$= 126 m

(a) The time at which work has to be determined is t = 10 s

Work = Force x displacement

= 7 x 126 = 882 J

(b) Work done by the friction in 10 s

W = -f x s = -1.96 x 126

(c) Work done by the net force in 10 s

W = (F – f)s = (7 – 1.96) 126 = 635 J

(d) From v = u + at

v = 0 + 2.52 x 10 = 25.2 m/s

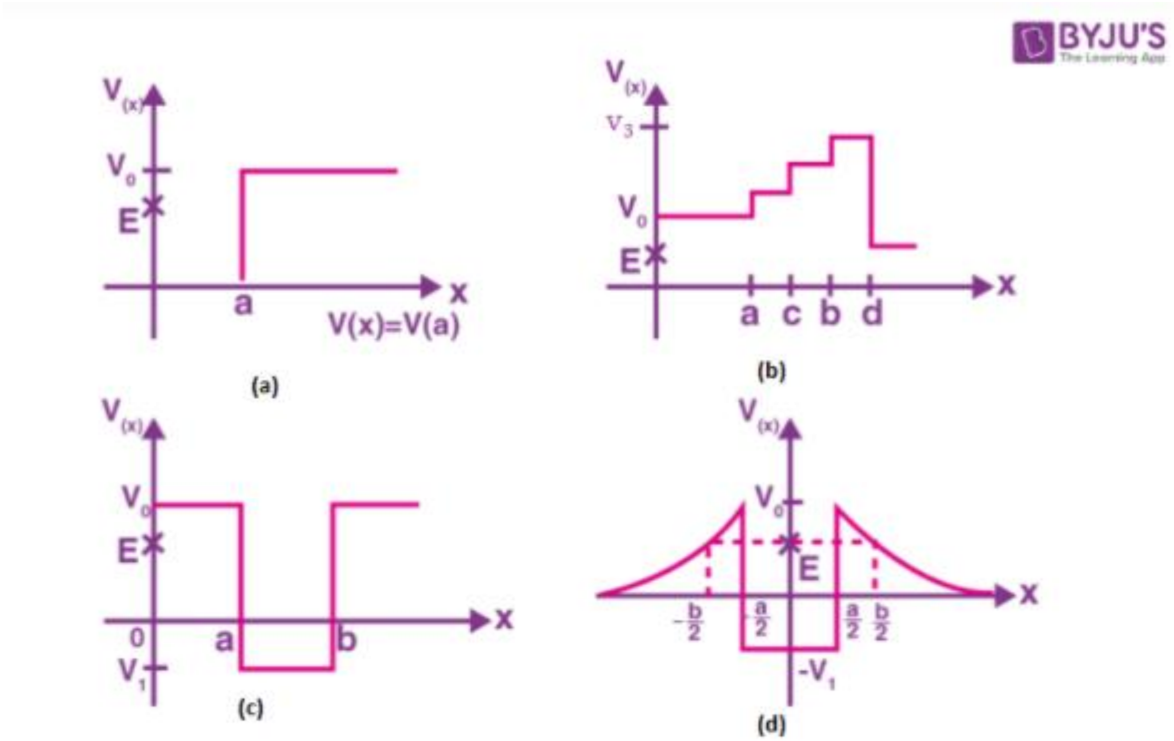
Final Kinetic Energy = (1/2) mv^2 = (1/2) x 2 x $(25.2)^2$ = 635 J

Initial Kinetic Energy= (1/2) mu^2 = 0

Change in Kinetic energy = 635 – 0 = 635 J

The work done by the net force is equal to the final kinetic energy

Q. 3. Given in Figure, are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.



Solution:

The total energy is given by E = K.E. + P.E.

K.E. = E – P.E.

Kinetic energy can never be negative. The particle cannot exist in the region, where K.E. would become negative.

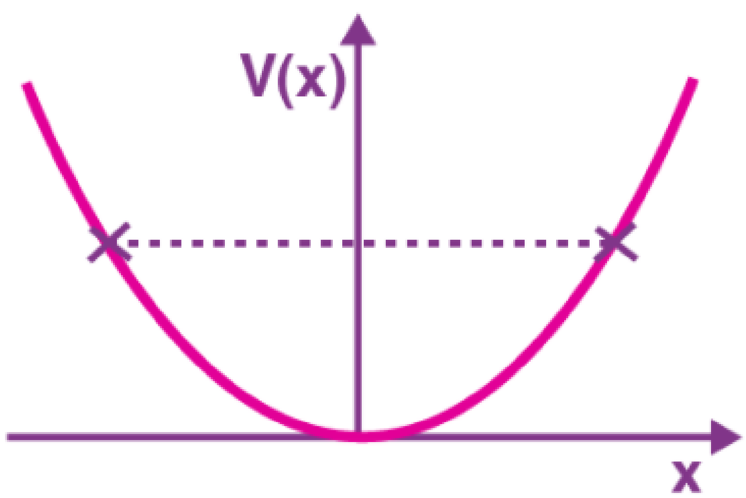
(a) For the region x = 0 and x = a, potential energy is zero. So, kinetic energy is positive. For, x > a, the potential energy has a value greater than E. So, kinetic energy becomes zero. Thus the particle will not exist in the region x > a.
The minimum total energy that the particle can have in this case is zero.

(b) For the entire x-axis, P.E. > E, the kinetic energy of the object would be negative. Thus the particle will not exist in this region.

(c) Here x = 0 to x = a and x > b, the P.E. is greater than E, so the kinetic energy is negative. The object cannot exist in this region.

(d) For x = -b/2 to x =-a/2 and x = a/2 to x = b/2. Kinetic energy is positive and the P.E. < E. The particle is present in this region.

Q.4. The potential energy function for a particle executing linear simple harmonic motion is given by V(x) = $kx^2/2$, where k is the force constant of the oscillator. For k = 0.5 N m^{-1}, the graph of V(x) versus x is shown in Fig. 6.12. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches x = ± 2 m.



Solution:

Particle energy E = 1 J

K = 0.5 N m^{-1}

K.E = $\frac{1}{2}$ mv^2

Based on law of conservation of energy:

E = V + K

1 = $\frac{1}{2}$ kx^2 + $\frac{1}{2}$ mv^2

Velocity becomes zero when it turns back

1 = $\frac{1}{2}$ kx^2

$\frac{1}{2}$ x 0.5x^2 = 1

X^2 = 4

X = ± 2

Thus, on reaching x = ± 2 m, the particle turns back.

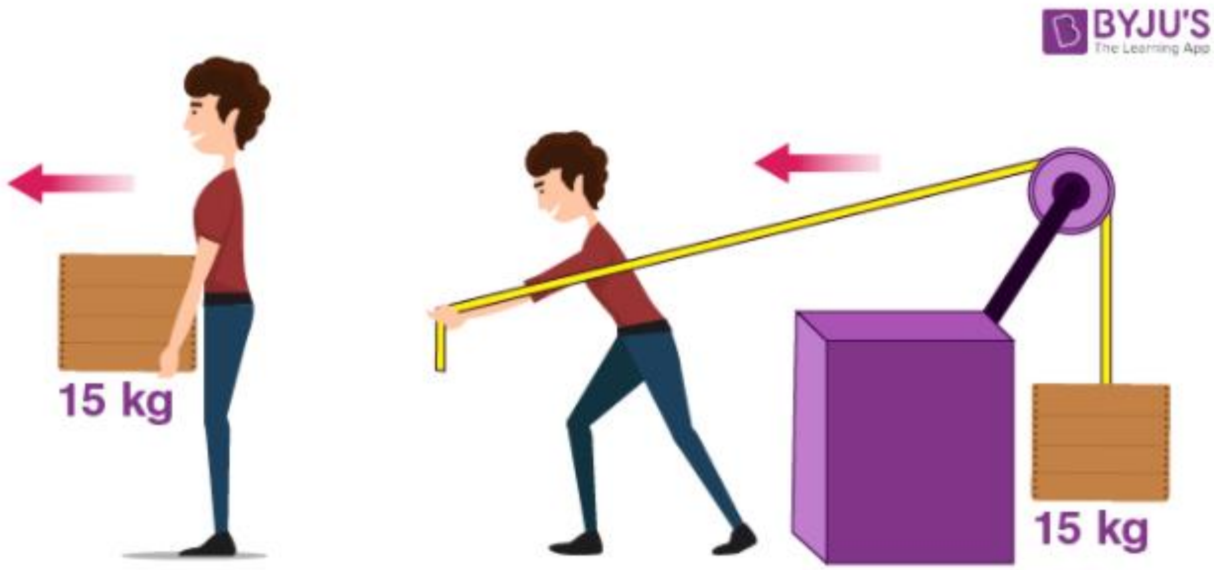
Q.5. Answer the following:

(a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?

(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why?

(c) An artificial satellite orbiting the earth in a very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth?

(d) In Fig. 6.13(i) the man walks 2 m carrying a mass of 15 kg on his hands. In Fig., he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater?



Solution:

(a) When the casing burns up due to the friction, the rocket's mass gets reduced.

As per the law of conservation of energy:

Total energy = kinetic energy + potential energy

$= mgh + \frac{1}{2} mv^2$

There will be a drop in total energy due to the reduction in the mass of the rocket. Hence, the energy which is needed for the burning of the casing is obtained from the rocket.

(b) The force due to gravity is a conservative force. The work done on a closed path by the conservative force is zero. Hence, for every complete orbit of the comet, the work done by the gravitational force is zero.

(c) The potential energy of the satellite revolving the Earth decreases as it approaches the Earth and since the system's total energy should remain constant, the kinetic energy increases. Thus, the satellite's velocity increases. In spite of this, the total energy of the system is reduced by a fraction due to the atmospheric friction.

(d)

Scenario I:

m = 20 kg

Displacement of the object, s = 4 m

$W = Fs \cos \theta$

θ = It is the angle between the force and displacement

Fs = mgs cos θ

W = mgs cos θ = 20 x 4 x 9.8 cos 90°

= 0 (cos 90° = 0)

Scenario II:

Mass = 20 kg

S = 4 m

The applied force direction is same as the direction of the displacement.

$\theta = 0^\circ$

Cos 0° = 1

W = Fs cos θ

= mgs θ

= 20 x 4 x 9.8 x cos 0°

= 784 J

Thus, the work done is more in the second scenario.

Q.6. Underline the correct alternative:

(a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.

(b) Work done by a body against friction always results in a loss of its kinetic/potential energy.

(c) The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system

(d) In an inelastic collision of two bodies, the quantities which do not change after the collision is the total kinetic energy/total linear momentum/total energy of the system of two bodies.

Solution:

(a) Decreases

When a body is displaced in the direction of the force, positive work is done on the body by the conservative force due to which the body moves to the centre of force. Thus, the separation between the two decreases and the potential energy of the body decreases.

(b) Kinetic energy

The velocity of the body is reduced when the work done is in the direction opposite to that of friction. Thus, the kinetic energy decreases.

(c) External force

Change in momentum cannot be produced by internal forces, irrespective of their directions. Thus, the change in total momentum is proportional to the external force on the system.

(d) Total linear momentum

Irrespective of elastic collision or an inelastic collision, the total linear momentum remains the same.

Q.7. State if each of the following statements is true or false. Give reasons for your answer

(a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.

(b) The total energy of a system is always conserved, no matter what internal and external forces on the body are present.

(c) Work done in the motion of a body over a closed loop is zero for every force in nature.

(d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

Solution:

(a) False

The momentum and the energy of both the bodies are conserved and not individually.

(b) False.

The external forces on the system can do work on the body and are able to change the energy of the system.

(c) False.

The work done by the conservative force on the moving body in a closed loop is zero.

(d) True

Q.8. Answer carefully, with reasons :

(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e. when they are in contact)?

(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls?

(c) What are the answers to (a) and (b) for an inelastic collision?

(d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic? (Note, we are talking here of potential energy corresponding to the force during a collision, not gravitational potential energy).

Solution:

(a) The initial and the final kinetic energy is equal in an elastic collision. When the two balls collide, there is no conservation of kinetic energy. It gets converted into potential energy.

(b) The total linear momentum of the system is conserved in an elastic collision.

(c) There will be a loss of kinetic energy in an inelastic collision. The K.E after the collision is always less than the K.E before the collision.

The total linear momentum of the system is conserved in an inelastic collision also.

(d) It is an elastic collision as the forces involved are conservative forces. It depends on the distance between the centres of the billiard balls.

Q.9. A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to

(i) $t^{\frac{1}{2}}$

(ii) $t^{\frac{3}{2}}$

(iii) t^2

(iv) t

Solution:

body mass = m

Acceleration = a

According to Newton's second law of motion:

F = ma (constant)

We know that a = $\frac{dv}{dt}$ = constant

dv = dt x constant

On integrating

v = α t $\rightarrow$ 1

Where, α is also a constant

v $\propto$ t $\rightarrow$ 2

The relation of power is given by:

P = F.v

From equation 1 & 2

P $\propto$ t

Thus, from the above, we conclude that power is proportional to time.

Q.10. A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to

(i) $t^{\frac{1}{2}}$

(ii) $t^{\frac{3}{2}}$

(iii) t^2

(iv) t

Solution:

We know that the power is given by:

P = Fv

= mav = mv $\frac{dv}{dt}$

= k (constant)

vdv = $\frac{k}{m}$ dt

On integration:

$\frac{v^2}{2}$ = $\frac{k}{m}dt$

v = $\sqrt{\frac{2kt}{m}}$

To get the displacement:

v = $\frac{dx}{dt}$ = $\sqrt{\frac{2k}{m}}$ $t^{\frac{1}{2}}$

dx = k' $t^{\frac{1}{2}}$ dt

where k' = $\sqrt{\frac{2k}{3}}$

x = $\frac{2}{3}$ k' $t^{\frac{2}{3}}$

Hence, from the above equation it is shown that x $\propto$ $t^{\frac{3}{2}}$

Q. 11. A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by

$F = -\hat{i} + 2\hat{j} + 3\hat{k}$ N

where i, j, k, are unit vectors along the x- y- and z-axis of the system respectively. What is the work done by this force in moving the body at a distance of 4 m along the z-axis?

Solution:

The body is displaced by 4 m along z-axis

$\vec{S} = 0\hat{i} + 0\hat{j} + 4\hat{k}$

$\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$

Work done,

W = $\vec{F}.\vec{S} = (0\hat{i} + 0\hat{j} + 4\hat{k})(-\hat{i} + 2\hat{j} + 3\hat{k})$

= $12(\hat{k}.\hat{k})$ Joule = 12 Joule

Q.12. An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds. (electron mass = 9.11×10^{-31} kg, proton mass = 1.67×10^{-27} kg, 1 eV = 1.60×10^{-19} J)

Solution:

Electron mass, $m_e = 9.11 \times 10^{-31}$ kg

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg

Electron's kinetic energy

$E_{ke} = 20$ keV = 20×10^3 eV

= $20 \times 10^3 \times 1.60 \times 10^{-19}$

= 3.2×10^{-15} J

Proton's kinetic energy,

$E_{kp} = 200$ kev = 2×10^5 eV

= 3.2×10^{-14} J

To find the velocity of electron v_e, kinetic energy is used.

$E_{ke} = \frac{1}{2} m v_e^2$

$V_e = \sqrt{\frac{2 \times E_{ke}}{m}}$

$= \sqrt{\frac{2 \times 3.2 \times 10^{-15}}{9.11 \times 10^{-31}}}$

$= 8.38 \times 10^7$ m/s

To find the velocity of proton v_p, the kinetic energy is used.

$E_{kp} = \frac{1}{2} m v_p^2$

$v_p = \sqrt{\frac{2 \times E_{kp}}{m}}$

$v_p = \sqrt{\frac{2 \times 3.2 \times 10^{-15}}{1.67 \times 10^{-27}}}$

$= 6.19 \times 10^6$ m/s

Thus, electron moves faster when compared with proton.

The speed ratios are:

$\frac{v_e}{v_p} = \frac{8.38 \times 10^7}{6.19 \times 10^6} = \frac{13.53}{1}$

Q. 13. A raindrop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 ms^{-1}?

Solution:

Radius of the drop = 2 mm = 2×10^{-3} m.
Height from which the raindrops fall, S=500 m.

The density of water, $\rho = 10^3$ kg/ m^3
Mass of rain drop = volume of drop x density
$m = (4/3)\pi r^3 \times \rho = (4/3) \times (22/7) \times (2 \times 10^{-3})^3 \times 10^3 = 3.35 \times 10^{-5}$ kg

The gravitational force experienced by the rain drop

F = mg

$= (4/3) \times (22/7) \times (2 \times 10^{-3})^3 \times 10^3 \times 9.8$ N

The work done by gravity on the drop is

$W = mg \times s = 3.35 \times 10^{-5} \times 9.8 \times 250 = 0.082$ J

The work done on the drop in the second half of the journey will remain the same.

The total energy of the raindrop will be conserved during the motion

Total energy at the top
$E_1 = mgh = 3.35 \times 10^{-5} \times 9.8 \times 500 = 0.164$ J

Due to resistive forces, energy of drop on reaching the ground.
$E_2 = 1/2mv^2 = 1/2 \times (10)^2 = 1.675 \times 10^{-3}$J

Work done by the resistive forces, $W = E_1 - E_2 = 0.164 - 1.675 \times 10^{-3}$ W
= 0.1623 joule.

Q.14. A molecule in a gas container hits a horizontal wall with speed 200 m s^{-1} and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?

Solution:

Momentum is always conserved for a elastic or inelastic collision.

The molecule approaches and rebounds with the same speed of 200 m/s.

$u = v = 200$ m s^{-1}

Therefore, Initial kinetic energy = $(1/2)\ mu^2 = (1/2)m(200)^2$

Final kinetic energy = $(1/2)\ mv^2 = (1/2)m(200)^2$

Therefore, kinetic energy is also conserved

Q.15. A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

Solution:

Volume of the tank= 30 m^3

time taken to fill the tank= 15 min = 15 x 60 = 900s
height of the tank above the ground, h = 40 m

Efficiency of the pump, η= 30%
Density of water, $\rho = 10^3$ kg m^{-3}
Mass of water pumped, m = volume x density = 30 x 10^3 kg
Power consumed or output power p_{0utput} = W/t = mgh/t

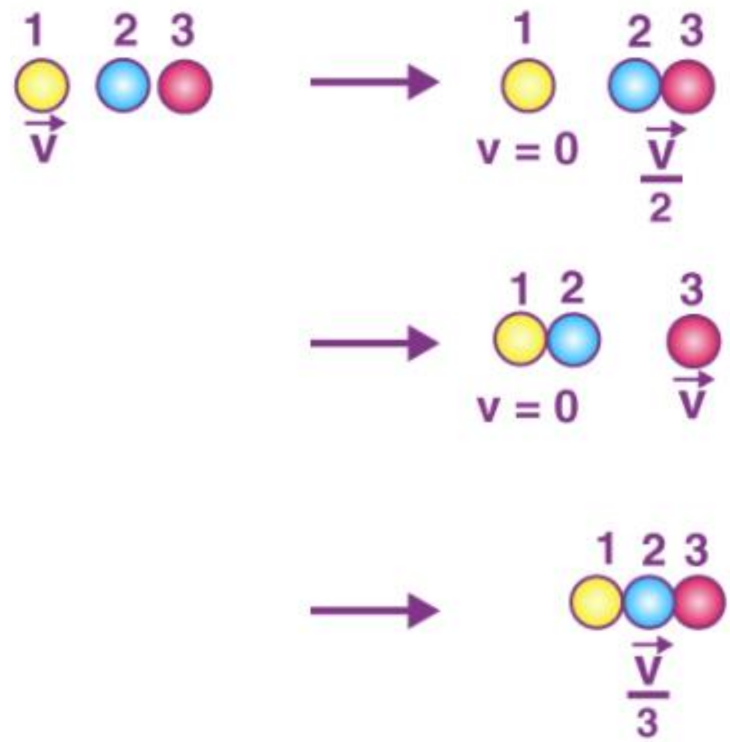
=(30 x 10^3 x 9.8 x 40)/900=13066 watt

Efficiency, $\eta = P_{output}/P_{input}$

P_{input} = $P_{output/\eta}$ = 13066/(30/100) = 1306600/30

= 43553 W = 43.6 kW

Q. 16. Two identical ball bearings in contact with each other and resting on a frictionless table is hit head-on by another ball bearing of the same mass moving initially with a speed V. If the collision is elastic, which of the following figure is a possible result after collision?



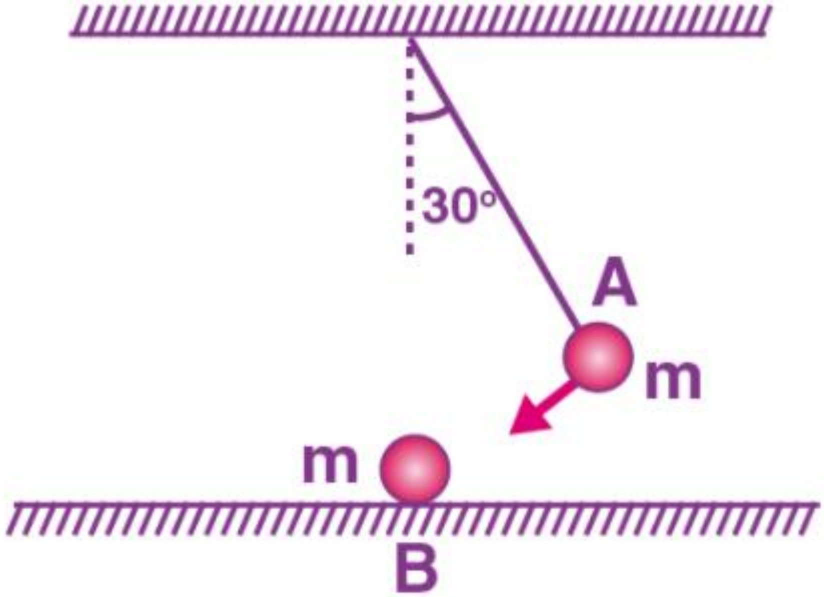
Solution:

The mass of the ball bearing is m

Before the collision, Total K.E. of the system
$=1/2mv^2 + 0 = 1/2\ mv^2$
After the collision, Total K.E. of the system is
Case I, $E_1 = (1/2)\ (2m)\ (v/2)^2 = 1/4\ mv^2$
Case II, $E_2 = (1/2)\ mv^2$

Case III, $E_3 = (1/2)(3m)(v/3)^2 = 3mv^2/18 = 1/6mv^2$
Thus, case II is the only possibility since K.E. is conserved in this case.

Q.17. A ball A which is at an angle 30^0 to the vertical is released and it hits a ball B of same mass which is at rest. Does the ball A rises after collision? The collision is an elastic collision.



Solution:

In an elastic collision when the ball A hits the ball B which is stationary, the ball B acquires the velocity of the ball A while the ball A comes to rest immediately after the collision. There is a transfer of momentum to the moving body from the stationary body. Thus, the ball A comes to rest after collision and ball B moves with the velocity of ball A.

Q.18. The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance?

Solution:

Length of the pendulum, l= 1.5 m

Potential of the bob at the horizontal position = mgh = mgl

The initial energy dissipated against air resistance when the bob moves from the horizontal position to the lowermost point= 5%

The total kinetic energy of the bob at the lowermost position = 95% of the total potential energy at the horizontal position

$(1/2)\ mv^2 = (95/100)\ mgl$

v^2 =2 [(95/100) x 9.8 x 1.5]

v^2 =2 (13.965) = 27.93

v = √27.93 = 5.28 m/s

Q.19. A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, the sand starts leaking out of a hole on the floor of the trolley at the rate of 0.05 kg s^{-1}. What is the speed of the trolley after the entire sandbag is empty?

Solution:

The sandbag is placed in the trolley that moves with a uniform velocity of 27 km/h. There is no external force acting system. Even if the sand starts leaking out of the bag there will not be any external force acting on the system. So, the speed of the trolley will not change. It will be equal to 27 km/h.

Q.20. A body of mass 0.5 kg travels in a straight line with velocity v =ax $^{3/2}$ where a = 5 $m^{-1/2}$ s^{-1}. What is the work done by the net force during its displacement from x = 0 to x = 2 m?

Solution:

Mass of the body, m = 0.5 kg
Velocity of the body, v=a$x^{3/2}$

here, a = 5 $m^{-1/2}$ s^{-1}.
Initial velocity at x = 0, v_1 = a x 0 = 0
Final velocity at x = 2, v_2 = a $(2)^{3/2}$ = 5 x $(2)^{3/2}$
Work done by the system= increase in K.E of the body
= 1/2 m(v_2^2-v_1^2) = 1/2 x 0.5 [(5 x $(2)^{3/2})^2$ – 0]

= (1/2) x 0.5 x (25 x 8) = 50 J

Q.21. The windmill sweeps a circle of area A with their blades. If the velocity of the wind is perpendicular to the circle, find the air passing through it in time t and also the kinetic energy of the air. 25 % of the wind energy is converted into electrical energy and v = 36 km/h, A = 30 m^2 and the density of the air is 1.2 kg m^{-3}. What is the electrical power produced?

Solution:

Area = A

Velocity = V

Density = ρ

(a) Volume of the wind through the windmill per sec = Av

Mass = ρ Av

Mass m through the windmill in time t = ρ Avt

(b) kinetic energy = $\frac{1}{2}$ mv^2

= $\frac{1}{2}$ (ρ Avt)v^2 = $\frac{1}{2}$ ρ Av^{3}t

(c) Area = 30 m^2

Velocity = 36 km/h

Density of air ρ = 1.2 kg m^{-3}

Electric energy = 25 % of wind energy

= $\frac{25}{100}$ x kinetic energy

= $\frac{1}{8}$ ρ Av^{3}t

Power = $\frac{Electric\ energy}{Time}$

= $\frac{1}{8}$ $\frac{\rho Av^3 t}{t}$ = $\frac{1}{8}$ ρ Av^3

= $\frac{1}{8}$ x 1.2 x 30 x 10^3

= 4.5 x 10^3 W =4.5 kW

Q. 22. A person trying to lose weight (dieter) lifts a 10 kg mass, one thousand times, to a height of 0.5 m each time. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force? (b) Fat supplies 3.8 × 10^{7}J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up?

Solution:

Mass, m = 10 kg

Height to which the mass is lifted, h = 0.5 m

Number of times, n = 1000
(a) Work done against gravitational force.
W = n(mgh) = 1000 x (10 x 9.8 x 0.5) = 49000J.

(b) Mechanical energy supplied by 1 kg of fat = $3.8 \times 10^7 \times 20/100 = 0.76 \times 10^7$ J/kg

Therefore, fat used up by the dieter $=\{1/(0.76 \times 10^7)\} \times 49000 = 49000/(0.76 \times 10^7) = 6.45 \times 10^{-3}$ kg

**Q.23. A family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW?
(b) Compare this area to that of the roof of a typical house.**

Solution:

(a) Power used by family, p = 8 KW = 8000 W

Solar energy received per square metre = 200 W/m^2

Percentage of energy converted to useful electrical energy = 20%

As solar energy is incident at a rate of 200 Wm^{-2}

The area required to generate the desired energy is A

Useful electrical energy produced per second

= (20/100) A x 200 = 8000

A= 4000 W/200 Wm^{-2} =200 m^2

(b) The area needed is comparable to the roof of a large house of dimension 14m × 14m

Q.24. A bullet of mass 0.012 kg and horizontal speed 70 m s^{-1} strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

Solution:

Mass of the bullet, m_1= 0.012 kg

Initial speed of the bullet, u_1 =70 m/s
Mass of the wooden block, m_2= 0.4 kg
Initial speed of the wooden block, u_2=0
Final speed of the system of the bullet and the block = v m/s
Applying the law of conservation of momentum:
$m_1u_1+m_2u_2 =(m_1+m_2)v$
(0.012×70)+(0.4×0)=(0.012+0.4)v
v=0.84/0.412
=2.04 m/s

Let h be the height to which the block rises

Applying the law of conservation of energy to this system:
Potential energy of the combination = Kinetic energy of the combination

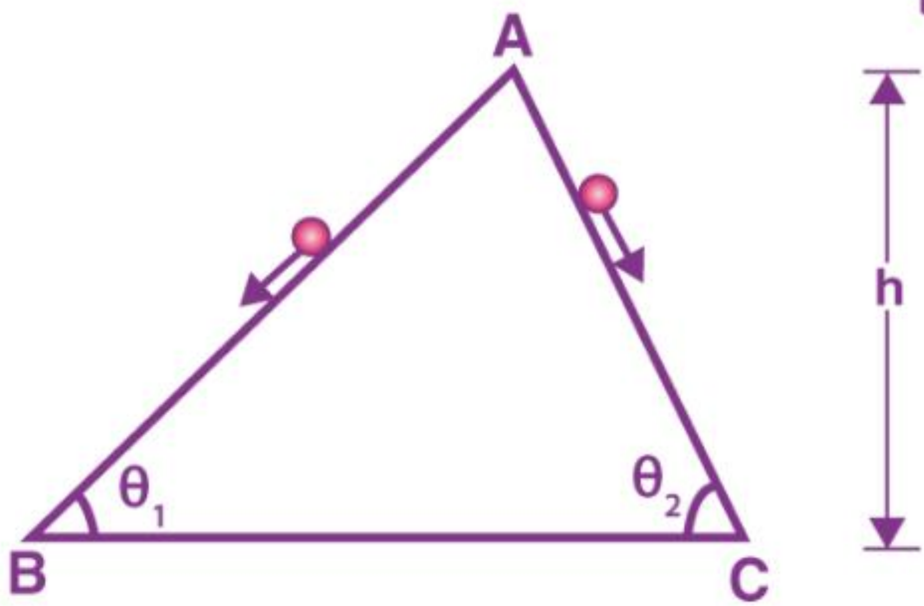
$(m_1 + m_2)\, gh = (1/2)\, (m_1 + m_2)\, v^2$

$h = v^2/2g$
$= (2.04)^2/2\times9.8$

$=0.212\text{m}$
The wooden block will rise to a height of 0.212m

The heat produced = Initial kinetic energy of the bullet – final kinetic energy of the combination
$=(1/2)m_1u_1^2 - (1/2)(m_1 + m_2)v^2$
$=(1/2)\times0.012\times(70)^2 - (1/2)\times(0.012 + 0.4)\times(2.04)^2$
$=29.4-0.857=28.54\text{J}$

Q. 25. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track Fig. Will the stones reach the bottom at the same time? Will they reach there with the same speed? Explain. Given $\theta_1 = 30^0$, $\theta_2 = 60^0$, and h = 10 m, what are the speeds and times taken by the two stones?



Solution:

In the figure, the sides AB and AC are inclined to the horizontal at $\angle\theta_1$ and $\angle\theta_2$ respectively.
According to law of conservation of mechanical energy,
PE at the top =KE at the bottom
$\therefore mgh=(1/2)mv_1^2$——(1)

and $mgh= (1/2)mv_2^2$——(2)
Since the height of both the sides is the same, therefore, both the stones will reach the bottom at the

same speed.
From (1) and (2), we get $v_1 = v_2$
Hence both the stones will reach the bottom with the same speed.

For the stone 1

Net force acting on the stone is given by

$F = ma_1 = mg \sin \theta_1$

$a_1 = g \sin \theta_1$

For stone 2

$a_2 = g \sin\theta_2$

As $\theta_2 > \theta_1$

Therefore, $a_2 > a_1$

From $v = u + at = 0 + at$

$\Rightarrow t = v/a$

For stone 1, $t_1 = v/a_1$
For stone 2, $t_2 = v/a_2$
As $t \propto 1/a$, and $a_2 > a_1$

Therefore, $t_2 < t_1$

Hence, stone 2 will reach faster than stone 1.

By applying the law of conservation of energy we get

$mgh = (1/2)\, mv^2$

When the height, h = 10 m, the speed of the stones are

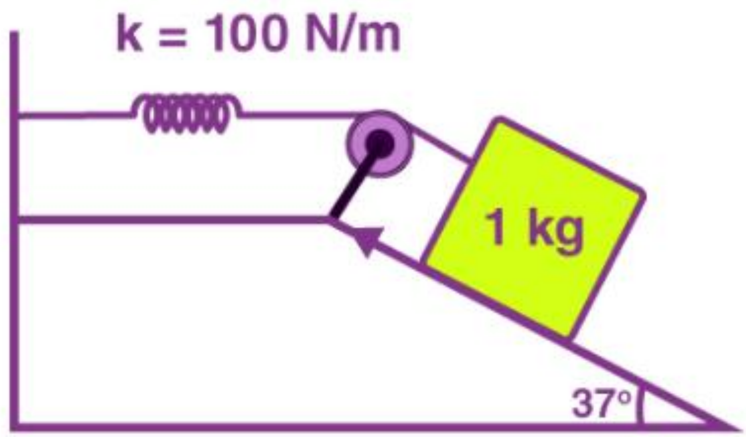
$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 10} = 14$ m/s

The time taken is given as

$t_1 = v/a_1 = v/g \sin \theta_1 = 14/(9.8 \times \sin 30) = 14/(9.8 \times 1/2) = 2.86$ s

$t_2 = v/a_2 = v/g \sin \theta_2 = 14/((9.8 \times \sin 30) = 14/(9.8 \times \sqrt{3}/2) = 1.65$ s

Q. 26. A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 N m^{-1} as shown in Fig. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has a negligible mass and the pulley is frictionless.



Solution:

Mass of the block, m = 1 kg
Spring constant, k = 100 N m^{-1}
Displacement in the block, x = 10 cm = 0.1 m

At equilibrium:
Normal reaction, R = mg cos 37°
Frictional force, F= μ R = mg Sin 37^0

μ is the coefficient of friction
Net force acting on the block down the incline= mg sin 37° – F
= mgsin 37° – μmgcos 37°

= mg(sin 37° – μcos 37°)
At equilibrium

Work done = Potential energy of the stretched string

mg(sin 37° – μcos 37°) x = (1/2)kx^2

1 × 10 x (sin 37^0 – μcos 37°) = (1/2) × 100 × (0.1)

10 (0.602 – μ (0.798)) = (1/2) x 100 x 0.1

0.602 – μ(0.798)= 0.5
Therefore, μ = (0.602- 0.5)/0.798 = 0.102 /0.798 = 0.127

μ = 0.127

Q. 27. A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 ms^{-1}. It hits the floor of the elevator (length of elevator = 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different if the elevator were stationary?

Solution:

Mass of the bolt, m = 0.3 kg

Potential energy of the bolt = mgh = 0.3 x 9.8 x 3 = 8.82 J

The bolt does not rebound. So the whole of the potential energy gets converted to heat energy. The heat produced will remain the same even if the lift is stationary, since the value of acceleration due to gravity is the same in all inertial system.

Q.28. On a frictionless track, a trolley moves with a speed of 36 km/h with a mass of 200 Kg. A child whose mass is 20 kg runs on the trolley with a speed of 4 m s^1 from one end to other which is 20 m. The speed is relative to the trolley in the direction opposite to its motion. Find the final speed of the trolley and the distance the trolley moved from the time the child began to run.

Solution:

Mass m = 200 Kg

Speed v = 36 km/h = 10 m/s

Mass of boy = 20 Kg

Initial momentum = (M + m)v

= (200 +20) x 10

= 2200 kg m/s

v' is final velocity of the trolley

Final velocity of boy = M v' +m(v' – 4)

= 200 v' + 20 v' – 80

= 220 v' – 80

According to law of conservation of energy:

Initial momentum = final momentum

2200 = 220 v' – 80

v' = $\frac{2280}{220}$ = 10.36 m/s

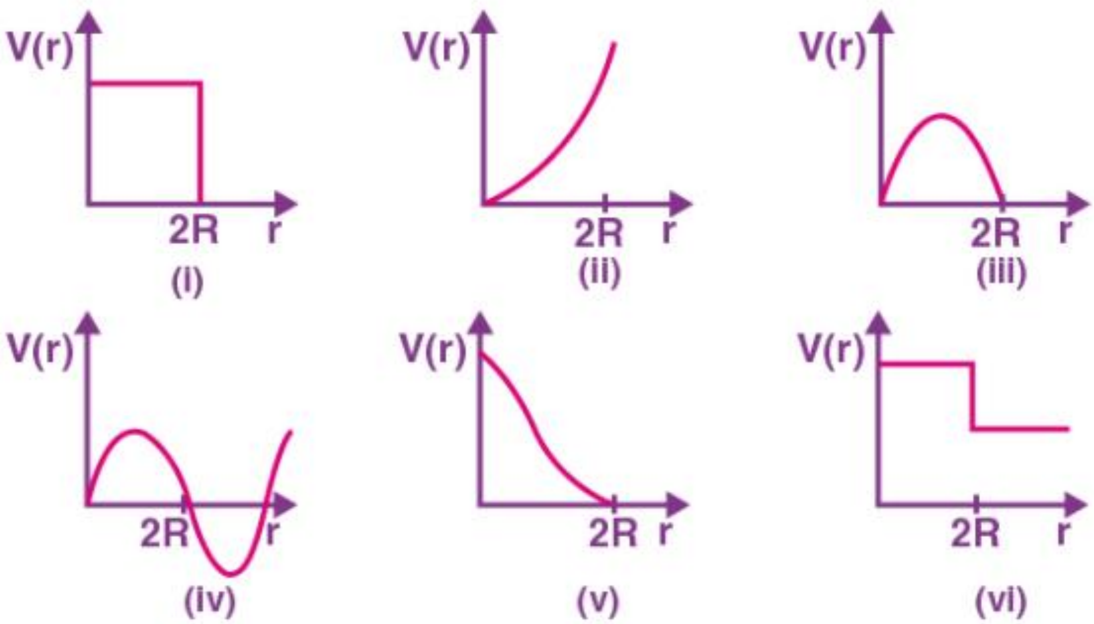
Length l = 20 m/s

v" = 4 m/s

t = $\frac{20}{4}$ = 5 s

Distance moved by the trolley = v" x t = 10.36 x 5 = 51.8 m

Que.29. Which of the following does not describe the elastic collision of two billiard balls? Distance between the centres of the balls is r.



Solution:

(i), (ii), (iii), (iv) and (vi).

The potential energy of two masses in a system is inversely proportional to the distance between them. The potential energy of the system of two balls will decrease as they get closer to each other. When the balls touch each other, the potential energy becomes zero, i.e. at r = 2R. The potential energy curve in (i), (ii), (iii), (iv) and (vi) do not satisfy these conditions. So, there is no elastic collision.