EXERCISE 1(A) PAGE: 4

1. Is zero a rational number? Can it be written in the form p/q, where p and q are integers and q ≠ 0?
Solution:

Yes, zero is a rational number.
It can be written in the form of p/q, where p and q are integers and q ≠ 0 ⇒ 0 = 0/1.

2. Are the following statements true or false? Give reasons for your answers.
(i) Every whole number is a natural number.
(ii) Every whole number is a rational number.
(iii) Every integer is a rational number.
(iv) Every rational number is a whole number.
Solution:

(i) False
Natural numbers- Numbers starting from 1 to infinity (without fractions or decimals)
i.e., Natural numbers = 1, 2, 3, 4 …
Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)
i.e., Whole numbers = 0, 1, 2,3, …
Or, we can say that whole numbers have all the elements of natural numbers and zero.
∴ Every natural number is a whole number; however, every whole number is not a natural number.

(ii) True
Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)
i.e., Whole numbers = 0, 1, 2, 3…
Rational numbers- All numbers in the form p/q, where p and q are integers and q ≠ 0.
i.e., Rational numbers = 0, 19/30, 2, 9/-3, -12/7 …
∴ Every whole number is a rational number; however, every rational number is not a whole number.

(iii) True
Integers- Integers are set of numbers that contain positive, negative and 0; excluding fractional and decimal numbers.
i.e., integers = {…-4,-3,-2,-1,0,1,2,3,4…}
Rational numbers- All numbers in the form p/q, where p and q are integers and q ≠ 0.
i.e., Rational numbers = 0, 19/30, 2, 9/-3, -12/7 …
∴ Every integer is a rational number; however, every rational number is not an integer.

(iv) False
Rational numbers- All numbers in the form p/q, where p and q are integers and q ≠ 0.
i.e., Rational numbers = 0, 19/30, 2, 9/-3, -12/7 …
Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)
i.e., Whole numbers = 0, 1, 2, 3, ...
Hence, we can say that integers include whole numbers as well as negative numbers.
∴ Every whole numbers are rational, however, every rational numbers are not whole numbers.

3. Arrange -5/9, 7/12, -2/3 and 11/18 in the ascending order of their magnitudes. Also, find the difference between the largest and the smallest of these rational numbers. Express this difference as a decimal fraction

correct to one decimal place.
Solution:

The given numbers are: -5/9, 7/12, -2/3 and 11/18
Now, the L.C.M of 9, 12 and 18 is 36
So, the given numbers are:
-5/9, 7/12, -2/3 and 11/18
= -5×4/9×4, 7×3/12×3, -2×12/3×12 and 11×2/18×2
= -20/36, 21/36, -24/36 and 22/36
Numbers in ascending order are:
-24/36, -20/36, 21/36 and 22/36
Hence, given numbers in ascending order are
-2/3, -5/9, 7/12 and 11/18
Now, to find the difference between the largest and smallest of the above number
Difference = 11/18 – (-2/3)
= 11/18 + 2/3
= 11/18 + (2×6)/(3×6)
= 11/18 + 12/18
= (11 + 12)/18
= 23/18
Now, to express this fraction as a decimal by correcting to one decimal place
Hence, 23/18 = 1.27777777… ≈ 1.3

4. Arrange 5/8, -3/16, -1/4 and 17/32 in the descending order of their magnitudes. Also, find the sum of the lowest and the largest of these rational numbers. Express the result obtained as a decimal fraction correct to two decimal places.
Solution:

Given numbers are: 5/8, -3/16, -1/4 and 17/32
The L.C.M of 8, 16, 4 and 32 is 32
So, the given numbers are:
5/8, -3/16, -1/4 and 17/32
= 5×4/8×4, -3×2/16×2, -1×8/4×8 and 17×1/32×1
= 20/32, -6/32, -8/32 and 17/32
Numbers in descending order are:
20/32, 17/32, -6/32, -8/32
Hence, given numbers in descending order are
5/8, 17/32, -3/16 and -1/4
Now, to find the sum of the largest and the smallest of the above numbers
Sum = 5/8 + (-1/4)
= 5/8 – 1/4
= 5/8 – (1×2)/(4×2)
= 5/8 – 2/8
= (5 - 2)/8
= 3/8
Now, to express this fraction as a decimal by correcting to two decimal place
Hence, 3/8 = 0.375 ≈ 0.38

5. Without doing any actual division, find which of the following rational numbers have terminating decimal representation:
(i) 7/16
(ii) 23/125
(iii) 9/14
(iv) 32/45
(v) 43/50
(vi) 17/40
(vii) 61/75
(viii) 123/250
Solution:

(i) Given number is 7/16
$16 = 2 \times 2 \times 2 \times 2 = 2^4 = 2^4 \times 5^0$
So, 16 can be expressed as $2^m \times 5^n$
Hence, 7/16 is convertible into the terminating decimal

(ii) Given number is 23/125
$125 = 5 \times 5 \times 5 = 5^3 = 2^0 \times 5^3$
So, 125 can be expressed as $2^m \times 5^n$
Hence, 23/125 is convertible into the terminating decimal

(iii) Given number is 9/14
$14 = 2 \times 7 = 2^1 \times 7^1$
So, 14 cannot be expressed as $2^m \times 5^n$
Hence, 9/14 is not convertible into the terminating decimal

(iv) Given number is 32/45
$45 = 3 \times 3 \times 5 = 3^2 \times 5^1$
So, 45 cannot be expressed as $2^m \times 5^n$
Hence, 32/45 is not convertible into the terminating decimal

(v) Given number is 43/50
$50 = 2 \times 5 \times 5 = 2^1 \times 5^2$
So, 50 can be expressed as $2^m \times 5^n$
Hence, 43/50 is convertible into the terminating decimal

(vi) Given number is 17/40
$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5^1$
So, 40 can be expressed as $2^m \times 5^n$
Hence, 17/40 is convertible into the terminating decimal

(vii) Given number is 61/75
$75 = 3 \times 5 \times 5 = 3^1 \times 5^2$
So, 75 cannot be expressed as $2^m \times 5^n$
Hence, 61/75 is not convertible into the terminating decimal

(viii) Given number is 123/250

$250 = 2 \times 5 \times 5 \times 5 = 2^1 \times 5^3$
So, 250 can be expressed as $2^m \times 5^n$
Hence, 123/250 is convertible into the terminating decimal

EXERCISE 1(B) PAGE: 13

1. State whether the following numbers are rational or not:

(i) $(2 + \sqrt{2})^2$

(ii) $(3 - \sqrt{3})^2$

(iii) $(5 + \sqrt{5})(5 - \sqrt{5})$

(iv) $(\sqrt{3} - \sqrt{2})^2$

(v) $(3/2\sqrt{2})^2$

(vi) $(\sqrt{7}/6\sqrt{2})^2$

Solution:

(i) $(2 + \sqrt{2})^2 = 2^2 + 2(2)(\sqrt{2}) + (\sqrt{2})^2$
$= 4 + 4\sqrt{2} + 2$
$= 6 + 4\sqrt{2}$

Therefore, it is irrational

(ii) $(3 - \sqrt{3})^2 = (3)^2 – 2(3)(\sqrt{3}) + (\sqrt{3})^2$
$= 9 - 6\sqrt{3} + 3$
$= 12 - 6\sqrt{3}$
$= 6(2 - \sqrt{3})$

Therefore, it is irrational.

(iii) $(5 + \sqrt{5})(5 - \sqrt{5}) = (5)^2 – (\sqrt{5})^2$
$= 25 – 5$
$= 20$

Therefore, it is rational.

(iv) $(\sqrt{3} - \sqrt{2})^2 = (\sqrt{3})^2 – 2(\sqrt{3})(\sqrt{2}) + (\sqrt{2})^2$
$= 3 - 2\sqrt{6} + 2$
$= 5 - 2\sqrt{6}$

Therefore, it is irrational.

(v) $(3/2\sqrt{2})^2 = 3^2/(2\sqrt{2})^2$
$= 9/(4 \times 2)$
$= 9/8$

Therefore, it is rational.

(vi) $(\sqrt{7}/6\sqrt{2})^2 = (\sqrt{7})^2/(6\sqrt{2})^2$
$= 7/(36 \times 2)$
$= 7/72$

Therefore, it is rational.

2. Find the square of:

(i) $3\sqrt{2}/5$

(ii) $\sqrt{3} + \sqrt{2}$

(iii) $\sqrt{5} – 2$

(iv) $3 + 2\sqrt{5}$

Solution:

(i) $(3\sqrt{2}/5)^2 = (3\sqrt{2})^2/5^2$
$= (9 \times 2)/25$
$= 18/25$
On further implication, we get
$= 1\frac{4}{5}$

(ii) $(\sqrt{3} + \sqrt{2})^2 = (\sqrt{3})^2 + (\sqrt{2})^2 + 2(\sqrt{3})(\sqrt{2})$
$= 3 + 2 + 2\sqrt{6}$
$= 5 + 2\sqrt{6}$

(iii) $(\sqrt{5} - 2)^2 = (\sqrt{5})^2 + (2)^2 - 2(\sqrt{5})(2)$
$= 5 + 4 - 4\sqrt{5}$
$= 9 - 4\sqrt{5}$

(iv) $(3 + 2\sqrt{5})^2 = 3^2 + 2(3)(2\sqrt{5}) + (2\sqrt{5})^2$
$= 9 + 12\sqrt{5} + (4x5)$
$= 9 + 20 + 12\sqrt{5}$
$= 29 + 12\sqrt{5}$

3. State, in each case, whether true or false:
(i) $\sqrt{2} + \sqrt{3} = \sqrt{5}$
(ii) $2\sqrt{4} + 2 = 6$
(iii) $3\sqrt{7} - 2\sqrt{7} = \sqrt{7}$
(iv) $2/7$ is an irrational number.
(v) $5/11$ is a rational number.
(vi) All rational numbers are real numbers.
(vii) All real numbers are rational numbers.
(viii) Some real numbers are rational numbers.
Solution:

(i) False
(ii) True
(iii) True
(iv) False
(v) True
(vi) True
(vii) False
(viii) True

4. Given universal set is {-6, -5¾, -√4, -3/5, -3/8, 0, 4/5, 1, 1⅔, √8, 3.01, π, 8.47}
From the given set, find:
(i) Set of Rational numbers
(ii) Set of irrational numbers
(iii) Set of integers
(iv) Set of non-negative integers
Solution:

(i) First find the set of rational numbers
Rational numbers are numbers of the form p/q, where q ≠ 0
U = {-6, -5¾, -√4, -3/5, -3/8, 0, 4/5, 1, 1⅔, √8, 3.01, π, 8.47}
Here, -5¾, -3/5, -3/8, 4/5 and 1⅔ are of the from p/q
Therefore, they are rational numbers
The set of integers is a subset of rational numbers, -6, 0 and 1 are also rational numbers
Here, decimal numbers 3.01 and 8.47 are also rational numbers as they are terminating decimals
Also, -√4 = -2 as square root of 4 is 2
Thus, -2 belongs to the set of integers
From the above set, the set of rational numbers is Q,
Q = {-6, -5¾, -√4, -3/5, -3/8, 0, 4/5, 1, 1⅔, 3.01, 8.47}

(ii) First find the set of irrational numbers
Irrational numbers are numbers which are not rational
From the above subpart, we know that the set of rational numbers is Q,
Q = {-6, -5¾, -√4, -3/5, -3/8, 0, 4/5, 1, 1⅔, 3.01, 8.47}
Here the set of irrational numbers is the set of complement of the rational numbers over real numbers
The set of irrational numbers is U – Q = {√8, π}

(iii) First find the set of integers
Set of integers consists of zero, the natural numbers and their additive inverses
Set of integers is Z
Z = {…, -3, -2, -1, 0, 1, 2, 3, …}
Here, the set of integers is U ⋂ Z = {-6, -√4, 0, 1}

(iv) First find the set of non-negative integers
Set of non-negative integers consists of zero and the natural numbers
Set of non-negative integers is Z^+ and
$Z^+ = \{0, 1, 2, 3, \dots\}$
Set of integers is $U \cap Z^+ = \{0, 1\}$

5. Use method of contradiction to show that √3 and √5 are irrational.
Solution:

Consider √3 and √5 as rational numbers
√3 = a/b and √5 = x/y (where a, b, x, y ∈ Z and b, y ≠ 0)
By squaring on both sides, we have
$3 = a^2/b^2$, $5 = x^2/y^2$
$3b^2 = a^2$, $5y^2 = x^2$ …. (a)
Here,
a^2 and x^2 are odd as $3b^2$ and $5y^2$ are odd.
a and x are odd …. (1)

Take a = 3c, x = 5z
By squaring on both sides
$a^2 = 9c^2$, $x^2 = 25z^2$

Using equation (a)
$3b^2 = 9c^2$, $5y^2 = 25z^2$
By further simplification
$b^2 = 3c^2$, $y^2 = 5z^2$
Here,
B^2 and y^2 are odd as $3c^2$ and $5z^2$ are odd.
b and y are odd …… (2)

Using equation (1) and (2) we know that a, b, x, y are odd integers.
a, b and x, y have common factors 3 and 5 which contradicts our assumption that a/b and x/y are rational
a, b and x, y do not have any common factors
a/b and x/y is not rational
$\sqrt{3}$ and $\sqrt{5}$ are irrational.

6. Prove that each of the following numbers is irrational:
(i) $\sqrt{3} + \sqrt{2}$
(ii) $3 - \sqrt{2}$
(iii) $\sqrt{5} - 2$
Solution:

(i) $\sqrt{3} + \sqrt{2}$
Consider $\sqrt{3} + \sqrt{2}$ be a rational number.
$\sqrt{3} + \sqrt{2} = x$
By squaring on both sides
$(\sqrt{3} + \sqrt{2})^2 = x^2$
$(\sqrt{3})^2 + (\sqrt{2})^2 + 2(\sqrt{3})(\sqrt{2}) = x^2$
$3 + 2 + 2\sqrt{6} = x^2$
$5 + 2\sqrt{6} = x^2$
$2\sqrt{6} = x^2 - 5$
$\sqrt{6} = (x^2 - 5)/2$
Now,
x is a rational number.
x^2 is a rational number.
$x^2 - 5$ is a rational number.
$(x^2 - 5)/2$ is also a rational number.
Considering the equation, $(x^2 - 5)/2 = \sqrt{6}$
$\sqrt{6}$ is an irrational number
But, $(x^2 - 5)/2$ is a rational number
So, $x^2 - 5$ has to be an irrational number.
Then, x^2 should also be an irrational number.
Also, x must be an irrational number.

We assumed that x is a rational number
So, we arrive at a contradiction.
Hence, our assumption that $\sqrt{3} + \sqrt{2}$ is a rational number is wrong.
Therefore, $\sqrt{3} + \sqrt{2}$ is an irrational number.

(ii) $3 - \sqrt{2}$
Consider $3 - \sqrt{2}$ as a rational number.
$3 - \sqrt{2} = x$
By squaring on both sides, we get
$(3 - \sqrt{2})^2 = x^2$
$(3)^2 + (\sqrt{2})^2 - 2(3)(\sqrt{2}) = x^2$
$9 + 2 - 6\sqrt{2} = x^2$
$11 - 6\sqrt{2} = x^2$
$6\sqrt{2} = 11 - x^2$
$\sqrt{2} = (11 - x^2)/6$
Now,
x is a rational number.
x^2 is a rational number.
$11 – x^2$ is a rational number.
$(11 - x^2)/6$ is also a rational number.
Considering the equation, $\sqrt{2} = (11 - x^2)/6$
$\sqrt{2}$ is an irrational number
But, $(11 - x^2)/2$ is a rational number
So, $11 - x^2$ has to be an irrational number.
Then, x^2 should also be an irrational number.
Also, x must be an irrational number.

We assumed that x is a rational number
So, we arrive at a contradiction.
Hence, our assumption that $3 - \sqrt{2}$ is a rational number is wrong.
Therefore, $3 - \sqrt{2}$ is an irrational number.

(iii) $\sqrt{5} – 2$
Consider $\sqrt{5} – 2$ as a rational number.
$\sqrt{5} – 2 = x$
By squaring on both sides
$(\sqrt{5} - 2)^2 = x^2$
$(\sqrt{5})^2 + (2)^2 - 2(\sqrt{5})(2) = x^2$
$5 + 4 - 4\sqrt{5} = x^2$
$9 - 4\sqrt{5} = x^2$
$4\sqrt{5} = 9 - x^2$
$\sqrt{5} = (9 - x^2)/4$
Now,
x is a rational number.
x^2 is a rational number.
$9 – x^2$ is a rational number.
$(9 - x^2)/4$ is also a rational number.
Considering the equation, $\sqrt{5} = (9 - x^2)/4$
$\sqrt{5}$ is an irrational number
But, $(9 - x^2)/4$ is a rational number
So, $9 - x^2$ has to be an irrational number.
Then, x^2 should also be an irrational number.

Also, x must be an irrational number.

We assumed that x is a rational number
So, we arrive at a contradiction.
Hence, our assumption that $\sqrt{5} - 2$ is a rational number is wrong.
Therefore, $\sqrt{5} - 2$ is an irrational number.

7. Write a pair of irrational numbers whose sum is irrational.
Solution:

$\sqrt{3} + 5$ and $\sqrt{5} - 3$ are irrational numbers whose sum is irrational.
Here,
Sum $= (\sqrt{3} + 5) + (\sqrt{5} - 3)$
$= \sqrt{3} + \sqrt{5} + 2$
Hence, the resultant is irrational.

8. Write a pair of irrational numbers whose sum is rational.
Solution:

$\sqrt{3} + 5$ and $4 - \sqrt{3}$ are irrational numbers whose sum is rational.
Here,
Sum $= (\sqrt{3} + 5) + (4 - \sqrt{3})$
$= \sqrt{3} - \sqrt{3} + 9$
$= 9$
Hence, the resultant is rational.

9. Write a pair of irrational numbers whose difference is irrational.
Solution:

$\sqrt{3} + 2$ and $\sqrt{2} - 3$ are irrational numbers whose sum is irrational.
Here,
Difference $= (\sqrt{3} + 2) - (\sqrt{2} - 3)$
$= \sqrt{3} - \sqrt{2} + 2 + 3$
$= \sqrt{3} - \sqrt{2} + 5$
Hence, the resultant is irrational.

10. Write a pair of irrational numbers whose difference is rational.
Solution:

$\sqrt{5} - 3$ and $\sqrt{5} + 3$ are irrational numbers whose sum is irrational.
Here,
Difference $= (\sqrt{5} - 3) - (\sqrt{5} + 3)$
$= \sqrt{5} - \sqrt{5} - 3 - 3$
$= -6$
Hence, the resultant is rational.

11. Write a pair of irrational numbers whose product is irrational.

Solution:

Let us take two irrational numbers $(5 + \sqrt{2})$ and $(\sqrt{5} - 2)$
Here the product $= (5 + \sqrt{2}) \times (\sqrt{5} - 2)$
By further calculation
$= 5\sqrt{5} - 10 + \sqrt{10} - 2\sqrt{2}$ which is irrational.

12. Write a pair of irrational numbers whose product is rational.
Solution:

Let us consider two irrational numbers $(2\sqrt{3} - 3\sqrt{2})$ and $(2\sqrt{3} + 3\sqrt{2})$
Here, the product $= (2\sqrt{3} - 3\sqrt{2}) \times (2\sqrt{3} + 3\sqrt{2})$
By further calculation, we get
$= (3\sqrt{2})^2 - (2\sqrt{3})^2$
$= 18 - 12$
$= 6$
Therefore, the resultant is rational.

13. Write in ascending order:
(i) $3\sqrt{5}$ *and* $4\sqrt{3}$
(ii) $2\sqrt[3]{5}$ *and* $3\sqrt[3]{2}$
(iii) $6\sqrt{5}, 7\sqrt{3}$ *and* $8\sqrt{2}$
Solution:

(i) $3\sqrt{5} = \sqrt{(3^2 \text{ x } 5)} = \sqrt{(9 \text{ x } 5)} = \sqrt{45}$
$4\sqrt{3} = \sqrt{(4^2 \text{ x } 3)} = \sqrt{(16 \text{ x } 3)}\ \sqrt{48}$
We know that, $45 < 48$
So, $\sqrt{45} < \sqrt{48}$
Therefore, $3\sqrt{5} < 4\sqrt{3}$

(ii) $2\sqrt[3]{5} = \sqrt[3]{(2^3 \text{ x } 5)} = \sqrt[3]{40}$
$3\sqrt[3]{2} = \sqrt[3]{(3^3 \text{ x } 2)} = \sqrt[3]{54}$
We know that, $40 < 54$
So, $\sqrt[3]{40} < \sqrt[3]{54}$
Therefore, $2\sqrt[3]{5} < 3\sqrt[3]{2}$

(iii) $6\sqrt{5} = \sqrt{(6^2 \text{ x } 5)} = \sqrt{(36 \text{ x } 5)} = \sqrt{180}$
$7\sqrt{3} = \sqrt{(7^2 \text{ x } 3)} = \sqrt{(49 \text{ x } 3)} = \sqrt{147}$
$8\sqrt{2} = \sqrt{(8^2 \text{ x } 2)} = \sqrt{(128 \text{ x } 2)} = \sqrt{128}$
We know that, $128 < 147 < 180$
So, $\sqrt{128} < \sqrt{147} < \sqrt{180}$
Therefore, $8\sqrt{2} < 7\sqrt{3} < 6\sqrt{5}$

14. Write in descending order:
(i) $2\sqrt[4]{6}$ and $3\sqrt[4]{2}$
(ii) $7\sqrt{3}$ and $3\sqrt{7}$

Solution:

(i) It can be written as
$2\sqrt[4]{6} = \sqrt[4]{(2^4 \times 6)} = \sqrt[4]{96}$
$3\sqrt[4]{2} = \sqrt[4]{(3^4 \times 2)} = \sqrt[4]{162}$
Here, $162 > 96$
So, $\sqrt[4]{162} > \sqrt[4]{96}$
Therefore, $3\sqrt[4]{2} > 2\sqrt[4]{6}$

(ii) It can be written as
$7\sqrt{3} = \sqrt{(7^2 \times 3)} = \sqrt{(49 \times 3)} = \sqrt{141}$
$3\sqrt{7} = \sqrt{(3^2 \times 7)} = \sqrt{(9 \times 7)} = \sqrt{63}$
Here, $141 > 63$
So, $\sqrt{141} > \sqrt{63}$
Thus, $7\sqrt{3} > 3\sqrt{7}$

15. Compare:
(i) $\sqrt[6]{15}$ **and** $\sqrt[4]{12}$
(ii) $\sqrt{24}$ **and** $\sqrt[3]{35}$
Solution:

(i)
$\sqrt[6]{15} = (15)^{\frac{1}{6}}$ and $\sqrt[4]{12} = (12)^{\frac{1}{4}}$
To make the powers $\frac{1}{6}$ and $\frac{1}{4}$ same,
We find the L.C.M. of 6, 4 is 12
$$\frac{1}{6} \times \frac{2}{2} = \frac{2}{12}$$
and
$$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$$
$$\sqrt[6]{15} = (15)^{\frac{1}{6}} = (15)^{\frac{2}{12}} = \left(15^2\right)^{\frac{1}{12}} = (225)^{\frac{1}{12}}$$
$$\text{and } \sqrt[4]{12} = (12)^{\frac{1}{4}} = (12)^{\frac{3}{12}} = \left(12^3\right)^{\frac{1}{12}} = (1728)^{\frac{1}{12}}$$
$1728 > 225$
$(1728)^{\frac{1}{12}} > (225)^{\frac{1}{12}}$
$\sqrt[4]{12} > \sqrt[6]{15}$

(ii) $\sqrt{24} = (24)^{1/2}$ and $\sqrt[3]{35} = (35)^{1/3}$
In order to make the powers ½ and 1/3 same,
We find L.C.M. of 2 and 3 i.e., 6
$\frac{1}{2} \times 3/3 = 3/6$ and $1/3 \times 2/2 = 2/6$
Now,

$(24)^{1/2} = (24)^{3/6} = (24^3)^{1/6} = (13824)^{1/6}$
$(35)^{1/3} = (35)^{2/6} = (35^2)^{1/6} = (1225)^{1/6}$
On comparing,
$13824 > 1225$
So, $(13824)^{1/6} > (1225)^{1/6}$
Therefore,
$\sqrt{24} > \sqrt[3]{35}$

16. Insert two irrational numbers between 5 and 6.
Solution:

Let's write 5 and 6 as square root
Then, $5 = \sqrt{25}$ and $6 = \sqrt{36}$
Now, take the numbers
$\sqrt{25} < \sqrt{26} < \sqrt{27} < \sqrt{28} < \sqrt{29} < \sqrt{30} < \sqrt{31} < \sqrt{32} < \sqrt{33} < \sqrt{34} < \sqrt{35} < \sqrt{36}$
Hence, any two irrational numbers between 5 and 6 is $\sqrt{29}$ and $\sqrt{30}$

17. Insert five irrational numbers between $2\sqrt{5}$ and $3\sqrt{3}$.
Solution:

Here, $2\sqrt{5} = \sqrt{(2^2 \text{ x } 5)} = \sqrt{(4 \text{ x } 5)} = \sqrt{20}$ and
$3\sqrt{3} = \sqrt{(3^2 \text{ x } 3)} = \sqrt{(9 \text{ x } 3)} = \sqrt{27}$
Now, take the numbers
$\sqrt{20} < \sqrt{21} < \sqrt{22} < \sqrt{23} < \sqrt{24} < \sqrt{25} < \sqrt{26} < \sqrt{27}$
Hence, any five irrational numbers between $2\sqrt{5}$ and $3\sqrt{3}$ are:
$\sqrt{21}$, $\sqrt{22}$, $\sqrt{23}$, $\sqrt{24}$ and $\sqrt{26}$

18. Write two rational numbers between $\sqrt{2}$ and$\sqrt{3}$.
Solution:

Let us take any two rational numbers between 2 and 3 which are perfect squares
For example, let us consider 2.25 and 2.56
Now, we have
$\sqrt{2.25} = 1.5$ and $\sqrt{2.56} = 1.6$
Now,
$\sqrt{2} < \sqrt{2.25} < \sqrt{2.56}\ \sqrt{3}$
$\sqrt{2} < 1.5 < 1.6 < \sqrt{3}$
$\sqrt{2} < 15/10 < 16/10 < \sqrt{3}$
$\sqrt{2} < 3/2 < 8/5 < \sqrt{3}$
Hence, any two rational numbers between $\sqrt{2}$ and $\sqrt{3}$ are: 3/2 and 8/5

19. Write three rational numbers between $\sqrt{3}$ and $\sqrt{5}$.
Solution:

Let us take any two rational numbers between 3 and 5 which are perfect squares
For example, let us consider 3.24, 3.61, 4, 4.41 and 4.84
Now, we have

$\sqrt{3.24} = 1.8$, $\sqrt{3.61} = 1.9$, $\sqrt{4} = 2$, $\sqrt{4.41} = 2.1$ and $\sqrt{4.84} = 2.2$

Now,

$\sqrt{3} < \sqrt{3.24} < \sqrt{3.61} < \sqrt{4} < \sqrt{4.41} < \sqrt{4.84} < \sqrt{5}$

$\sqrt{3} < 1.8 < 1.9 < 2 < 2.1 < 2.2 < \sqrt{5}$

$\sqrt{3} < 18/10 < 19/10 < 2 < 21/10 < 22/10 < \sqrt{5}$

$\sqrt{3} < 9/5 < 19/10 < 2 < 21/10 < 11/5 < \sqrt{5}$

Hence, any three rational numbers between $\sqrt{3}$ and $\sqrt{5}$ are: 9/5, 21/10 and 11/5

20. Simplify each of the following:

(i) $\sqrt[5]{16} \times \sqrt[5]{2}$

(ii) $\dfrac{\sqrt[4]{243}}{\sqrt[4]{3}}$

(iii) $(3 + \sqrt{2})(4 + \sqrt{7})$

(iv) $(\sqrt{3} - \sqrt{2})^2$

Solution:

(i) It can be rewritten as $16^{1/5} \times 2^{1/5}$

By further simplification, we have

$= (2^4)^{1/5} \times 2^{1/5}$

$= 2^{4/5} \times 2^{1/5}$

$= 2^{4/5 + 1/5}$

$= 2^1$

$= 2$

(ii) It can be rewritten as $\sqrt[4]{3^5}/\sqrt[4]{3}$

By further simplification, we have

$= (3)^{1/4 \times 5}/(3)^{1/4}$

$= 3^{5/4}/3^{1/4}$

$= (3)^{5/4 - 1/4}$

$= (3)^{4/4}$

$= 3^1$

$= 3$

(iii) $(3 + \sqrt{2})(4 + \sqrt{7})$

By further calculation,

$= 3 \times 4 + 3 \times \sqrt{7} + 4 \times \sqrt{2} + \sqrt{2} \times \sqrt{7}$

So, we get

$= 12 + 3\sqrt{7} + 4\sqrt{2} + \sqrt{14}$

(iv) $(\sqrt{3} - \sqrt{2})^2$

It can be written as

$= (\sqrt{3})^2 + (\sqrt{2})^2 - 2 \times \sqrt{3} \times \sqrt{2}$

By further calculation, we get

$= 3 + 2 - 2\sqrt{6}$

$= 5 - 2\sqrt{6}$

EXERCISE 1(C)

1. State, with reason, which of the following are surds and which are not:

(i) $\sqrt{180}$
(ii) $\sqrt[4]{27}$
(iii) $\sqrt[5]{128}$
(iv) $\sqrt[3]{64}$
(v) $\sqrt[3]{23}.\sqrt[3]{40}$
(vi) $\sqrt[3]{-125}$
(vii) $\sqrt{\pi}$
(viii) $\sqrt{3+\sqrt{2}}$

Solution:

(i) $\sqrt{180} = \sqrt{(2 \times 2 \times 5 \times 3 \times 3)} = 6\sqrt{5}$
It is irrational
Therefore, $\sqrt{180}$ is a surd.

(ii) $\sqrt[4]{27} = \sqrt[4]{(3 \times 3 \times 3)}$
It is irrational
Therefore, $\sqrt[4]{27}$ is a surd

(iii)
$\sqrt[5]{128} = \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = 2\sqrt[5]{4}$
It is irrational.
Therefore, $\sqrt[5]{128}$ is a surd

(iv) $\sqrt[3]{64} = \sqrt[3]{(4 \times 4 \times 4)} = 4$
It is rational
Therefore, $\sqrt[3]{64}$ is not a surd

(v) $\sqrt[3]{25}.\sqrt[3]{40} = \sqrt[3]{(25 \times 40)} = \sqrt[3]{(5 \times 5 \times 2 \times 2 \times 5 \times 2)} = 2 \times 5 = 10$
It is rational
Therefore, $\sqrt[3]{23}.\sqrt[3]{40}$ is not a surd

(vi) $\sqrt[3]{-125} = \sqrt[3]{(-5 \times -5 \times -5)} = -5$
It is rational
Therefore, $\sqrt[3]{-125}$ is not a surd

(vii) π is irrational.
Therefore, $\sqrt{\pi}$ is not a surd.

(viii) $3 + \sqrt{2}$ is irrational
Therefore, $\sqrt{3+\sqrt{2}}$ is not a surd

2. Write the lowest rationalizing factor of:
(i) $5\sqrt{2}$

(ii) $\sqrt{24}$
(iii) $\sqrt{5} - 3$
(iv) $7 - \sqrt{7}$
(v) $\sqrt{18} - \sqrt{50}$
(vi) $\sqrt{5} - \sqrt{2}$
(vii) $\sqrt{13} + 3$
(viii) $15 - 3\sqrt{2}$
(ix) $3\sqrt{2} + 2\sqrt{3}$
Solution:

(i) $5\sqrt{2}$
It can be written as
$5\sqrt{2} \times \sqrt{2} = 5 \times 2 = 10$
It is rational.
Therefore, lowest rationalizing factor is $\sqrt{2}$.

(ii) $\sqrt{24}$
It can be written as
$\sqrt{24} = \sqrt{(2 \text{ x } 2 \text{ x } 2 \text{ x } 3)} = 2\sqrt{6}$
Therefore, lowest rationalizing factor is $\sqrt{6}$.

(iii) $\sqrt{5} - 3$
It can be written as
$(\sqrt{5} - 3)(\sqrt{5} + 3) = (\sqrt{5})^2 - 3^2 = 5 - 9 = -4$
Therefore, lowest rationalizing factor is $(\sqrt{5} + 3)$.

(iv) $7 - \sqrt{7}$
It can be written as
$(7 - \sqrt{7})(7 + \sqrt{7}) = 49 - 7 = 42$
Therefore, lowest rationalizing factor is $(7 + \sqrt{7})$.

(v) $\sqrt{18} - \sqrt{50}$
It can be written as
$\sqrt{18} - \sqrt{50} = \sqrt{(2 \text{ x } 3 \text{ x } 3)} - \sqrt{(5 \text{ x } 5 \text{ x } 2)}$
$= 3\sqrt{2} - 5\sqrt{2}$
$= -2\sqrt{2}$
Therefore, lowest rationalizing factor is $\sqrt{2}$.

(vi) $\sqrt{5} - \sqrt{2}$
It can be written as
$(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 3$
Therefore, lowest rationalizing factor is $\sqrt{5} + \sqrt{2}$.

(vii) $\sqrt{13} + 3$
It can be written as
$(\sqrt{13} + 3)(\sqrt{13} - 3) = (\sqrt{13})^2 - 3^2 = 13 - 9 = 4$
Therefore, lowest rationalizing factor is $\sqrt{13} - 3$.

(viii) $15 - 3\sqrt{2}$
It can be written as
$15 - 3\sqrt{2} = 3\,(5 - \sqrt{2})$
By further simplification
$= 3\,(5 - \sqrt{2})\,(5 + \sqrt{2})$
$= 3\,[5^2 - (\sqrt{2})^2]$
So, we get
$= 3 \times [25 - 2]$
$= 3 \times 23$
$= 69$
Therefore, lowest rationalizing factor is $(5 + \sqrt{2})$.

(ix) $3\sqrt{2} + 2\sqrt{3}$
It can be written as
$3\sqrt{2} + 2\sqrt{3} = (3\sqrt{2} + 2\sqrt{3})\,(3\sqrt{2} - 2\sqrt{3})$
By further calculation
$= (3\sqrt{2})^2 - (2\sqrt{3})^2$
So, we get
$= 9 \times 2 - 4 \times 3$
$= 18 - 12$
$= 6$
Therefore, lowest rationalizing factor is $3\sqrt{2} - 2\sqrt{3}$.

3. Rationalize the denominators of:

(i) $\frac{3}{\sqrt{5}}$

(ii) $\frac{2\sqrt{3}}{\sqrt{5}}$

(iii) $\frac{1}{\sqrt{3}-\sqrt{2}}$

(iv) $\frac{3}{\sqrt{5}+\sqrt{2}}$

(v) $\frac{2-\sqrt{3}}{2+\sqrt{3}}$

(vi) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(vii) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(viii) $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

(ix) $\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$

Solution:

(i) $(3/\sqrt{5}) \times (\sqrt{5}/\sqrt{5}) = 3\sqrt{5}/5$

(ii) $(2\sqrt{3}/\sqrt{5}) \times (\sqrt{5}/\sqrt{5}) = 2\sqrt{15}/5$

(iii)

$$\frac{1}{\sqrt{3}-\sqrt{2}} \times \left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)$$

It can be written as

$$= \frac{\sqrt{3}+\sqrt{2}}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$

So we get

$$= \frac{\sqrt{3}+\sqrt{2}}{3-2}$$

$$= \sqrt{3}+\sqrt{2}$$

(iv)

$$\frac{3}{\sqrt{5}+\sqrt{2}} \times \left(\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}\right)$$

It can be written as

$$= \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2 - \left(\sqrt{2}\right)^2}$$

So we get

$$= \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}$$

$$= \sqrt{5}-\sqrt{2}$$

(v)

$$\frac{2-\sqrt{3}}{2+\sqrt{3}} \times \left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)$$

It can be written as

$$= \frac{\left(2-\sqrt{3}\right)^2}{(2)^2 - \left(\sqrt{3}\right)^2}$$

So we get

$$= \frac{4+3-4\sqrt{3}}{4-3}$$

$$= \frac{7-4\sqrt{3}}{1}$$

$$= 7-4\sqrt{3}$$

(vi)

$\frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

It can be written as

$= \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2-(1)^2}$

$= \frac{3+1+2\sqrt{3}}{3-1}$

$= \frac{4+2\sqrt{3}}{2}$

So we get

$= \frac{\cancel{2}(2+\sqrt{3})}{\cancel{2}}$

$= 2+\sqrt{3}$

(vii)

$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

It can be written as

$= \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$

So we get

$= \frac{3+2-2\sqrt{6}}{3-2}$

$= 5 - 2\sqrt{6}$

(viii)

$\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$

It can be written as

$= \frac{6+5-2\sqrt{30}}{(\sqrt{6})^2-(\sqrt{5})^2}$

So we get

$= \frac{11-2\sqrt{30}}{6-5}$

$= 11-2\sqrt{30}$

(ix)

$$\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$$

It can be written as

$$= \frac{\left(2\sqrt{5}+3\sqrt{2}\right)^2}{\left(2\sqrt{5}\right)^2 - \left(3\sqrt{2}\right)^2}$$

$$= \frac{4\times 5 + 9 \times 2 + 12\sqrt{10}}{20-18}$$

So we get

$$= \frac{20+18+12\sqrt{10}}{2}$$

$$= \frac{38+12\sqrt{10}}{2}$$

$$= \frac{\cancel{2}\left(19+6\sqrt{10}\right)}{\cancel{2}}$$

$$= 19+6\sqrt{10}$$

4. Find the values of 'a' and 'b' in each of the following:

(i) $\frac{2+\sqrt{3}}{2-\sqrt{3}} = a + b\sqrt{3}$

(ii) $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7} + b$

(iii) $\frac{3}{\sqrt{3}-\sqrt{2}} = a\sqrt{3} + b\sqrt{2}$

(iv) $\frac{5+3\sqrt{2}}{5-3\sqrt{2}} = a + b\sqrt{2}$

Solution:

(i)

$$\frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = a + b\sqrt{3}$$

It can be written as

$$\frac{\left(2+\sqrt{3}\right)^2}{(2)^2 - \left(\sqrt{3}\right)^2} = a+b\sqrt{3}$$

$$\frac{4+3+4\sqrt{3}}{4-3} = a+b\sqrt{3}$$

So we get

$$7+4\sqrt{3} = a+b\sqrt{3}$$

$$a = 7,\ b = 4$$

(ii)

$$\frac{\sqrt{7}-2}{\sqrt{7}+2} \times \frac{\sqrt{7}-2}{\sqrt{7}-2} = a\sqrt{7}+b$$

It can be written as

$$\frac{\left(\sqrt{7}-2\right)^2}{\left(\sqrt{7}\right)^2-(2)^2} = a\sqrt{7}+b$$

$$\frac{7+4-4\sqrt{7}}{7-4} = a\sqrt{7}+b$$

So we get

$$\frac{11-4\sqrt{7}}{3} = a\sqrt{7}+b$$

$$a = \frac{-4}{3},\ b = \frac{11}{3}$$

(iii)

$$\frac{3}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = a\sqrt{3}-b\sqrt{2}$$

It can be written as

$$\frac{3\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2} = a\sqrt{3}-b\sqrt{2}$$

$$\frac{3\left(\sqrt{3}+\sqrt{2}\right)}{3-2} = a\sqrt{3}-b\sqrt{2}$$

So we get

$$\left(3\sqrt{3}+3\sqrt{2}\right) = a\sqrt{3}-b\sqrt{2}$$

$$a = 3, b = -3$$

(iv)

$$\frac{5+3\sqrt{2}}{5-3\sqrt{2}} \times \frac{5+3\sqrt{2}}{5+3\sqrt{2}} = a+b\sqrt{2}$$

It can be written as

$$\frac{\left(5+3\sqrt{2}\right)^2}{(5)^2-\left(3\sqrt{2}\right)^2} = a+b\sqrt{2}$$

$$\frac{25+18+30\sqrt{2}}{25-18} = a+b\sqrt{2}$$

So we get

$$\frac{43+30\sqrt{2}}{7} = a+b\sqrt{2}$$

$$a = \frac{43}{7},\quad b = \frac{30}{7}$$

5. Simplify:

(i) $\frac{22}{2\sqrt{3}+1}+\frac{17}{2\sqrt{3}-1}$

(ii) $\frac{\sqrt{2}}{\sqrt{6}-\sqrt{2}}+\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

Solution:

(i)

$$\frac{22}{2\sqrt{3}+1}+\frac{17}{2\sqrt{3}-1}$$

It can be written as

$$=\frac{22\left(2\sqrt{3}-1\right)+17\left(2\sqrt{3}+1\right)}{\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right)}$$

By further calculation

$$=\frac{44\sqrt{3}-22+34\sqrt{3}+17}{\left(2\sqrt{3}\right)^2-1}$$

So we get

$$=\frac{78\sqrt{3}-5}{12-1}$$

$$=\frac{78\sqrt{3}-5}{11}$$

(ii)

$$\frac{\sqrt{2}}{\sqrt{6}-2}-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}$$

It can be written as

$$=\frac{\sqrt{2}\left(\sqrt{6}+\sqrt{2}\right)-\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2}$$

By further calculation

$$=\frac{\sqrt{12}+2-\sqrt{18}+\sqrt{6}}{6-2}$$

So we get

$$=\frac{2\sqrt{3}+2-3\sqrt{2}+\sqrt{6}}{4}$$

6. If $x = \frac{\sqrt{5}-2}{\sqrt{5}+2}$ and $y = \frac{\sqrt{5}+2}{\sqrt{5}-2}$; Find:

(i) x^2

(ii) y^2

(iii) xy

(iv) $x^2 + y^2 = xy$

Solution:

(i)

$$x^2 = \left(\frac{\sqrt{5}-2}{\sqrt{5}+2}\right)^2$$

It can be written as

$$= \frac{5+4-4\sqrt{5}}{5+4+4\sqrt{5}} = \frac{9-4\sqrt{5}}{9+4\sqrt{5}}$$

By further calculation

$$= \frac{9-4\sqrt{5}}{9+4\sqrt{5}} \times \left(\frac{9-4\sqrt{5}}{9-4\sqrt{5}}\right) = \frac{\left(9-4\sqrt{5}\right)^2}{(9)^2-\left(4\sqrt{5}\right)^2}$$

So we get

$$= \frac{81+80-72\sqrt{5}}{81-80} = 161-72\sqrt{5}$$

(ii)

$$y^2 = \left(\frac{\sqrt{5}+2}{\sqrt{5}-2}\right)^2$$

It can be written as

$$= \frac{5+4+4\sqrt{5}}{5+4-4\sqrt{5}} = \frac{9+4\sqrt{5}}{9-4\sqrt{5}}$$

Bv further calculation

$$= \frac{9+4\sqrt{5}}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}} = \frac{\left(9+4\sqrt{5}\right)^2}{(9)^2-\left(4\sqrt{5}\right)^2}$$

So we get

$$= \frac{81+80+72\sqrt{5}}{81-80} = 161+72\sqrt{5}$$

(iii) We know that

$$xy = \frac{(\sqrt{5}-2)}{(\sqrt{5}+2)}\frac{(\sqrt{5}+2)}{(\sqrt{5}-2)} = 1$$

(iv) $x^2 + y^2 = xy$
By substituting the values
$= 161 - 72\sqrt{5} + 161 + 72\sqrt{5} + 1$
So, we get
$= 322 + 1$
$= 323$

7. If $m = 1/(3 - 2\sqrt{2})$ and $n = 1/(3 + 2\sqrt{2})$, find:
(i) m^2
(ii) n^2
(iii) mn
Solution:

(i)
$$m = \frac{1}{3-2\sqrt{2}}$$
It can be written as
$$= \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$
By further calculation
$$= \frac{3+2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$$
So we get
$$= \frac{3+2\sqrt{2}}{9-8}$$
$$= 3+2\sqrt{2}$$
Here
$$m^2 = (3+2\sqrt{2})^2$$
Expanding using the formula
$$= (3)^2 + 2 \times 3 \times 2\sqrt{2} + (2\sqrt{2})^2$$
$$= 9 + 12\sqrt{2} + 8$$
$$= 17 + 12\sqrt{2}$$

(ii)

$n = \frac{1}{3+2\sqrt{2}}$

It can be written as

$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

By further calculation

$= \frac{3-2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$

So we get

$= \frac{3+2\sqrt{2}}{9-8}$

$= 3-2\sqrt{2}$

Here

$n^2 = (3-2\sqrt{2})^2$

Expanding using the formula

$= (3)^2 - 2 \times 3 \times 2\sqrt{2} + (2\sqrt{2})^2$

$= 9 - 12\sqrt{2} + 8$

$= 17 - 12\sqrt{2}$

(iii) We know that

$mn = (3 + \sqrt{2})(3 - \sqrt{2})$

By further calculation, we get

$mn = 3^2 – (2\sqrt{2})^2$

So, we get

$= 9 – 8$

$= 1$

8. If $x = 2\sqrt{3} + 2\sqrt{2}$, find:

(i) 1/x

(ii) x + 1/x

(iii) $(x + 1/x)^2$

Solution:

(i)

$\frac{1}{x} = \frac{1}{2\sqrt{3}+2\sqrt{2}} \times \frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}-2\sqrt{2}}$

By further calculation

$= \frac{2\sqrt{3}-2\sqrt{2}}{12-8}$

So we get

$$= \frac{\cancel{2}(\sqrt{3}-\sqrt{2})}{\cancel{4}2}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{2}$$

(ii)

$$x + \frac{1}{x} = 2\sqrt{3} + 2\sqrt{2} + \frac{\sqrt{3}-\sqrt{2}}{2}$$

By further calculation

$$= 2(\sqrt{3}+\sqrt{2}) + \frac{(\sqrt{3}-\sqrt{2})}{2}$$

$$= \frac{4(\sqrt{3}+\sqrt{2}) + (\sqrt{3}-\sqrt{2})}{2}$$

So we get

$$= \frac{4\sqrt{3}+4\sqrt{2}+\sqrt{3}-\sqrt{2}}{2}$$

$$= \frac{5\sqrt{3}+3\sqrt{2}}{2}$$

(iii)

$$\left(x + \frac{1}{x}\right)^2 = \left(\frac{5\sqrt{3}+3\sqrt{2}}{2}\right)^2$$

By further calculation

$$= \frac{75+18+30\sqrt{6}}{4}$$

So we get

$$= \frac{93+30\sqrt{6}}{4}$$

9. If $x = 1 - \sqrt{2}$, find the value of $(x + 1/x)^3$.
Solution:

It is given that
$x = 1 - \sqrt{2}$
We should find the value of $(x + 1/x)^3$
So, $x = 1 - \sqrt{2}$, we get

$$\frac{1}{x} = \frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$$

Using the formula $(a - b)(a + b) = a^2 - b^2$

$$\frac{1}{x} = \frac{1+\sqrt{2}}{1^2 - \left(\sqrt{2}\right)^2}$$

$$\frac{1}{x} = \frac{1+\sqrt{2}}{1-2}$$

$$\frac{1}{x} = \frac{1+\sqrt{2}}{-1}$$

$$\frac{1}{x} = -\left(1+\sqrt{2}\right) \ldots (1)$$

Here

$(x - 1/x) = (1 - \sqrt{2}) - (-(1 + \sqrt{2}))$

$= 1 - \sqrt{2} + 1 + \sqrt{2}$

$= 2$

By cubing on both sides, we get

$(x - 1/x)^3 = 2^3$

$= 8$

10. If $x = 5 - 2\sqrt{6}$, find: $x^2 + 1/x^2$

Solution:

It is given that

$x = 5 - 2\sqrt{6}$

We should find the value of $(x^2 + 1/x^2)$

So, $x = 5 - 2\sqrt{6}$, we get

$$\frac{1}{x} = \frac{1}{5 - 2\sqrt{6}} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}}$$

$$\frac{1}{x} = \frac{5+2\sqrt{6}}{\left(5-2\sqrt{6}\right)\left(5+2\sqrt{6}\right)}$$

Using the formula $(a - b)(a + b) = a^2 - b^2$

$$\frac{1}{x} = \frac{5+2\sqrt{6}}{5^2 - \left(2\sqrt{6}\right)^2}$$

$$\frac{1}{x} = \frac{5+2\sqrt{6}}{25-24}$$

$$\frac{1}{x} = 5 + 2\sqrt{6} \ldots (1)$$

Here,

$(x - 1/x) = (5 - 2\sqrt{6}) - (5 + 2\sqrt{6})$

$= 5 - 2\sqrt{6} - 5 - 2\sqrt{6}$

$= -4\sqrt{6} \ldots (2)$

Now,

Consider $(x - 1/x)^2$
Using the equation $(a - b)^2 = a^2 + b^2 - 2ab$

$(x - 1/x)^2 = x^2 + 1/x^2 - 2(x)(1/x)$
$(x - 1/x)^2 = x^2 + 1/x^2 - 2$
$(x - 1/x)^2 + 2 = x^2 + 1/x^2$ ... (3)
From equations (2) and (3), we get
$x^2 + 1/x^2 = (-4\sqrt{6})^2 + 2$
$= 96 + 2$
$= 98$

11. Show that:

$$\frac{1}{3-2\sqrt{2}} - \frac{1}{2\sqrt{2}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} = 5$$

Solution:

Consider

$$\text{L.H.S.} = \frac{1}{3-2\sqrt{2}} - \frac{1}{2\sqrt{2}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$

It can be written as

$$= \frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$

$$= \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$$
$$- \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$$

Using the formula $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{3+\sqrt{8}}{(3)^2-(\sqrt{8})^2} - \frac{\sqrt{8}+\sqrt{7}}{(\sqrt{8})^2-(\sqrt{7})^2} + \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2} - \frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6})^2-(\sqrt{5})^2} + \frac{\sqrt{5}+2}{(\sqrt{5})^2-(2)^2}$$

$$= \frac{3+\sqrt{8}}{9-8} - \frac{\sqrt{8}+\sqrt{7}}{8-7} + \frac{\sqrt{7}+\sqrt{6}}{7-6} - \frac{\sqrt{6}+\sqrt{5}}{6-5} + \frac{\sqrt{5}+2}{5-4}$$

So, we get
$= 3 + \sqrt{8} - \sqrt{8} - \sqrt{7} + \sqrt{7} + \sqrt{6} - \sqrt{6} - \sqrt{5} + \sqrt{5} + 2$
$= 3 + 2$
$= 5$
= R.H.S.

12. Rationalize the denominator of:

$$\frac{1}{\sqrt{3}-\sqrt{2}+1}$$

Solution:

We know that,

$$\frac{1}{\sqrt{3}-\sqrt{2}+1}$$

$$= \frac{1}{\left(\sqrt{3}-\sqrt{2}\right)+1} \times \frac{\left(\sqrt{3}-\sqrt{2}\right)-1}{\left(\sqrt{3}-\sqrt{2}\right)-1}$$

Using the formula $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{\left(\sqrt{3}-\sqrt{2}\right)^2-(1)^2}$$

Using the formula $(a - b)^2 = a^2 + b^2 - 2ab$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{\left(\sqrt{3}\right)^2-2\sqrt{6}+\left(\sqrt{2}\right)^2-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{3-2\sqrt{6}+2-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{4-2\sqrt{6}}$$

It can be written as

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)-1}{2\left(2-\sqrt{6}\right)}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{2\left(2-\sqrt{6}\right)} \times \frac{2+\sqrt{6}}{2+\sqrt{6}}$$

Using the formula $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{2\sqrt{3}-2\sqrt{2}-2+\sqrt{18}-\sqrt{12}-\sqrt{6}}{2\left[(2)^2-\left(\sqrt{6}\right)^2\right]}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}-2+3\sqrt{2}-2\sqrt{3}-\sqrt{6}}{2(4-6)}$$

So, we get

$$= \frac{\sqrt{2}-2-\sqrt{6}}{2(-2)}$$

$$= \frac{\sqrt{2}-2-\sqrt{6}}{-4}$$

$$= \frac{1}{4}\left(2+\sqrt{6}-\sqrt{2}\right)$$

13. If $\sqrt{2} = 1.4$ and $\sqrt{3} = 1.7$, find the value of each of the following, correct to one decimal place:
(i) $1/(\sqrt{3} - \sqrt{2})$
(ii) $1/(3 + 2\sqrt{2})$

(iii) (2 - √3)/√3
Solution:

(i)

$$\frac{1}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

It can be written as

$$= \frac{\sqrt{3}+\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{3}+\sqrt{2}}{3-2}$$

So, we get
= √3 + √2
= 1.7 + 1.4
= 3.1

(ii)

$$\frac{1}{3+2\sqrt{2}}$$

$$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

It can be written as

$$= \frac{3-2\sqrt{2}}{(3)^2-\left(2\sqrt{2}\right)^2}$$

$$= \frac{3-2\sqrt{2}}{9-8}$$

So, we get
= 3 - 2√2
= 3 – 2(1.4)
= 3 – 2.8
= 0.2

(iii)

$$\frac{2-\sqrt{3}}{\sqrt{3}}$$

It can be written as

$$\frac{2-\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

By further calculation

$$\frac{2\sqrt{3}-3}{3} = \frac{(2 \times 1.7) - 3}{3}$$

So, we get

(3.4 - 3)/3 = 0.4/3

= 0.133333...

≈ 0.1

14. Evaluate:

(4 - √5)/(4 + √5) + (4 + √5)/(4 - √5)

Solution:

We have,

$$\frac{4-\sqrt{5}}{4+\sqrt{5}} + \frac{4+\sqrt{5}}{4-\sqrt{5}}$$

$$= \frac{4-\sqrt{5}}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}} + \frac{4+\sqrt{5}}{4-\sqrt{5}} \times \frac{4+\sqrt{5}}{4+\sqrt{5}}$$

Using the formula $(a^2 - b^2) = (a + b)(a - b)$

$$= \frac{\left(4-\sqrt{5}\right)^2}{4^2-\left(\sqrt{5}\right)^2} + \frac{\left(4+\sqrt{5}\right)^2}{4^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{16+5-8\sqrt{5}}{16-5} + \frac{16+5+8\sqrt{5}}{16-5}$$

By further calculation

$$= \frac{21-8\sqrt{5}}{11} + \frac{21+8\sqrt{5}}{11}$$

$$= \frac{21-8\sqrt{5}+21+8\sqrt{5}}{11}$$

$$= \frac{42}{11}$$

$$= 3\frac{9}{11}$$

15. If (2 + √5)/(2 - √5) = x and (2 - √5)/(2 + √5) = y; find the value of $x^2 - y^2$.

Solution:

We have,

$$x = \frac{2+\sqrt{5}}{2-\sqrt{5}}$$

$$= \frac{2+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}}$$

Using the formula $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{(2+\sqrt{5})^2}{2^2-(\sqrt{5})^2}$$

$$= \frac{4+4\sqrt{5}+5}{4-5}$$

So, we get

$$= \frac{9+4\sqrt{5}}{-1}$$

$$= -9-4\sqrt{5}$$

Similarly,

$$y = \frac{2-\sqrt{5}}{2+\sqrt{5}}$$

$$= \frac{2-\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

Using the formula $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{(2-\sqrt{5})^2}{2^2-(\sqrt{5})^2}$$

By further calculation

$$= \frac{4-4\sqrt{5}+5}{4-5}$$

$$= \frac{9-4\sqrt{5}}{-1}$$

$$= -9+4\sqrt{5}$$

Here,

$x^2 - y^2 = (-9 - 4\sqrt{5})^2 - (-9 + 4\sqrt{5})^2$

Expanding using the formula, we get

$= 81 + 72\sqrt{5} + 80 - (81 - 72\sqrt{5} + 80)$

$= 81 + 72\sqrt{5} + 80 - 81 + 72\sqrt{5} - 80$

$= 144\sqrt{5}$

EXERCISE 1D

1. Simplify:

$$\frac{\sqrt{18}}{5\sqrt{18}+3\sqrt{72}+2\sqrt{162}}$$

Solution:

We have,

$$\frac{\sqrt{18}}{5\sqrt{18}+3\sqrt{72}-2\sqrt{162}}$$

It can be written as

$$=\frac{\sqrt{18}}{5\sqrt{18}+3\sqrt{4\times 18}-2\sqrt{9\times 18}}$$

$$=\frac{\sqrt{18}}{5\sqrt{18}+(3\times 2\sqrt{18})-(2\times 3\sqrt{18})}$$

So, we get

$$=\frac{\sqrt{18}}{5\sqrt{18}+6\sqrt{18}-6\sqrt{18}}$$

$$=\frac{\sqrt{18}}{5\sqrt{18}}=\frac{1}{5}$$

2. Simplify:

$$\frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}\div\frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y}$$

Solution:

We have,

$$\frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}\div\frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y}$$

It can be written as

$$=\frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}\times\frac{\sqrt{x^2+y^2}+y}{\sqrt{x^2-y^2}+x}$$

$$=\frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}\times\frac{\sqrt{x^2+y^2}+y}{x+\sqrt{x^2-y^2}}$$

Using the formula, $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{(\sqrt{x^2+y^2})^2 - y^2}{x^2 - (\sqrt{x^2-y^2})^2}$$

$$= \frac{x^2+y^2-y^2}{x^2-x^2+y^2}$$

So, we get

$= x^2/y^2$

3. Evaluate, correct to one place of decimal. The expression 5/(√20 - √10), if √5 = 2.2 and √10 = 3.2.

Solution:

We have,

$$\frac{5}{\sqrt{20}-\sqrt{10}} = \frac{5}{\sqrt{4\times 5}-\sqrt{10}}$$

It can be written as

= 5/(2√5 - √10)

= 5/[(2 x 2.2) – 3.2)]

So, we get

= 5/(4.4 – 3.2)

= 5/1.2

= 4.2

[Note: In textual answer, the value of √20 has been directly taken, which is 4.5. Hence the answer 3.8!]

4. If x = √3 – √2. Find the value of:

(i) x + 1/x

(ii) $x^2 + 1/x^2$

(iii) $x^3 + 1/x^3$

(iv) $x^3 + 1/x^3 – 3(x^2 + 1/x^2) + x + 1/x$

Solution:

(i) We have,

x + 1/x

= (√3 - √2) + 1/(√3 - √2)

$$= \frac{(\sqrt{3}-\sqrt{2})^2+1}{(\sqrt{3}-\sqrt{2})}$$

$$= \frac{3-2\sqrt{3}\sqrt{2}+2+1}{(\sqrt{3}-\sqrt{2})}$$

$$= \frac{6-2\sqrt{3}\sqrt{2}}{(\sqrt{3}-\sqrt{2})}$$

$$= \frac{6-2\sqrt{6}}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$$

$$= \frac{6\sqrt{3}-2\sqrt{6}\sqrt{3}+6\sqrt{2}-2\sqrt{6}\sqrt{2}}{1}$$

$= 6\sqrt{3} - 2\sqrt{18} + 6\sqrt{2} - 2\sqrt{12}$
$= 6\sqrt{3} - 2\sqrt{(9 \times 2)} + 6\sqrt{2} - 2\sqrt{(4 \times 3)}$
$= 6\sqrt{3} - 2 \times 3\sqrt{2} + 6\sqrt{2} - 2 \times 2\sqrt{3}$
$= 6\sqrt{3} - 6\sqrt{2} + 6\sqrt{2} - 4\sqrt{3}$
$= 6\sqrt{3} - 4\sqrt{3}$
$= 2\sqrt{3}$

(ii) $x^2 + 1/x^2$
We have,
$= (\sqrt{3} - \sqrt{2})^2 + 1/(\sqrt{3} - \sqrt{2})^2$

$$= (3 - 2\sqrt{3}\sqrt{2} + 2) + \frac{1}{(3 - 2\sqrt{3}\sqrt{2} + 2)}$$
$$= (5 - 2\sqrt{6}) + \frac{1}{(5 - 2\sqrt{6})}$$
$$= \frac{25 - 10\sqrt{6} - 10\sqrt{6} + 4 \times 6 + 1}{(5 - 2\sqrt{6})}$$
$$= \frac{25 - 20\sqrt{6} + 25}{(5 - 2\sqrt{6})}$$
$$= \frac{50 - 20\sqrt{6}}{(5 - 2\sqrt{6})}$$
$$= \frac{10(5 - 2\sqrt{6})}{(5 - 2\sqrt{6})}$$
$$= 10$$

(iii) We have,
$x^3 + 1/x^3$
$= (\sqrt{3} - \sqrt{2})^3 + 1/(\sqrt{3} - \sqrt{2})^3$
We know that, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
$(\sqrt{3} - \sqrt{2})^3 = (\sqrt{3})^3 - (\sqrt{2})^3 - 3(\sqrt{3})(\sqrt{2})(\sqrt{3} - \sqrt{2})$
$= 3\sqrt{3} - 2\sqrt{2} - 3\sqrt{6}(\sqrt{3} - \sqrt{2})$
$= 3\sqrt{3} - 2\sqrt{2} - 3\sqrt{18} + 3\sqrt{12}$
$= 3\sqrt{3} - 2\sqrt{2} - 3\sqrt{(3^2 \times 2)} + 3\sqrt{(2^2 \times 3)}$
$= 3\sqrt{3} - 2\sqrt{2} - 3 \times 3\sqrt{2} + 3 \times 2\sqrt{3}$
$= 3\sqrt{3} - 2\sqrt{2} - 9\sqrt{2} + 6\sqrt{3}$
$= 9\sqrt{3} - 11\sqrt{2}$

$$\therefore \left(\sqrt{3} - \sqrt{2}\right)^3 + \frac{1}{(\sqrt{3} - \sqrt{2})^3} = \left(9\sqrt{3} - 11\sqrt{2}\right) + \frac{1}{(9\sqrt{3} - 11\sqrt{2})}$$

$$Considering\ \frac{1}{(9\sqrt{3}-11\sqrt{2})}$$
$$\frac{1}{(9\sqrt{3}-11\sqrt{2})} \times \frac{(9\sqrt{3}+11\sqrt{2})}{(9\sqrt{3}+11\sqrt{2})}$$
$$= \frac{(9\sqrt{3}+11\sqrt{2})}{(81\times 3)-(121\times 2)}$$
$$= \frac{(9\sqrt{3}+11\sqrt{2})}{(243)-(242)}$$
$$= (9\sqrt{3}+11\sqrt{2})$$

Now, $(9\sqrt{3} - 11\sqrt{2}) + 1/(9\sqrt{3} - 11\sqrt{2}) = (9\sqrt{3} - 11\sqrt{2}) + (9\sqrt{3} + 11\sqrt{2})$
$= 9\sqrt{3} - 11\sqrt{2} + 9\sqrt{3} + 11\sqrt{2}$
$= 9\sqrt{3} + 9\sqrt{3}$
$= 18\sqrt{3}$

(iv) $x^3 + 1/x^3 – 3(x^2 + 1/x^2) + x + 1/x$
According to the results obtained in (i), (ii) and (iii), we get
$x^3 + 1/x^3 – 3(x^2 + 1/x^2) + x + 1/x = 18\sqrt{3} – 3(10) + 2\sqrt{3}$
$= 20\sqrt{3} – 30$
$= 10(2\sqrt{3} - 3)$

5. Show that:
(i) Negative of an irrational number is irrational.
Solution:

Let the irrational number be $\sqrt{2}$
Considering the negative of $\sqrt{2}$, we get $-\sqrt{2}$
We know that $-\sqrt{2}$ is an irrational number
Hence, negative of an irrational number is irrational

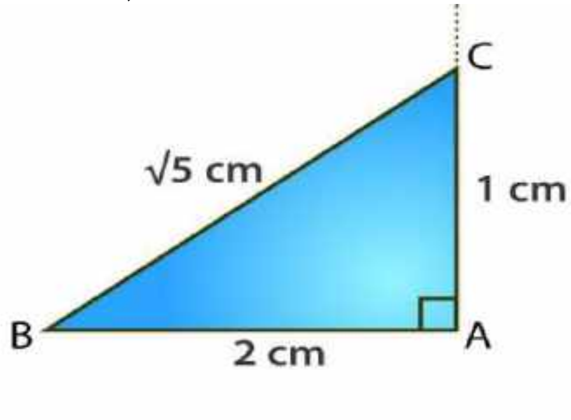
(ii) The product of a non-zero rational number and an irrational number is an irrational number.
Solution:

Let the non-zero rational number be 3
Let the irrational number be $\sqrt{5}$
Then, according to the question
$3 \times \sqrt{5} = 3\sqrt{5} = 3 \times 2.2 = 6.6$, which is irrational

6. Draw a line segment of length √5 cm.
Solution:

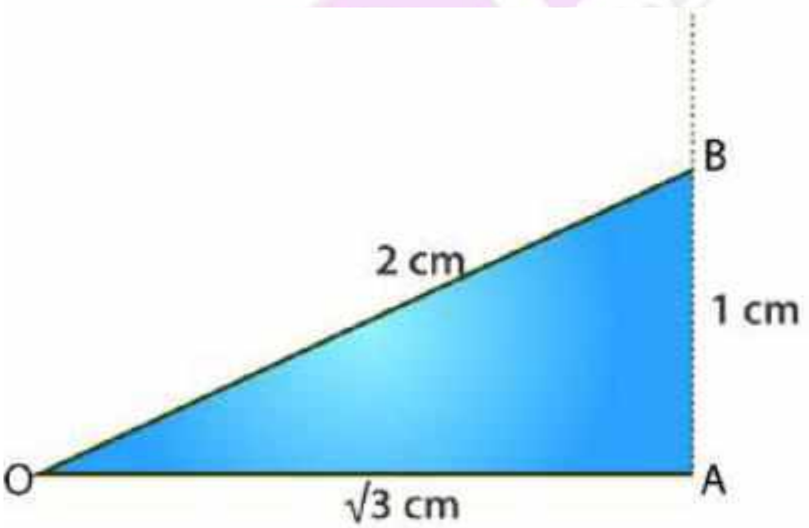
We know that, $\sqrt{5} = \sqrt{(2^2 + 1^2)}$
Which relates to: Hypotenuse $= \sqrt{[(\text{side }1)^2 + (\text{side }2)^2]}$ … [Pythagoras theorem]

Hence, considering
Side 1 = 2 and Side 2 = 1,
We get a right-angled triangle such that:
$\angle A = 90°$, AB = 2 cm and AC = 1 cm



7. Draw a line segment of length √3 cm.
Solution:

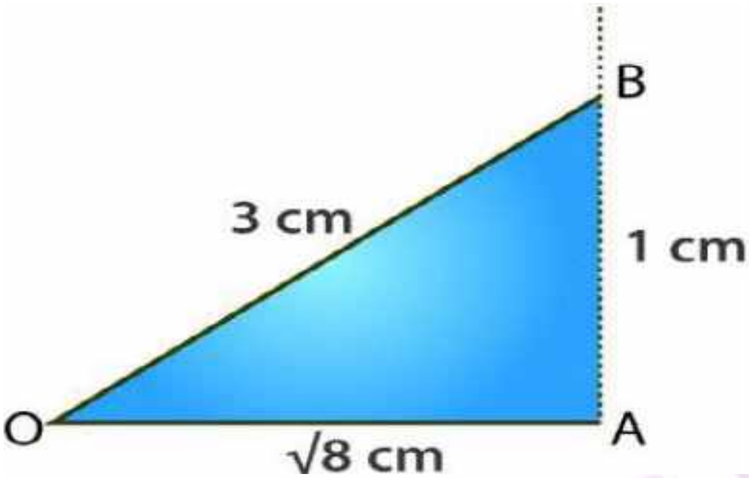
We know that, $\sqrt{3} = \sqrt{(2^2 - 1^2)}$
Which relates to: Hypotenuse= $\sqrt{[(\text{side } 1)^2 + (\text{side } 2)^2]}$ … [Pythagoras theorem]
Hypotenuse^2 – Side 1^2 = Side 2^2
Hence, considering Hypotenuse = 2 cm and Side 1 = 1 cm,
We get a right-angled triangle OAB such that:
$\angle O = 90°$, OB = 2 cm and AB = 1 cm



8. Draw a line segment of length √8 cm.
Solution:

We know that, $\sqrt{8} = \sqrt{(3^2 - 1^2)}$
Which relates to: Hypotenuse = $\sqrt{[(\text{side } 1)^2 + (\text{side } 2)^2]}$ … (Pythagoras theorem)

$\text{Hypotenuse}^2 - (\text{Side } 1)^2 = (\text{Side } 2)^2$
Hence, considering Hypotenuse = 3
cm and Side 1 =1 cm,
We get a right-angled triangle OAB such that:
$\angle A = 90°$, OB = 3 cm and AB=1 cm



9. Show that:

$$\frac{4-\sqrt{5}}{4+\sqrt{5}}+\frac{2}{5+\sqrt{3}}+\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{2}{5-\sqrt{3}}=\frac{52}{11}$$

Solution:

We have,

$$\frac{4-\sqrt{5}}{4+\sqrt{5}}+\frac{2}{5+\sqrt{3}}+\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{2}{5-\sqrt{3}}=\frac{52}{11}$$

$Here,$

$Considering\ \frac{4-\sqrt{5}}{4+\sqrt{5}}$

$$\Rightarrow \frac{4-\sqrt{5}}{4+\sqrt{5}}\times\frac{4-\sqrt{5}}{4-\sqrt{5}}=\frac{(4-\sqrt{5})^2}{16-5}=\frac{(4-\sqrt{5})^2}{11}$$

$Now,\ Considering\ \frac{2}{5+\sqrt{3}}$

$$\Rightarrow \frac{2}{5+\sqrt{3}}\times\frac{5-\sqrt{3}}{5-\sqrt{3}}=\frac{10-2\sqrt{3}}{25-3}=\frac{10-2\sqrt{3}}{22}$$

$Now,\ Considering\ \frac{4+\sqrt{5}}{4-\sqrt{5}}$

$$\Rightarrow \frac{4+\sqrt{5}}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}}=\frac{(4+\sqrt{5})^2}{16-5}=\frac{(4+\sqrt{5})^2}{11}$$

$Now,\ Considering\ \frac{2}{5-\sqrt{3}}$

$$\Rightarrow \frac{2}{5-\sqrt{3}}\times\frac{5+\sqrt{3}}{5+\sqrt{3}}=\frac{10+2\sqrt{3}}{25-3}=\frac{10+2\sqrt{3}}{22}$$

$$\therefore \frac{4-\sqrt{5}}{4+\sqrt{5}}+\frac{2}{5+\sqrt{3}}+\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{2}{5-\sqrt{3}}$$

$$=\frac{(4-\sqrt{5})^2}{11}+\frac{10-2\sqrt{3}}{22}+\frac{(4+\sqrt{5})^2}{11}+\frac{10+2\sqrt{3}}{22}$$

$$=\frac{(4-\sqrt{5})^2}{11}+\frac{5-\sqrt{3}}{11}+\frac{(4+\sqrt{5})^2}{11}+\frac{5+\sqrt{3}}{11}$$

$$=\frac{16-8\sqrt{5}+5+5-\sqrt{3}+16+8\sqrt{5}+5+5+\sqrt{3}}{11}$$

$$=\frac{52}{11}$$

$Hence\ proved$

10. Show that:

(i) $x^3 + 1/x^3 = 52$, if $x = 2 + \sqrt{3}$

(ii) $x^2 + 1/x^2 = 34$, if $x = 3 + 2\sqrt{2}$

(iii) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}=11$

Solution:

(i) We know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$x^3 + 1/x^3 = (2 + \sqrt{3})^3 + 1/(2 + \sqrt{3})^3$

Here, taking

$$\begin{aligned}(2 + \sqrt{3})^3 &= 2^3 + (\sqrt{3})^3 + 3(2)(\sqrt{3})(2 + \sqrt{3})\\ &= 8 + 3\sqrt{3} + 6\sqrt{3}(2 + \sqrt{3})\\ &= 8 + 3\sqrt{3} + 12\sqrt{3} + 6(\sqrt{3})^2\\ &= 8 + 3\sqrt{3} + 12\sqrt{3} + (6 \times 3)\\ &= 8 + 15\sqrt{3} + 18\\ &= 26 + 15\sqrt{3}\end{aligned}$$

$$Now,\ (2+\sqrt{3})^3 + \frac{1}{(2+\sqrt{3})^3} = 26 + 15\sqrt{3} + \frac{1}{26+15\sqrt{3}}$$

$$Taking\ \frac{1}{26+15\sqrt{3}},$$

$$\Rightarrow \frac{1}{26+15\sqrt{3}} \times \frac{26-15\sqrt{3}}{26-15\sqrt{3}} = \frac{26-15\sqrt{3}}{676-675} = 26-15\sqrt{3}$$

$$= 26 + 15\sqrt{3} + 26 - 15\sqrt{3} = 52$$

- Hence, proved.

(ii) We know that, $(a + b)^2 = a^2 + b^2 + 2ab$

$$\begin{aligned}x^2 + 1/x^2 &= (3 + 2\sqrt{2})^2 + 1/(3 + 2\sqrt{2})^2\\ &= (9 + 8 + 2 \times 3 \times 2\sqrt{2}) + 1/(9 + 8 + 2 \times 3 \times 2\sqrt{2})\\ &= (17 + 12\sqrt{2}) + 1/(17 + 12\sqrt{2})\end{aligned}$$

$$Taking\ \frac{1}{(17+12\sqrt{2})}\ we\ get:$$

$$\frac{1}{(17+12\sqrt{2})} \times \frac{(17-12\sqrt{2})}{(17-12\sqrt{2})} = \frac{(17-12\sqrt{2})}{289-288} = 17-12\sqrt{2}$$

$$\therefore (17+12\sqrt{2}) + \frac{1}{(17+12\sqrt{2})} = 17+12\sqrt{2}+17-12\sqrt{2} = 34$$

- Hence, proved.

(iii) We have,

$$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

$$First, taking\ \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}},$$

$$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}\times\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=\frac{(3\sqrt{2}-2\sqrt{3})^2}{18-12}=\frac{18-12\sqrt{6}+12}{6}$$

$$=\frac{6(3-2\sqrt{6}+2)}{6}=5-2\sqrt{6}$$

$$Now, taking\ \frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}},$$

$$\frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\frac{6+2\sqrt{6}}{3-2}=6+2\sqrt{6}$$

$$\therefore\ \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}=5-2\sqrt{6}+6+2\sqrt{6}=11$$

- Hence, proved.

11. Show that x is rational if:

(i) $x^2 = 6$

(ii) $x^2 = 0.009$

(iii) $x^2 = 27$

Solution:

(i) $x^2 = 6$

$x = \sqrt{6} = 2.449$ … which is irrational.

(ii) $x^2 = 0.009$

$x = \sqrt{0.009} = 0.0948$ … which is irrational.

(iii) $x^2 = 27$

$x = \sqrt{27} = 5.1961$ … which is irrational.

12. Show that x is rational if:

(i) $x^2 = 16$

(ii) $x^2 = 0.0004$

(iii) $x^2 = 1\frac{7}{9}$

Solution:

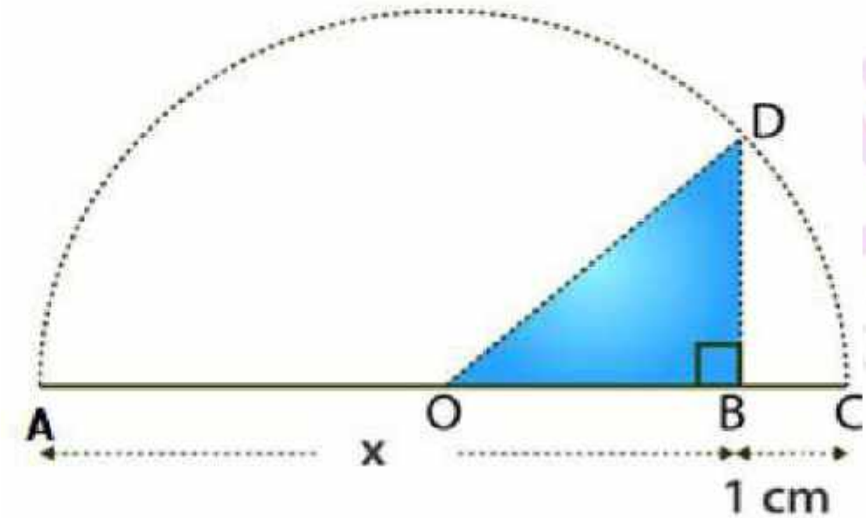
(i) $x^2 = 16$

$x = \sqrt{16} = 4$, which is rational.

(ii) $x^2 = 0.0004$
$x = \sqrt{0.0004} = 0.02$, which is rational.

(iii)
$x^2 = 1\frac{7}{9}$
$x = \sqrt{1\frac{7}{9}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$, which is rational.

13. Using the following figure, show that BD = √x.



Solution:

Let's assume AB = x, BC = 1 and AC = x + 1
Here, AC is diameter and O is the centre
OA = OC = OD = radius = (x + 1)/2
And,
OB = OC – BC
= (x + 1)/2 – 1
= (x + 1 - 2)/2
= (x - 1)/2
Now, using Pythagoras theorem, we have
$OD^2 = OB^2 + BD^2$

$$\left(\frac{x+1}{2}\right)^2 = \left(\frac{x-1}{2}\right)^2 + BD^2$$

$$\Rightarrow BD^2 = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2$$

$$= \frac{x^2 + 2x + 1 - x^2 + 2x - 1}{4}$$

= 4x/x

$= x$

$\therefore BD = \sqrt{x}$

- Hence, proved.