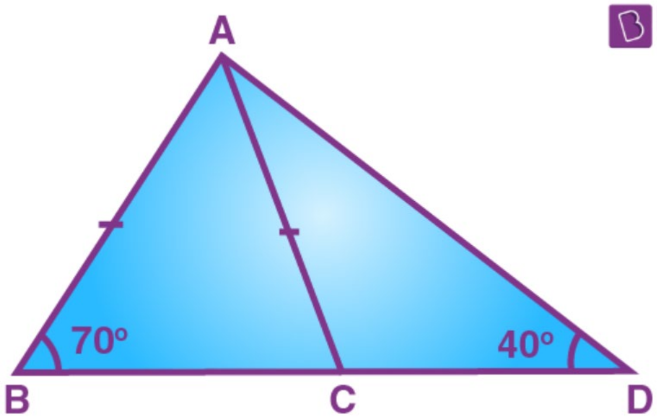
EXERCISE 11

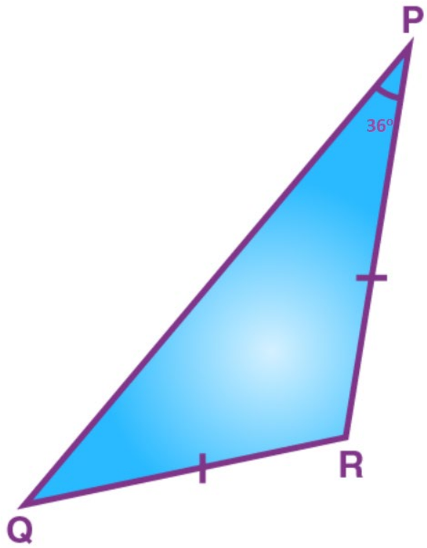
1. From the following figure, prove that: AB > CD.



Solution:

In Δ ABC,
AB = AC[Given]
Therefore, ∠ACB = ∠B [angles opposite to equal sides are equal]
∠B = 70°[Given]
∠ACB = 70° .... (i)
Now,
∠ACB +∠ACD = 180°[BCD is a straight line]
70° + ∠ACD = 180°
∠ACD = 110° ....(ii)
In Δ ACD,
∠CAD + ∠ACD + ∠D = 180°
∠CAD + 110° + ∠D = 180° [From (ii)]
∠CAD + ∠ D = 70°
But ∠D = 40° [Given]
∠CAD + 40°= 70°
∠CAD = 30° ....(iii)
In Δ ACD,
∠ACD = 110° [From (ii)]
∠CAD = 30° [From (iii)]
∠ D = 40° [Given]
∠D > ∠CAD
AC > CD [Greater angle has greater side opposite to it]
Also,
AB = AC[Given]
Therefore, AB > CD.

2. In a triangle PQR; QR = PR and ∠P = 36°. Which is the largest side of the triangle?



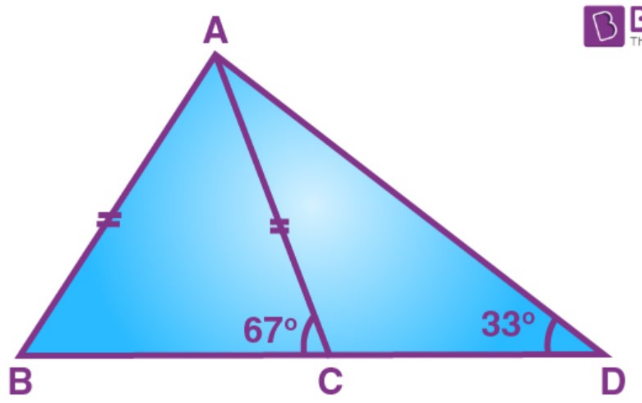
Solution:
In ΔPQR,
QR = PR[Given]
Therefore ∠P = ∠Q [angles opposite to equal sides are equal]
∠P = 36°[Given]
∠Q = 36°
In Δ PQR,
∠P + ∠Q + ∠R = 180°
36° + 36° + ∠R = 180°
∠R + 72° = 180°
∠R = 108°
Now,
∠R = 108°
∠P = 36°
∠Q = 36°
Since ∠R is the greatest, therefore, PQ is the largest side.

3. If two sides of a triangle are 8 cm and 13 cm, then the length of the third side is between a cm and b cm. Find the values of a and b such that a is less than b.

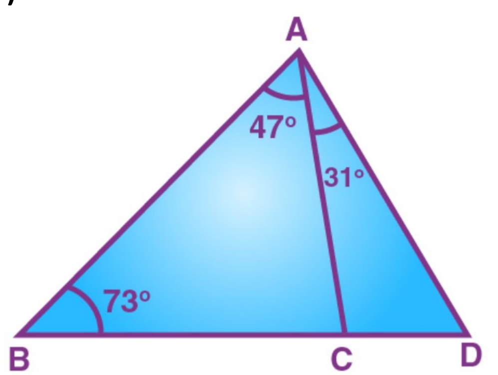
Solution:
The sum of any two sides of the triangle is always greater than third side of the triangle.
Third side < 13 + 8 = 21 cm.
The difference between any two sides of the triangle is always less than the third side of the triangle.
Third side > 13 - 8 = 5 cm.
Therefore, the length of the third side is between 5 cm and 9 cm, respectively.
The value of a = 5 cm and b = 21 cm.

4. In each of the following figures, write BC, AC and CD in ascending order of their lengths.

(i)



(ii)



Solution:

In Δ ABC,

AB = AC

∠ABC = ∠ACB [angles opposite to equal sides are equal]

∠ABC = ∠ACB = 67°

∠BAC = 180° - ∠ABC - ∠ACB (angle sum property of triangle)

∠BAC = 180° – 67° – 67° = 46°

Since ∠BAC < ∠ABC we have

BC < AC ……….. (1)

Now,

∠ACD = 180° - ∠ACB (linear pair)

∠ACD = 180° – 67° = 113°

In Δ ACD,

∠CAD = 180° - ∠ACD - ∠ADC

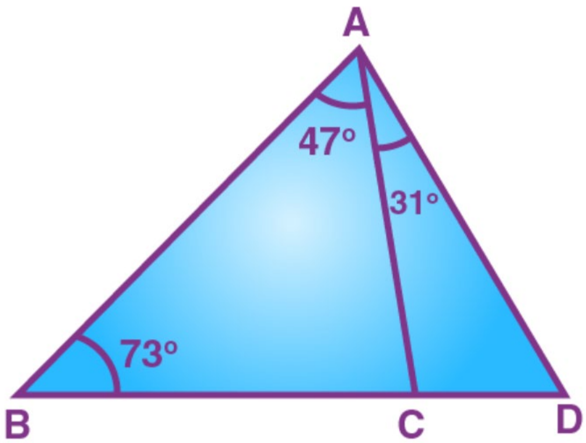
∠CAD = 180° – 113° – 33° = 34°

Since ∠ADC < ∠CAD we have

AC < CD … (2)

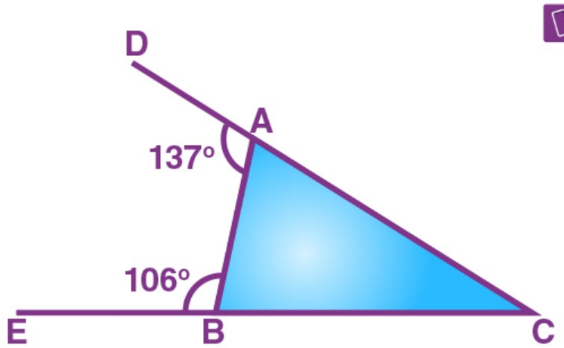
From (1) and (2) we have

BC < AC < CD

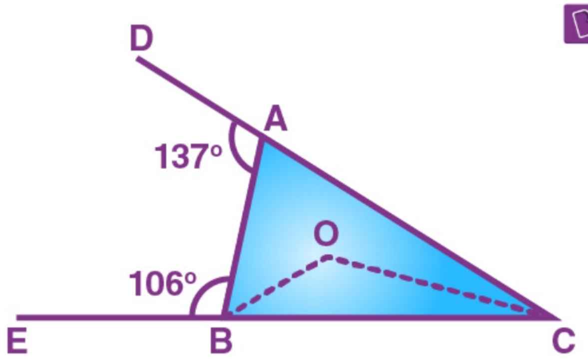


In Δ ABC,
∠BAC = ∠ABC
BC < AC ……… (1)
∠ACB = 180° - ∠ABC - ∠BAC
∠ACB = 180° – 73° – 47° = 60°
Now,
∠ACD = 180° - ∠ACB
∠ACD = 180° – 60° = 120°
In Δ ACD,
∠ADC = 180° - ∠ACD - ∠CAD
∠ADC = 180° – 120° – 31° = 29°
Since ∠ADC < ∠CAD we have
AC < CD … (2)
From (1) and (2) we have
BC < AC < CD

5. Arrange the sides of ΔBOC in descending order of their lengths. BO and CO are bisectors of angles ABC and ACB respectively.

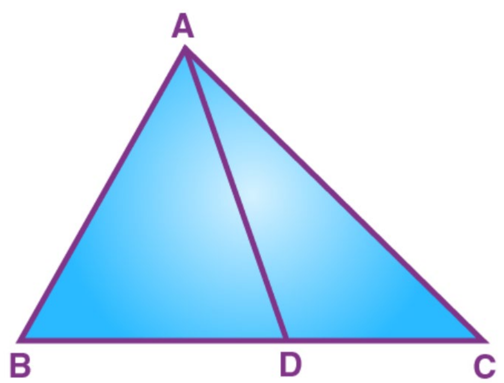


Solution:



In triangle BEC,
∠B + ∠BEC + ∠BCE = 180^0
∠B = 65^0 [Given]
∠BEC = 90^0[CE is perpendicular to AB]
65^0 + 90^0 + ∠BCE = 180^0
∠BCE = 180^0 - 155^0
∠BCE = 25^0 = ∠DCF …………(i)
In triangle CDF,
∠DCF + ∠FDC + ∠CFD = 180^0
∠ DCF = 25^0 [From (i)]
∠ FDC = 90^0[AD is perpendicular to BC]
25^0 + 90^0 + ∠CFD = 180^0
∠CFD = 180^0 - 115^0
∠CFD = 65^0

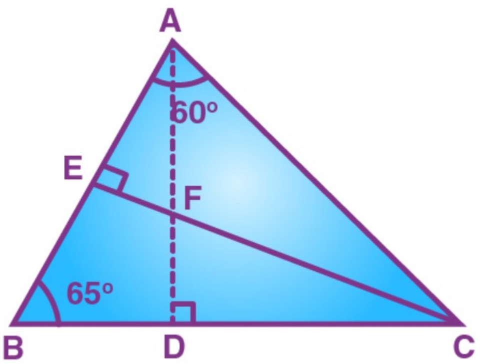
6. D is a point in side BC of triangle ABC. If AD > AC, show that AB > AC.



Solution:
AD > AC (given)
∠C > ∠ADC …… (1)
∠ADC > ∠B + ∠BAC (exterior angle property)
∠ADC > ∠B ……… (2)
From (1) and (2) we have

∠C > ∠ADC > ∠B
∠C > ∠B
AB > AC

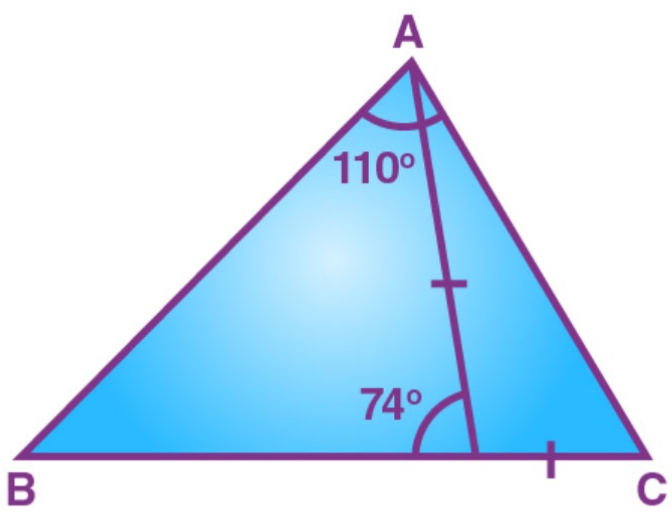
7. In the following figure, ∠BAC = 60° and ∠ABC = 65°.
Prove that:
(i) CF > AF
(ii) DC > DF



Solution:
In ΔBEC,
∠B + ∠BEC + ∠BCE = 180^0
∠B = 65^0 [Given]
∠BEC = 90^0[CE is perpendicular to AB]
65^0 + 90^0 + ∠BCE = 180^0
∠BCE = 180^0 - 155^0
∠BCE = 25^0 = ∠DCF …………(i)
In ΔCDF,
∠DCF + ∠FDC + ∠CFD = 180^0
∠DCF = 25^0 [From (i)]
∠FDC = 90^0[AD is perpendicular to BC]
25^0 + 90^0 + ∠CFD = 180^0
∠CFD = 180^0 - 115^0
∠CFD = 65^0 …………(ii)
Now, ∠ AFC + ∠CFD = 180^0[AFD is a straight line]
∠AFC + 65^0 = 180^0
∠AFC = 115^0 ………(iii)
In ΔACE,
∠ACE + ∠CEA + ∠BAC = 180^0
∠BAC = 60^0 [Given]
∠CEA = 90^0[CE is perpendicular to AB]
∠ACE + 90^0 + 60^0 = 180^0

$\angle ACE = 180^0 - 150^0$
$\angle ACE = 30^0$ …………(iv)
In ΔAFC,
$\angle AFC + \angle ACF + \angle FAC = 180^0$
$\angle AFC = 115^0$ [From (iii)]
$\angle ACF = 30^0$[From (iv)]
$115^0 + 30^0 + \angle FAC = 180^0$
$\angle FAC = 180^0 - 145^0$
$\angle FAC = 35^0$ …………(v)
In ΔAFC,
$\angle FAC = 35^0$[From (v)]
$\angle ACF = 30^0$[From (iv)]
FAC > ACF
CF > AF
In ΔCDF,
$\angle DCF = 25^0$[From (i)]
$\angle CFD = 65^0$[From (ii)]
CFD > DCF
DC > DF

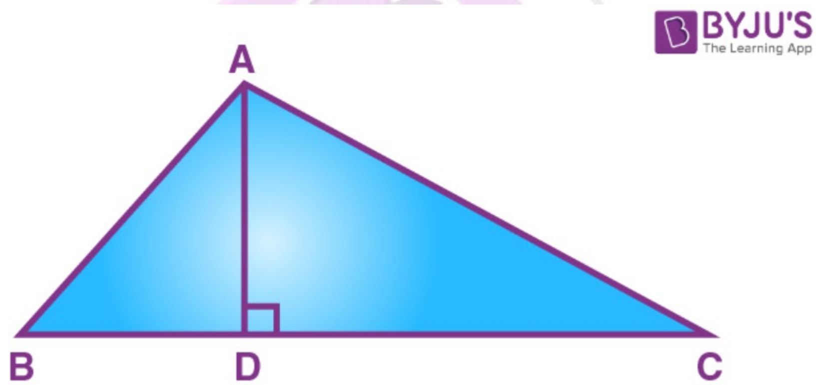
8. In the following figure; AC = CD; ∠BAD = 110° and ∠ACB = 74°.
Prove that: BC > CD.



Solution:
$\angle ACB = 74^0$ …..(i)[Given]
$\angle ACB + \angle ACD = 180^0$[BCD is a straight line]
$74^0 + \angle ACD = 180^0$
$\angle ACD = 106^0$ ……..(ii)
In ΔACD,
$\angle ACD + \angle ADC + \angle CAD = 180^0$

Given that AC = CD
∠ADC= ∠CAD
106^0 + ∠CAD + ∠CAD = 180^0[From (ii)]
2∠CAD = 74^0
∠CAD = 37^0 = ∠ADC………..(iii)
Now,
∠BAD = 110^0[Given]
∠BAC + ∠CAD = 110^0
∠BAC + 37^0 = 110^0
∠BAC = 73^0 ……..(iv)
In ΔABC,
∠B + ∠BAC+ ∠ACB = 180^0
∠B + 73^0 + 74^0 = 180^0[From (i) and (iv)]
∠B + 147^0 = 180^0
∠B = 33^0 ………..(v)
BAC > B
BC > AC
But
AC = CD
BC > CD

9. From the following figure; prove that:
(i) AB > BD
(ii) AC > CD
(iii) AB + AC > BC

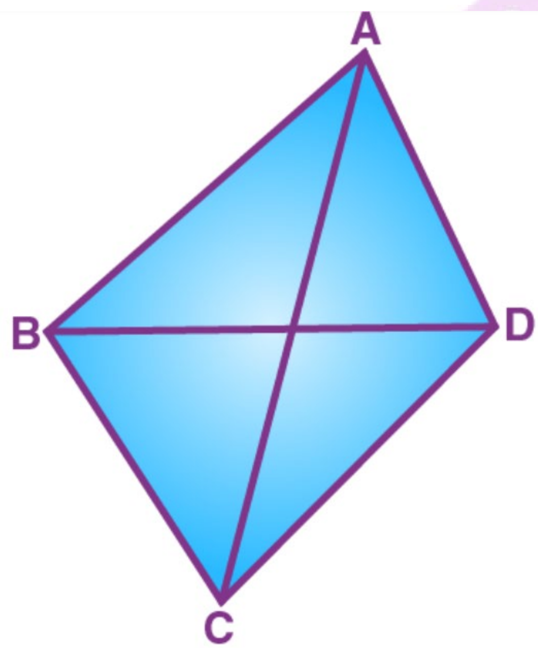


Solution:
(i) ∠ADC + ∠ADB = 180^0[BDC is a straight line]
∠ADC = 90^0[Given]
90^0 + ∠ADB = 180^0
∠ADB = 90^0 …………(i)
In ΔADB,

∠ADB = 90^0[From (i)]
∠B + ∠BAD = 90^0
Therefore, ∠B and ∠BAD are both acute, that is less than 90^0.
AB > BD ……. (ii)[Side opposite 90^0 angle is greater than side opposite acute angle]

(ii) In ΔADC,
ADB = 90^0
∠C + ∠DAC = 90^0
Therefore, ∠C and ∠DAC are both acute, that is less than 90^0.
AC > CD ……..(iii)[Side opposite 90^0 angle is greater than side opposite acute angle]
Adding (ii) and (iii)
AB + AC > BD + CD
AB + AC > BC

10. In a quadrilateral ABCD; prove that:
(i) AB+ BC + CD > DA
(ii) AB + BC + CD + DA > 2AC
(iii) AB + BC + CD + DA > 2BD



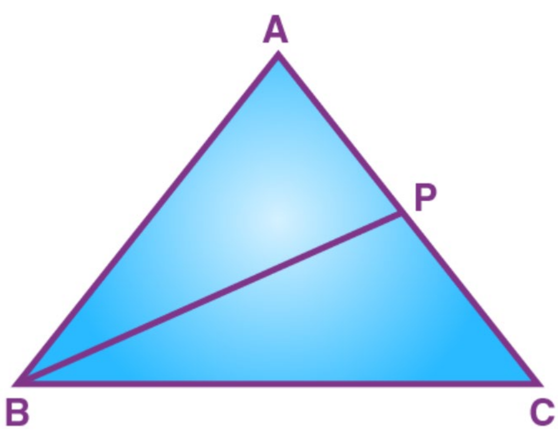
Solution:
Construction: Join AC and BD.
(i) In ΔABC,
AB + BC > AC …. (i)[Sum of two sides is greater than the third side]

In ΔACD,
AC + CD > DA …. (ii)[Sum of two sides is greater than the third side]
Adding (i) and (ii)
AB + BC + AC + CD > AC + DA
AB + BC + CD > AC + DA - AC
AB + BC + CD > DA ……. (iii)

(ii)In ΔACD,
CD + DA > AC …. (iv)[Sum of two sides is greater than the third side]
Adding (i) and (iv)
AB + BC + CD + DA > AC + AC
AB + BC + CD + DA > 2AC

(iii) In ΔABD,
AB + DA > BD …. (v)[Sum of two sides is greater than the third side]
In ΔBCD,
BC + CD > BD …. (vi)[Sum of two sides is greater than the third side]
Adding (v) and (vi)
AB + DA + BC + CD > BD + BD
AB + DA + BC + CD > 2BD

11. In the following figure, ABC is an equilateral triangle and P is any point in AC; prove that:
(i) BP > PA
(ii) BP > PC

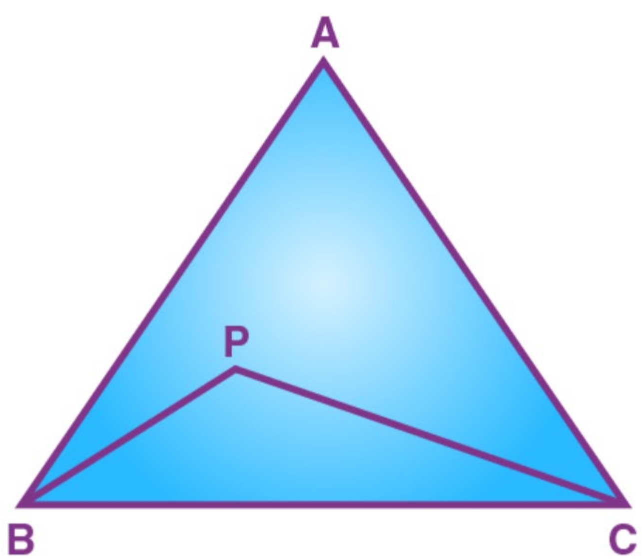


Solution:

(i) In ΔABC,
AB = BC = CA [ABC is an equilateral triangle]
∠A = ∠B = ∠C
∠A = ∠B = ∠C = $180^o/3$
∠A = ∠B = ∠C = 60^o
In ΔABP,
∠A = 60^0
∠ABP< 60^0
∠A > ∠ABP
BP > PA
[Side opposite to greater side is greater]

(ii) In ΔBPC,
∠C = 60^0
∠CBP< 60^0
∠C > ∠CBP
BP > PC
[Side opposite to greater side is greater]

12. P is any point inside the triangle ABC. Prove that:
∠BPC > ∠BAC.
Solution:

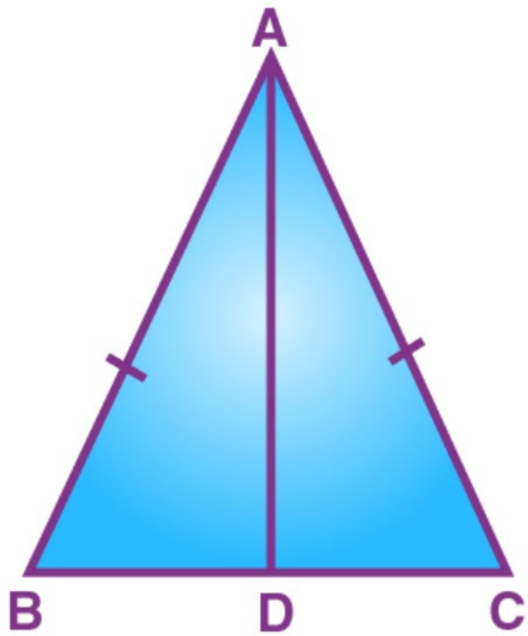


Let ∠PBC = x and ∠PCB = y
then,
∠BPC = 180^0 - (x + y) ....(i)
Let ∠ABP = a and ∠ACP = b
then,

∠BAC = 180^0 - (x + a) - (y + b)
∠BAC = 180^0 - (x + y) - (a + b)
∠BAC =∠BPC - (a + b)
∠BPC = ∠BAC + (a + b)
∠BPC > ∠BAC

13. Prove that the straight line joining the vertex of an isosceles triangle to any point in the base is smaller than either of the equal sides of the triangle.

Solution:

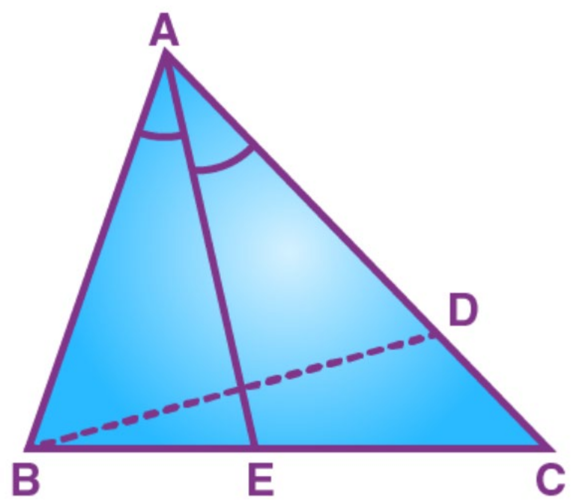


We know that exterior angle of a triangle is always greater than each of the interior opposite angles.
In ΔABD,
∠ADC > ∠B ……..(i)
In ΔABC,
AB = AC
∠B = ∠C ….. (ii)
From (i) and (ii)
∠ADC > ∠C

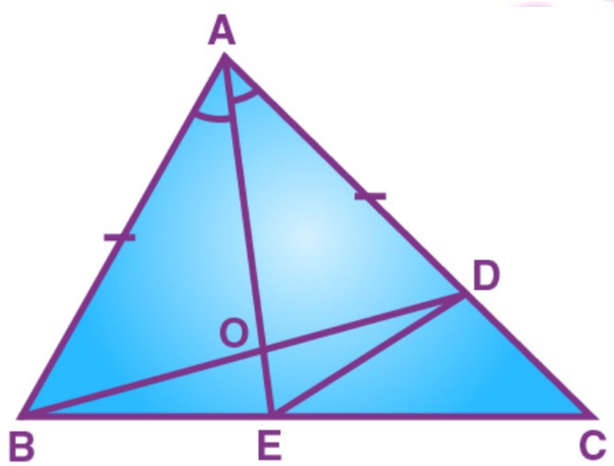
(i) In ΔADC,
∠ADC > ∠C
AC > AD ………(iii) [side opposite to greater angle is greater]

(ii) In ΔABC,
AB = AC
⇒AB > AD [From (iii)]

14. In the following diagram; AD = AB and AE bisect angle A. Prove that:
(i) BE = DE
(ii) ∠ABD > ∠C



Solution:



Construction: Join ED.
In ΔAOB and ΔAOD,
AB = AD [Given]
AO = AO [Common]
∠BAO = ∠ DAO [AO is bisector of ∠A]
Therefore ΔAOB ≅ ΔAOD [SAS criterion]
Hence,
BO = OD………(i) [c.p.c.t]
∠AOB = ∠AOD …….(ii) [c.p.c.t]
∠ABO = ∠ADO
∠ABD = ∠ADB ………(iii) [c.p.c.t]

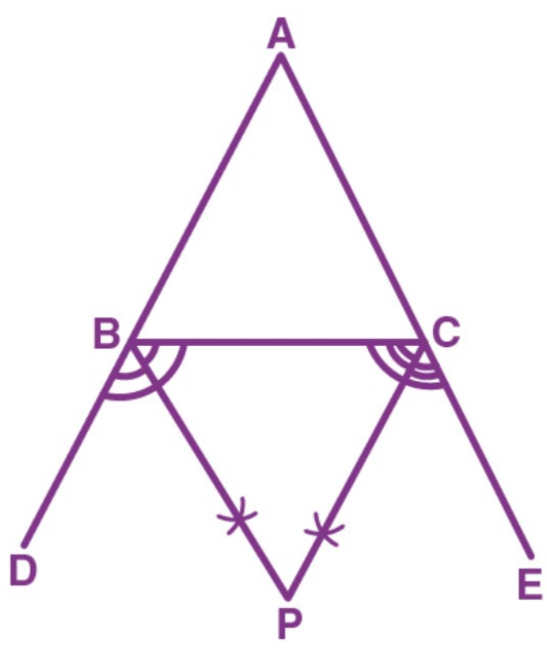
Now,
∠AOB = ∠DOE [Vertically opposite angles]
∠AOD = ∠BOE [Vertically opposite angles]
∠BOE = ∠DOE ……(iv) [From (ii)]

(i) In ΔBOE and ΔDOE,
BO = CD [From (i)]
OE = OE [Common]
∠BOE = ∠DOE [From (iv)]
Therefore ΔBOE ≅ ΔDOE [SAS criterion]
Hence, BE = DE [c.p.c.t]

(ii) In ΔBCD,
∠ADB = ∠C + ∠CBD [Ext. angle = sum of opp. int. angles]
∠ADB > ∠C
∠ABD > ∠C [From (iii)]

15. The sides AB and AC of a triangle ABC are produced; and the bisectors of the external angles at B and C meet at P. Prove that if AB > AC, then PC > PB.

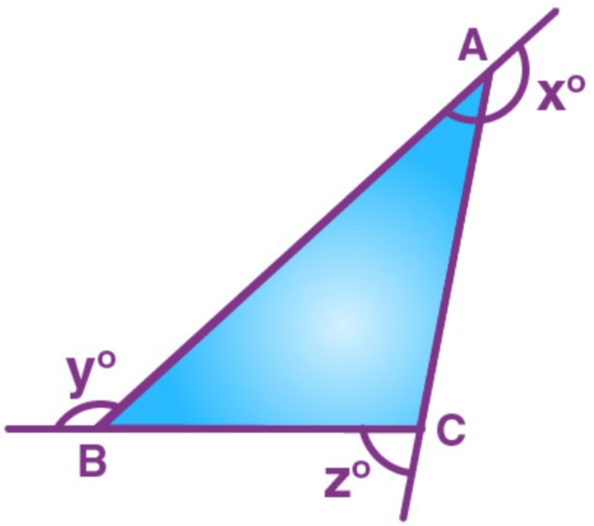
Solution:



In ΔABC,
AB > AC,
∠ABC < ∠ACB

$180^0 - \angle ABC > 180^0 - \angle ACB$
$(180^0 - \angle ABC)/2 > (180^0 - \angle ABC)/2$
$90^0 – ½ \angle ABC > 90^0 – ½ \angle ACB$
$\angle CBP > \angle BCP$ (BP is bisector of $\angle CBD$ and CP is bisector of $\angle BCE$)
PC > PB (side opposite to greater angle is greater)

16. In the following figure; AB is the largest side and BC is the smallest side of triangle ABC.

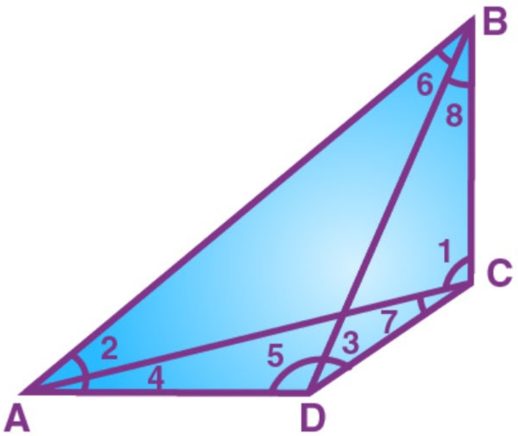


Write the angles x°, y° and z° in ascending order of their values.

Solution:
Since AB is the largest side and BC is the smallest side of the triangle ABC
AB > AC > BC
$180^0 – Z^0 > 180^0 – y^0$
$-Z^0 > -y^0 > -x^0$
$Z^0 < y^0 < x^0$

17. In quadrilateral ABCD, side AB is the longest and side DC is the shortest.
Prove that:
(i) $\angle C > \angle A$
(ii) $\angle D > \angle B$.

Solution:

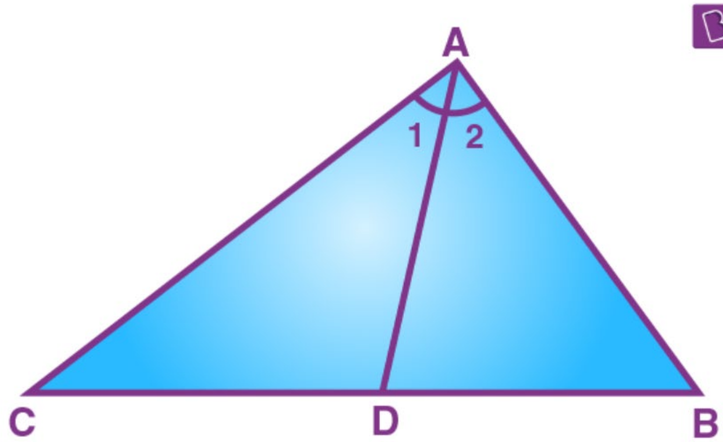


In the quadrilateral ABCD,
Since AB is the longest side and DC is the shortest side.
(i) ∠1 > ∠2 [AB > BC]
∠7 > ∠4 [AD > DC]
∠1 + ∠7 > ∠2 + ∠4
∠C > ∠A

(ii) ∠5 > ∠6[AB > AD]
∠3 > ∠8[BC > CD]
∠5 + ∠3 > ∠6 + ∠8
∠D > ∠B

18. In triangle ABC, side AC is greater than side AB. If the internal bisector of angle A meets the opposite side at point D, prove that: ∠ADC is greater than ∠ADB.

Solution:

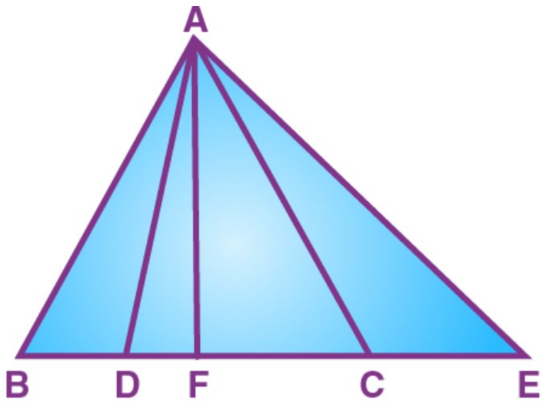


In ΔADC,
∠ADB = ∠1 + ∠C............(i)
In ΔADB,
∠ADC = ∠2 + ∠B................(ii)

But AC > AB [Given]
∠B > ∠C
Also given, ∠2 = ∠1 [AD is bisector of ∠A]
∠2 + ∠B > ∠1 + ∠C ……. (iii)
From (i), (ii) and (iii)
∠ADC > ∠ADB

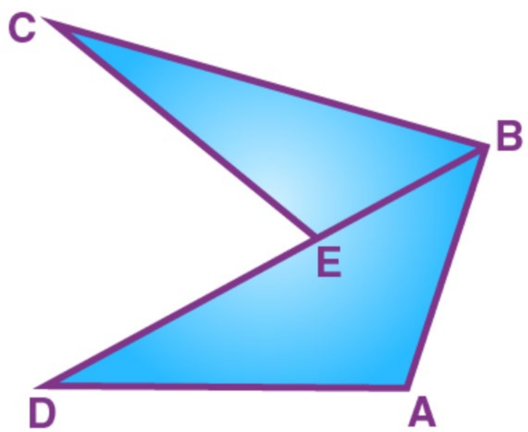
19. In isosceles triangle ABC, sides AB and AC are equal. If point D lies in base BC and point E lies on BC produced (BC being produced through vertex C), prove that:
(i) AC > AD
(ii) AE > AC
(iii) AE > AD

Solution:



We know that the bisector of the angle at the vertex of an isosceles triangle bisects the base at right angle.
Using Pythagoras theorem in Δ AFB,
$AB^2 = AF^2 + BF^2$…………. (i)
In Δ AFD,
$AD^2 = AF^2 + DF^2$…………. (ii)
We know ABC is isosceles triangle and AB = AC
$AC^2 = AF^2 + BF^2$ …….. (iii)b[From (i)]
Subtracting (ii) from (iii)
$AC^2 - AD^2 = AF^2 + BF^2 - AF^2 - DF^2$
$AC^2 - AD^2 = BF^2 - DF^2$
Let 2DF = BF
$AC^2 - AD^2 = (2DF)^2 - DF^2$
$AC^2 - AD^2 = 4DF^2 - DF^2$
$AC^2 = AD^2 + 3DF^2$
$AC^2 > AD^2$
AC > AD
Similarly, AE > AC and AE > AD.

20. Given: ED = EC
Prove: AB + AD > BC.

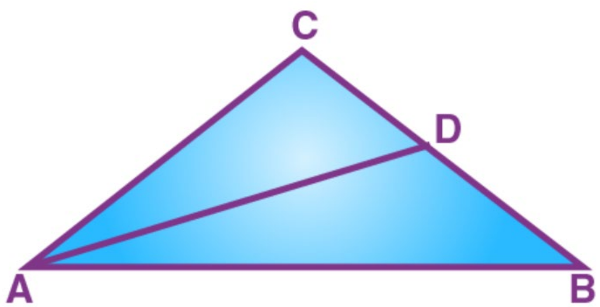


Solution:
In ΔCEB
CE + EB > BC
DE + EB > BC (CE = DE)
DB > BC ……… (1)

In ΔADB,
Ad + AB > BD
AD + AB > BD > BC (from 1)
AD + AB > BC

21. In triangle ABC, AB > AC and D is a point in side BC. Show that: AB > AD

Solution:



Given that AB > AC
∠C > ∠B ………. (1)

In ΔADC

$\angle ADB = \angle DAC + \angle C$ (exterior angle)
$\angle ADB > \angle C$
$\angle ADB > \angle C > \angle B$ (from 1)
$\angle ADB > \angle B$
$AB > AD$